**Time-Dependent Dial-a-Ride Problems: Formulation Development and Numerical Experiments**

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**Abstract:** In order to satisfy the transportation demand for elderly and disabled people, providing efficient dial-a-ride systems is an important issue. Appropriate vehicle routes for dial-a-ride systems can increase transportation efficiency and reduce the operation cost. Most literatures focus on improving the vehicle routing and scheduling and assume that the travel time is a constant value. However, the travel time can fluctuate dramatically due to traffic congestion or incident. Using the constant travel time to solve dial-a-ride problems (DARP) cannot obtain the efficient vehicle routes. To consider the variation of the travel time, this paper formulates a mixed integer programming formulation for the time-dependent DARP with time window. In the formulation, the travel time variations are treated as step functions. The simulation-assignment model, DynaTAIWAN is applied to generate time-dependent travel time matrices and simulate traffic flows in the real network. Numerical experiments are conducted in a Kaohsiung network.

**Key Words:** Dial-a-Ride Problem, Time-dependent, DynaTAIWAN

**1. INTRODUCTION**

In order to meet the transportation needs for elderly and disabled people, providing efficient dial-a-ride systems is an important issue. Appropriate vehicle routes for dial-a-ride system can increase transportation efficiency and reduce the operation cost. The dial-a-ride problem (DARP) is defined as follows: A fleet of vehicles with fixed capacities serve customers with specific pickup and delivery requests. Each vehicle has not to violate the limitation of the time window for each customer. Based on the requests of the customers, the dispatcher assigns vehicles to serve all customers and designs vehicle routes according to the minimum travel costs. Most literatures focus on improving the vehicle routing and scheduling and assume that the travel time is a constant value to design the vehicle routes. However, the travel time can fluctuate dramatically due to traffic congestion or incident. Using the constant travel time to solve DARP cannot obtain the efficient vehicle routes. To consider the variation of the travel time, this paper treats the travel time as step functions and formulates a mixed integer programming formulation for the time-dependent DARP with time window.

Srobvel (1982) has identified three different operating modes for DARP, including one-to-many operation mode, many-to-one operation mode and many-to-many operation mode. In this paper, the many-to-one operation mode including many origins and one destination, such as home-to-hospital trip, is concerned in constructing the mathematical formulation.
This research aims at constructing the time-dependent DARP formulation with time window to explicitly consider variation of link travel times in a traffic network. In the formulation, the travel time variations are treated as step functions. In the numerical experiments, the many-to-one operation modes for DARP are designed and solved to illustrate the flexibility of the proposed formulation. The simulation-assignment model, DynaTAIWAN (Hu et al., 2007) is applied to generate time-dependent travel time matrices and simulate traffic flows in the real network. Numerical experiments are conducted in a Kaohsiung network.

This paper is organized as follows: Section 2 presents a literature review on DARP. The time-dependent DARP is formulated and discussed in Section 3. Section 4 discussed the solution framework. In Section 5, the numerical experiments are conducted in a Kaohsiung network. The last section discusses our conclusions and comments.

2. LITERATURE REVIEW

Based on the classification of DARP by Psarafits(1980), DARP includes two different types of problems, static problems and dynamic problems. For the static case, all requests are known in advance and, for the dynamic case, partial requests reveal in real time. Cordeau and Laporte (2007) reviewed literatures pertaining to static and dynamic DARP and enumerated the practical applications for DARP such as, courier service and door-to-door transportation services for elderly and disabled people.

For the static DARP, different approaches have been proposed. Psarafits (1980) used an exact approach which is developed by dynamic programming to solve DARP. The objective function in dynamic program is the minimization of the weighted sum of route completion time and customer dissatisfaction to solve the multi-vehicle static DARP with time window. The algorithm which introduced by Psarafits was only able to handle small sizes of instances (n=9). Xiang et al. (2006) proposed a heuristic-based algorithm to solve a large-scale static DARP. In this algorithm, insertions and inter-route exchanges are applied to construct the routes. Wong and Bell (2006) developed a parallel insertion procedure which considers time window restriction and solves under the minimization of a linear combination of total operating time, passenger ride time and taxi cost for unassigned requests. Cordeau (2006) and Roke et al. (2007) formulated the problem as an integer linear programming and propose an exact branch-and-cut algorithm for DARP. Jørgensen et al. (2007) explored the DARP under a multi-criteria objective and developed genetic algorithm for solving DARP. The algorithm consisted of assigning problem and route improvement. Using genetic algorithm dealt with assigning problem. Route improvement was solved by a nearest neighbor procedure. The heuristics which is developed by Jørgensen et al. has solved bigger sizes of instances (n=144).
In order to consider possible real-time requests, Psarafits (1980) applied dynamic programming to develop the algorithm for the dynamic single-vehicle DARP. Madsen et al. (1995) introduced an insertion algorithm which takes into account new requests to tackle the multi-vehicle dynamic DARP. Coslovich et al. (2006) used a two-phase strategy for the insertion of a new request. An off-line phase was used to generate an initial feasible solution and an on-line phase was used to insert a new request.

To consider travel time variations, Malandraki and Daskin (1992) introduced a new concept of step functions to deal with time-dependent travel time and developed a heuristic approach to solve Vehicle Routing Problems (VRP). Fu (2002) considered a time-dependent, stochastic travel time model to construct the algorithm for solving DARP. Haghani and Jung (2005) extended the Malandraki and Daskin’s work and treated the travel time variation as a continuous function. They constructed a formulation for the dynamic VRP with time-dependent travel time. Chen et al. (2006) pointed out that the traffic congestion can seriously affect the vehicle routing and scheduling and developed heuristic which consists of routes construction and routes improvement to solve the vehicle routing problems with real-time and time-dependent travel times.

In the research on the DARP, real-time requests and the variation of the travel time are two critical factors to modify vehicle routes and scheduling under the minimum travel cost. The familiar approach to deal with DARP is either developing heuristic-based algorithm or formulating mathematical model. Seldom literatures consider the variation of the travel time to solve for DARP. This paper aims to consider the variation of the travel time to formulate mathematical model and conduct experiments in the Kaohsiung network.

3. PROBLEM STATEMENT AND FORMULATION

The DARP is defined as follow: a fleet of vehicles satisfy transportation service for all customers under the minimum total travel time. Each vehicle has capacity limitation and must depart from the depot and return to the depot during the vehicle routing period. Each customer has expected service time, specific starting node and specific destination. In the dial-a-ride system, customer might reserve transportation service at the vehicle in advance. In Taiwan, the most common example for DARP arises in the medical transportation service for the elderly or disabled people, such as medical service bus in Taipei.

In the mathematical formulation, the travel time functions extend the concept of the step functions proposed by Malandraki and Daskin (1992) to formulate a mixed integer programming formulation for the time-dependent DARP with time window. In order to explicitly establish the relationship between the link and the vehicle, the mathematical formulation treats the link and the vehicle as the decision variables.

The network in the mathematical formulation is assumed to be a complete graph and travel costs between nodes i and j at time interval m are assumed to be \( c_{ij}^m \). Travel time step function for link (i, j) is shown in Figure 2.
The time dependent DARP with time window can be formulated as the following mixed integer program:

Minimize:

\[
\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{m=1}^{M} c_{ij}^m x_{ij}^m
\]

Subject to

\[
\sum_{k=1}^{K} v_i^k = 1, \quad i = 2, \ldots, n
\]

\[
\sum_{m=1}^{M} x_{ij}^m = 1, \quad j = 2, \ldots, n
\]

\[
\sum_{m=1}^{M} x_{ji}^m = 1, \quad i = 2, \ldots, n
\]

\[
\sum_{m=1}^{M} x_{ij}^m = K
\]

\[
\sum_{m=1}^{M} x_{ji}^m = K
\]

\[
\sum_{i=2}^{n} v_i^k d_i \leq b_k, \quad k = 1, \ldots, K
\]

\[
t_i = t
\]

\[
t_j - t_i - B x_{ij}^m \geq c_{ij}^m - B
\]

\[
t_j - t_i + B x_{ij}^m \leq c_{ij}^m + B
\]

\[
t_i - T_{ij}^{m-1} x_{ij}^m \geq 0, \quad i = 1, \ldots, n, \quad j = 2, \ldots, n, \quad i \neq j, \quad m = 1, \ldots, M
\]

\[
t_i + B x_{ij}^m \leq T_{ij}^m + B, \quad i = 1, \ldots, n, \quad j = 2, \ldots, n, \quad i \neq j, \quad m = 1, \ldots, M
\]

\[
v_i^k - v_j^k - B \sum_{m=1}^{M} x_{ij}^m \geq -B, \quad i = 2, \ldots, n, \quad j = 2, \ldots, n, \quad i \neq j, \quad m = 1, \ldots, M
\]
\[ v_i^j - v_j^j + B \sum_{m=1}^{M} m_{ij}^m \leq B, \quad i = 2...n, \quad j = 2...n, \quad i \neq j, \quad m = 1...M \] \quad (14)

\[ \sum_{k=1}^{K} v_i^k = 1, \quad i = 2,...,n \] \quad (15)

\[ L_i \leq t_i \leq U_i, \quad i = 2...n \] \quad (16)

\[ m_{ij}^m = \{0,1\}; \quad \forall \ i, \ j, \ m \] \quad (17)

\[ v_i^k = \{0,1\}; \quad \forall \ i, \ k \] \quad (18)

\[ t_i \geq 0 \] \quad (19)

The notation is defined as follows:

**Constants:**
- \( n \) = number of nodes (\( n = 1 \): depot; \( n=2...N \): demand),
- \( k \) = number of vehicles,
- \( m \) = number of time interval,
- \( m_{ij}^m \) = travel time from node \( i \) to node \( j \) at the time interval \( m \),
- \( d_i \) = volume of demand \( i \),
- \( b_k \) = capacity of vehicle \( k \),
- \( T_{ij}^m \) = upper bound for time interval \( m \) for link(\( i, j \)),
- \( t \) = the starting time of the depot node.
- \( B \) = a large number,
- \( L_i \) = earliest time that the vehicle can arrive at node \( i \),
- \( U_i \) = latest time that the vehicle can arrive at node \( i \),

**Decision variables:**
- \( m_{ij}^m \) \( \{1 \text{ : if any vehicle travel form node } i \text{ to node } j \text{ during time interval } m \} \),
- \( v_i^k \) \( \{1 \text{ : if node } i \text{ served by vehicle } k \} \),
- \( t_i \) = departure time of any vehicle from node \( i \).

Several assumptions are made: (1) the number of the vehicle is fixed; (2) the departure times for all vehicles are assumed to be the same. The detailed descriptions for objective function and constraints are described as follows: The equation (1) is the objective function which minimizes total travel times of all vehicles. Constraints (2) ensure that each demand must be visited precisely once and each demand is only allowed to be served by one vehicle. Constraints (3) to (6) are the flow conservation equations. Constraints (3) and (4) ensure that each of demands can be visited exactly once. Constraints (5) and (6) set all vehicles starting at the depot and returning at the depot. Constraints (7) impose the capacity restrictions. Constraint (8) set the starting time from the depot. Based on the setting in the constraint (8), the starting time from the depot for all vehicles is the same. Constraints (9) and (10) calculate the departure time to node \( j \). Constraints (11) and (12) are the temporal constraints. If the
vehicle travels from demand i to demand j during time interval m, the departure time of the vehicle from node i is between upper bound for time interval m-1 and upper bound for time interval m. Constraints (13) and (14) define the relation between \( x^{m}_{ij} \) and \( v^k_{ij} \). Constraints (15) ensure that each demand must be served by one vehicle. In the Constraints (13) and (14), \( v^k_{ij} \) are equal to \( v^k_{ij} \) when the vehicle k move from demand i to demand j during time interval m. Due to the restrictions of the constraints (7) and constraints (15), \( v^k_{ij} \) and \( v^k_{ij} \) will be forced equal to 1. Constraints (16) impose the time windows restrictions. Combining the constraints (8) to (10) are the subtour elimination constraints (Parragh et al., 2008).

4. SOLUTION FRAMEWORK

The solution framework is depicted in Figure 3. The main steps are described as follows:

Step 1: Input data
In this step, the network geometric data and origin-destination demand data are the necessary input. The simulation-assignment model, DynaTAIWAN, is applied to simulate traffic flows in the real network. Ideally, the traffic flow data can be collected from the surveillance and monitoring systems. Then, the time-dependent travel time matrices in the network are generated.

Step 2: Construct and Solve Mathematical Formulation
According to the time-dependent travel time matrices, vehicle data and demand data, the mathematical programming is formulated. The mathematical formulation is solved by CPLEX. After the solving process in CPLEX, the vehicles routes and service sequences are obtained.

Step 3: Numerical Experiments
Based on the results, experiments are conducted under the different scenarios.
5. NUMERICAL EXPERIMENTS

The numerical experiments in a Kaohsiung network are conducted. As shown in Figure 4, the network consists of a freeway and urban streets and the network includes 132 nodes and 363 links.

The basic setups of experiments are summarized as follows: the maximum simulation time is 600 minutes; the warm-up time is 20 minutes; the instance in the numerical experiment incorporates 3 vehicles, 12 demand nodes and 1 depot. Each demand node has expected service time.
Two experimental factors, including time window and OD demand, are considered.

1. Time Window
The time window setup is shown in Figure 5. In the numerical experiments, an expected service time (x) for each demand is generated randomly. According to expected service time, the length of the time window equals to the difference between the earliest time (x-t) and the latest time (x+t). Four time-window constraints, including 10, 15, 20 and infinity, are considered.

![Figure 5 Time Window setup](image)

2. OD demand:
Two different OD demand scenarios are considered: 1.0 and 1.5 termed as scenarios I and II. In the scenario I, there are 75,311 vehicles in the network; the average travel time is 20.04 minutes and the average stopped time is 12.31 minutes. In the scenario II, there are 108,800 vehicles; the average travel time is 45.72 minutes and the average stopped time is 28.44 minutes. The scenario II is more congested than the scenario I.

Average schedule delay is used to evaluate the customer satisfaction. If the average schedule delay is equal to zero, the dial-a-ride service can meet the customer expected time. The definition of the average schedule delay is described as follows:

$$\text{Average schedule delay} = \frac{\sum (|\text{expected service time} - \text{actual service time}|)}{\text{numbers of customers}}$$

The results are summarized in Tables 1 and 2. In Table 1 and Table 2, travel time for the
vehicle routes, objective values, computational time and average schedule delay are listed.

1. Scenario I
   In Table 1, the total travel times are 56.52 minutes, 54.18 minutes, 53.79 minutes, and 49.92 minutes for different sizes of time window, respectively. The relationship between computational time and time window are illustrated in Figure 6(a). The results reveal that the computational times increase with respect to the time window.

   The route sequences and travel times for each vehicle are listed in Table 1. The travel time depends on how the vehicle serves demands. The relationship between objective values and time windows are depicted in Figure 6(b). The results show that the solution is better with larger time-window, which means that the feasible region is relative large with respect to time windows. In general, vehicle travel times in the I-Inf scenario are much lower than other scenarios.

   The values of average schedule delay vary slightly and it looks like the average schedule delay does not improve with respect to vehicle travel time. One of the reasons might be that the objective function does not include the average schedule delay.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Vehicle ID (route sequence)</th>
<th>Travel time (minutes)</th>
<th>Objective (minutes)</th>
<th>Computational time (CPU seconds)</th>
<th>Average schedule delay (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-10</td>
<td>1(16, 47, 83, 63)</td>
<td>16.50</td>
<td>56.52</td>
<td>3.70</td>
<td>7.88</td>
</tr>
<tr>
<td></td>
<td>2(73, 60, 24, 29)</td>
<td>20.41</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3(85, 132, 50, 41)</td>
<td>19.61</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I-15</td>
<td>1(50, 47, 16, 41)</td>
<td>14.37</td>
<td>54.18</td>
<td>45.23</td>
<td>8.12</td>
</tr>
<tr>
<td></td>
<td>2(60, 63, 29, 24)</td>
<td>16.39</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3(85, 132, 50, 41)</td>
<td>23.42</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I-20</td>
<td>1(29, 63, 83, 73)</td>
<td>14.96</td>
<td>53.79</td>
<td>89.48</td>
<td>6.53</td>
</tr>
<tr>
<td></td>
<td>2(50, 47, 16, 41)</td>
<td>14.34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3(85, 132, 60, 24)</td>
<td>24.49</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>I-Inf.</td>
<td>1(29, 24, 16, 47)</td>
<td>14.35</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2(63, 83, 73, 60)</td>
<td>15.98</td>
<td>49.92</td>
<td>1434.00</td>
<td>8.62</td>
</tr>
<tr>
<td></td>
<td>3(85, 132, 50, 41)</td>
<td>19.59</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Scenario II
   The Scenario II generates more vehicles in the network and is more congested than the Scenario I. Basically, the results exhibit similar patterns. In Table 2, the total travel times are 61.49 minutes, 55.61 minutes, 55.61 minutes, and 54.62 minutes, respectively. In Figure 7(a), the computational times increase when the range of the time window expands gradually. In Figure 7(b), the results also show that the solution is better with larger time-window.
Table 2. The results under the 1.5 OD demands

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Vehicle ID (route sequence)</th>
<th>Travel time (min)</th>
<th>Objective (min)</th>
<th>Computational time (CPU seconds)</th>
<th>Average schedule delay (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>II-10</td>
<td>1(16, 47, 29, 24)</td>
<td>15.45</td>
<td>61.49</td>
<td>25.61</td>
<td>5.44</td>
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<tr>
<td></td>
<td>2(83, 50, 73, 60)</td>
<td>23.17</td>
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</tr>
<tr>
<td></td>
<td>3(85, 132, 41, 63)</td>
<td>22.87</td>
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</tr>
<tr>
<td>II-15</td>
<td>1(29, 60, 85, 132)</td>
<td>28.52</td>
<td>55.61</td>
<td>14.25</td>
<td>6.43</td>
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<tr>
<td></td>
<td>2(50, 41, 47, 16)</td>
<td>13.19</td>
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<tr>
<td></td>
<td>3(73, 83, 63, 24)</td>
<td>13.90</td>
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</tr>
<tr>
<td>II-20</td>
<td>1(29, 60, 85, 132)</td>
<td>28.50</td>
<td>55.61</td>
<td>138.26</td>
<td>6.43</td>
</tr>
<tr>
<td></td>
<td>2(50, 41, 47, 16)</td>
<td>13.17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3(73, 83, 63, 24)</td>
<td>13.94</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>II-Inf.</td>
<td>1(50, 41, 47, 16)</td>
<td>13.19</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>2(60, 85, 132, 83)</td>
<td>27.03</td>
<td>54.62</td>
<td>717.00</td>
<td>8.11</td>
</tr>
<tr>
<td></td>
<td>3(73, 63, 29, 24)</td>
<td>14.40</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Overall Discussions
   Several observations are summarized as follows:
   a. Although the network condition is much more congested in the Scenario II, the vehicle travel time only increases slightly. One of the reasons might be that the departing times of vehicles are assumed fixed in the formulation. If the departure times of vehicles are treated as decision variables, the complexity of the formulation might increase.
   b. One of the critical factors in the formulation is time window. The feasible region increases with respect to the size of time window.

Figure 6 Results for different time windows under the 1.0 OD demand.
6. CONCLUSION

In this paper, the time-dependent DARP formulation is proposed and numerical experiments are conducted in the Kaohsiung network. Two main factors: time window and OD demand, are experimented. The numerical results reveal that the range of the time window obviously affects the computational time and objective value for the proposed formulation. When the range of the time window increases, the computational times increase exponentially and the objective values decrease slowly. In these experiments, the time window is an important factor to design vehicle routes under the minimum travel cost and the increasing OD demand level only can increase objective value slightly.

Several modifications might be considered and enhanced in the formulation. Firstly, the objective function should be re-designed to include travel cost and average schedule delay simultaneously. Secondly, the departure times of vehicles need to be re-considered; however, the complexity might increase if the departure time is treated as decision variables. Thirdly, more numerical examples should be tested to fully understand possible benefits of the formulation.

REFERENCES


