Bus Coordination Model for Passive Signal Priority

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Abstract: This research suggests the coordination model for the passive bus signal priority strategy. This model is based on the arterial signal optimization tool, MAXBAND and is given a name to MAXBAND MILP-Bus. MILP-Bus model calculates both general vehicles and buses bandwidth to improve that the general priority strategies impose limitations on the general vehicle travels. This priority model can compute the dual bandwidth in view of a lower bus speed than general vehicles and a midblock dwell time.

Key Words: Passive Signal Priority, Coordination, MAXBAND, Median Bus Lane

1. INTRODUCTION

The quest for low carbon emissions and green growth aims at a sustainable economic success considering the natural environment. It recognizes environmental stability as a new value because of the United Nations Framework Convention on Climate Change. The transportation sector is also responsible for the lower carbon dioxide emissions. Between 1971 and 2007, the global transportation emissions jumped 20 percent to 23 percent (IEA, 2009). The energy consumption has shown a similar trend with CO2 emissions. The transportation component in U.S. energy consumption was about 28 percent in 2008 (EIA, 2010), and the road traffic sector forms over 80 percent among transportation sectors (Stacy and Susan, 2008). This situation is natural because the transportation policy has focused on road traffic in most countries, as well as in the U.S. The total CO2 emission of the transportation sector in Korea is assumed to be an economic value of over one billion dollars according to the Emission Trading Scheme (Park, 2008). So, we must consider the balanced and sustainable transportation. The national policy already focuses on the transit in Korea. It contains a wide variety from the median bus lane to the bus information & management system.

Bus signal priority is divided into the passive and active strategies. The passive priority uses the signal timing favoring transit, and the active priority is bus preemption through a combination of technologies (Skabardonis, 2000). In this paper, we developed a coordination model for the integrated signal operation of buses and general vehicle under the passive transit signal priority. This bus signal priority strategy deals with the optimal progression based on the traditional arterial signal optimization model, MAXBAND. The bus generally has a slower speed than the general vehicles and midblock dwell time. Also, a left turn phase sequence is important constraints in the process of signal timing optimization because of exclusive median bus lane. This research suggests the signal coordination model which is reflected these bus travel characteristics of median bus lane for the passive signal priority strategies.
2. THE BASIC BANDWIDTH MAXIMIZATION PROBLEM: MAXBAND MODEL

MAXBAND can determine the maximized bandwidth of total n signal intersections in arterial. \( S_i \) is the signal intersections at node \( i \) as shown in figure 1, where \( i = 1, \ldots, n \). All time variables are in units of the cycle time (Gartner and Assman, 1991).

First, the directional interference constraints make sure that the outbound and inbound bandwidths must use only the green time and cannot violate the red time. We see that:

\[
\begin{align*}
     w_i + b &\leq 1 - r_i \\
    \bar{w}_i + \bar{b} &\leq 1 - \bar{r}_i
\end{align*}
\]

Next is the loop integer constraint. This constraint results from that each intersection \( i \) of the arterial is synchronized. If we start at the center of the outbound red at \( S_i \) from Figure 1, we must end up at a point that is removed an integral number of cycle times from the point of departure. Thus we obtain that:

\[
\phi(i,i+1) + \bar{\phi}(i,i+1) + \Delta_i - \Delta_{i+1} = m(i,i+1)
\]

Where, \( m(i,i+1) \) is the corresponding loop integer variable.

We can eliminate \( \phi(i,i+1) \) and \( \bar{\phi}(i,i+1) \), and define \( x_i = x(i,i+1) \) in the following loop integer constraint. We can obtain as follow:

\[
(t_i + \bar{r}_i) + (w_i + \bar{w}_i) - (w_{i+1} + \bar{w}_{i+1}) + \Delta_i - \Delta_{i+1}
= -\frac{1}{2}(r_i + \bar{r}_i) + \frac{1}{2}(r_{i+1} + \bar{r}_{i+1}) + (\bar{t}_i + \tau_{i+1}) + m_i
\]

\[ (3) \]

**Figure 1** Time-space diagram for MAXBAND
The extended formulation of MAXBAND can determine the cycle length, progression speed and speed change using each lower and upper limit value. Another important decision point is to determine the order of left turn phases. The intra-node offset \( \Delta_i \) is expressed in a binary variable \( \delta_i \) and \( \bar{\delta}_i \). Figure 2 shows the possible combinations of left turn phases and \( \Delta_i \). All cases can be expressed in the following equation:

\[
\Delta_i = \frac{1}{2} \left[ (2\delta_i - 1)L_i - (2\bar{\delta}_i - 1)\bar{L}_i \right]
\]  

(4)

If only a certain pattern is permitted, the user can designate the variable \( \delta_i \), \( \bar{\delta}_i \) as 0 or 1. MAXBAND model provides the best progression in the condition of the restricted phase sequence. MILP-1 and additional constraints provide a more useful and powerful mixed-integer linear program for the arterial progression.

MAXBAND MILP-2. Find \( b, \bar{b}, z, w_i, t_i, \bar{t}_i, \delta_i, \bar{\delta}_i, m_i \)

Max \( (b + k\bar{b}) \)

Subject to

\( (1-k)\bar{b} \geq (1-k)kb \)

\( 1/C_2 \leq z \leq 1/C_1 \)

\( w_i + b \leq 1 - r_i ; i = 1, ..., n \)

\( \bar{w}_i + \bar{b} \leq 1 - \bar{r}_i ; i = 1, ..., n \)

\( (t_i + \bar{t}_i) + (w_i + \bar{w}_i) - (w_i + \bar{w}_i) + \delta_i L_i - \bar{\delta}_i \bar{L}_i - \delta_i L_i \bar{t}_i + \bar{\delta}_i \bar{L}_i \bar{t}_i \)

(5)

\( = (r_i - r_i) + (t_i + \bar{t}_i) + m_i ; i = 1, ..., n - 1 \)

\( (d_i / f_i)z \leq t_i \leq (d_i / e_i)z ; i = 1, ..., n - 1 \)

\( (\bar{d}_i / \bar{f}_i)z \leq \bar{t}_i \leq (\bar{d}_i / \bar{e}_i)z ; i = 1, ..., n - 1 \)

\( (d_i / h_i)z \leq (d_i / d_i) t_i - t_i \leq (d_i / g_i)z ; i = 1, ..., n - 2 \)

\( (\bar{d}_i / \bar{f}_i)z \leq (\bar{d}_i / \bar{d}_i) \bar{t}_i - \bar{t}_i \leq (\bar{d}_i / \bar{g}_i)z ; i = 1, ..., n - 2 \)

\( b, \bar{b}, z, w_i, t_i, \bar{t}_i, \delta_i, \bar{\delta}_i \geq 0 \)

\( m_i \) Integer

\( \delta_i, \bar{\delta}_i \) 0 or 1, Binary variable

MAXBAND MILP-2 is an extended model of MILP-1, and can calculate the optimal cycle length, progression speed, speed change of coordination and order of left turn phases. The user can determine the signal parameters for the improved progression (Little and Kelson, 1981).

3. BUS COORDINATION MODEL FOR PASSIVE SIGNAL PRIORITY

This research suggests the modified MAXBAND model, MILP-Bus. This model accommodates the bandwidth maximization function of basic MAXBAND, can calculate the additional bandwidth from Figure 2:

First, the bus bandwidths of outbound and inbound must use only the green time and cannot violate the red time as shown in figure 3. It is the same relation with \( w_i, b \) and \( r_i \) in
MAXBAND.

\[ \begin{align*}
  tw_i + tb & \leq 1 - tr_i \\
  r\bar{w}_i + r\bar{b} & \leq 1 - r\bar{r}_i
\end{align*} \]  

Next is the loop integer constraint between the bus bandwidth. We obtain that:

\[ t\phi(i,i+1) + t\phi(i,i+1) + \Delta_i - \Delta_{i+1} = tm(i,i+1) \]  

Where, \( tm(i,i+1) \) is the corresponding loop integer variable

The next constraint is a restriction for left turn phase sequences. This constraint needs \( d_i \) and \( \bar{d}_i \) that mean the offset between the general vehicle and bus red time. If the inbound and outbound left turn green have same time length, the bus red time \( pr_i \) can be expressed using the general vehicle red time \( r_i \), binary variable \( \alpha_i \) and left turn phase \( L_i \) as follows:

\[ \begin{align*}
  pr_i & = r_i + \alpha_i L_i \\
  \bar{p}r_i & = r_i + \alpha_i \bar{L}_i
\end{align*} \]  

At this point, we can eliminate \( \phi \) and \( \bar{\phi} \) in the loop integer constraint as follows:

\[ \begin{align*}
  tt_i + \bar{t}i_i + tw_i + r\bar{w}_i - tw_{i+1} - r\bar{w}_{i+1} + \frac{1}{2}[(r_i + \bar{r}_i) - (r_{i+1} + \bar{r}_{i+1})] \\
  + \frac{1}{2}[(\alpha_i(L_i + \bar{L}_i) - \alpha_{i+1}(L_{i+1} + \bar{L}_{i+1})) = tm_i
\end{align*} \]  

The next constraint determines the bus travel time using the maximum and minimum speed, \( tf_i, te_i \) and \( \bar{t}f_i, \bar{t}e_i \). The bus needs the more travel time than the general vehicles because of lower speed and dwell time. This constraint is made up the station location \( \rho_i \) and the average dwell time \( st_i \) as follows.

\[ \begin{align*}
  (d_i / tf_i + p_i st_i)z & \leq ut_i \leq (d_i / te_i + p_i st_i)z \\
  (d_i / \bar{t}f_i + \bar{p}_i \bar{st}_i)z & \leq \bar{t}i_i \leq (d_i / \bar{t}e_i + \bar{p}_i \bar{st}_i)z
\end{align*} \]  

The bus dwell time at each station is formal distribution under the passive priority strategy. But this model can determine the optimal dwell time using a fixed cycle. If the dwell time can be rescheduled at the only specific station, this model obtains more maximized bandwidth. The last restriction is the speed changes for the coordination speed. This is expressed in lower and upper change limits \( tg_i, \bar{t}g_i, tb_i \) and \( \bar{t}g_i, \bar{t}b_i \) such that:

\[ \begin{align*}
  (d_i / tb_i)z & \leq (d_i / d_{i+1})(t_{i+1} - p_{i+1} st_{i+1})z - (tt_i - p_i st_i z) \leq (d_i / tg_i)z \\
  (\bar{d}_i / \bar{t}b_i)z & \leq (\bar{d}_i / \bar{d}_{i+1})(\bar{u}_{i+1} - \bar{p}_{i+1} \bar{st}_{i+1})z - (\bar{tt}_i - \bar{p}_i \bar{st}_i z) \leq (\bar{d}_i / \bar{t}g_i)z
\end{align*} \]  

4
The MAXBAND MILP-Bus model provides the dual bandwidth for the bus and general vehicles coordination as follows:

MAXBAND MILP-Bus

Find \( b, \bar{b}, z, w, \bar{w}, t, \bar{t}, \delta, \bar{\delta}, tw, \bar{tw}, tI, \bar{tI}, \alpha, m, tM, qM, rm \)

Max \( (b + k\bar{b}) \)

Subject to

\[
(1 - k)\bar{b} \geq (1 - k)kb
\]

\[
1/C_i \leq z \leq 1/C_i
\]

\[
w_j + b \leq 1 - r_i; i = 1, ..., n
\]

\[
\bar{w_i} + \bar{b} \leq 1 - \bar{r_i}; i = 1, ..., n
\]

\[
tw_i + tb \leq 1 - (r_i + \alpha_i L_i); i = 1, ..., n
\]

\[
\bar{w_i} + t\bar{b} \leq 1 - (\bar{r_i} + \alpha_i \bar{L_i}); i = 1, ..., n
\]

\[
(t_i + \bar{t_i}) + (w_i + \bar{w_i}) - (w_{i+1} + \bar{w_{i+1}}) + \delta_i L_i - \bar{\delta_i} \bar{L_i} - \delta_{i+1} L_{i+1} + \bar{\delta_{i+1}} \bar{L_{i+1}}
\]

\[
= (r_{i+1} - r_i) + (t_i - r_i) + m_i; i = 1, ..., n - 1
\]

\[
(t_i + \bar{t_i}) + (tw_i + \bar{tw_i}) - (tw_{i+1} + \bar{tw_{i+1}}) + \frac{1}{2}(r_i + \bar{r_i}) - \frac{1}{2}(r_{i+1} + \bar{r_{i+1}})
\]

\[
+ \frac{1}{2} \alpha_i (L_i + \bar{L_i}) - \frac{1}{2} \alpha_{i+1} (L_{i+1} + \bar{L_{i+1}}) = tm_i; i = 1, ..., n - 1
\]

\[
(t_i + \bar{t_i}) + (tw_i - w_i) - (tw_{i+1} - w_{i+1}) + \tau_{i+1} + \frac{1}{2}(\alpha_i L_i - \alpha_{i+1} L_{i+1}) - \frac{1}{2}[\delta_i L_i - \bar{\delta_i} \bar{L_i} - \frac{1}{2}(r_i - \bar{r_i})]
\]

\[
+ \frac{1}{2}[\delta_{i+1} L_{i+1} - \bar{\delta_{i+1}} \bar{L_{i+1}} - \frac{1}{2}(r_{i+1} - \bar{r_{i+1}})] = qM_i; i = 1, ..., n - 1
\]

\[
(t_i + \bar{t_i}) + (\bar{w_i} - w_i) - (\bar{w_{i+1}} - w_{i+1}) + \bar{\tau_i} + \frac{1}{2}(\alpha_i \bar{L_i} - \alpha_{i+1} \bar{L_{i+1}}) - \frac{1}{2}[\delta_i \bar{L_i} - \bar{\delta_i} \bar{L_i} - \frac{1}{2}(r_i - \bar{r_i})]
\]

\[
+ \frac{1}{2}[\delta_{i+1} \bar{L_{i+1}} - \bar{\delta_{i+1}} \bar{L_{i+1}} - \frac{1}{2}(r_{i+1} - \bar{r_{i+1}})] = rm_i; i = 1, ..., n - 1
\]

\[
(d_i / f_i)z \leq t_i \leq (d_i / e_i)z; i = 1, ..., n - 1
\]

\[
(d_i / h_i)z \leq \bar{t_i} \leq (d_i / \bar{e_i})z; i = 1, ..., n - 1
\]

\[
(d_i / \bar{h_i})z \leq (d_i / \bar{e_i})\bar{t_i} \leq (d_i / \bar{e_i})z; i = 1, ..., n - 2
\]

\[
(d_i / \bar{h_i})z \leq (d_i / \bar{e_i})\bar{t_i} \leq (d_i / \bar{e_i})z; i = 1, ..., n - 2
\]

\[
(d_i / tf_i + p_i st_i)z \leq t_i \leq (d_i / te_i + p_i st_i)z; i = 1, ..., n - 1
\]
sequence constraint. The bus travel characteristics, for example the low speed, dwell time and left turn phase are considered in the traditional arterial signal optimization model, signal priority strategy deals with the optimal progression strategy of median bus lane. This model can reflect the bus travel characteristics, for example the low speed, dwell time and left turn phase sequence constraint.

4. CONCLUSION

This research developed a progression strategy of median bus lane for the passive transit signal priority. This bus signal priority strategy works on the traditional arterial signal optimization model, MAXBAND. Also, this model can reflect the bus travel characteristics. The increased decision capabilities of MILP-bus require a corresponding increase in the size of the mathematical program.

MILP-Bus involves (24n-25) constraints and (18n-9) variables to consider the bus and general vehicle bandwidths. The increased decision capabilities of MILP-bus require a corresponding increase in the size of the mathematical program.

\[
(d_i / t_i) z \leq (d_i / t_i) (u_{1i} - p_{1i} s_{1i}) z - (u_i - p_i s_i) z \leq (d_i / t_i) z ; i = 1, ..., n-1
\]

\[
(d_i / t_i) z \leq (d_i / t_i) (u_{1i} - p_{1i} s_{1i}) z - (u_i - p_i s_i) z \leq (d_i / t_i) z ; i = 1, ..., n-2
\]

\[
\delta_j + \bar{\delta}_j + \alpha_i \leq 2; i = 1, ..., n
\]

\[
\delta_j - \bar{\delta}_j + \alpha_i \geq 0; i = 1, ..., n
\]

\[
\delta_j - \bar{\delta}_j - \alpha_i \leq 0; i = 1, ..., n
\]

\[
\delta_j - \bar{\delta}_j - \alpha_i \geq 0; i = 1, ..., n
\]

\[
b, \bar{b}, z, w_i, \bar{w}_i, t_i, \bar{t}_i, t_w_i, t_l_i, t_l_i, t_l_i, t_l_i, t_l_i, t_l_i, t_l_i, t_l_i, t_l_i \geq 0
\]

\[
m_i, l_m_i, q_m_i, r_m_i \text{ integer variable}
\]

\[
\delta_j, \bar{\delta}_j, \alpha_i \text{ 0 or 1 Binary variable}
\]

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