Valuing Expansion Opportunities in BOT Highway Projects: A Branching and Folding Back (BFB) Method

Nakhon KOKKAEW
Lecturer
Department of Civil Engineering
Walailak University
Nakhon Si Thammarat
80160, Thailand
Email: knakhon@wu.ac.th

Nicola CHIARA
Assistant Professor
Department of Civil Engineering and Engineering Mechanics
Columbia University
New York, NY 10027, USA
Email: nc2112@columbia.edu

Jirapon SUNKPHO
Assistant Professor
Department of Civil Engineering
Walailak University
Nakhon Si Thammarat
80160, Thailand
Email: sjirapon@wu.ac.th

Feng DONG
Research Analyst
Private Equity Fund, CCB International Asset Management Ltd, Hong Kong,
Email: dongfeng@ccbinl.com

Abstract: Public Private Partnerships (PPPs) around the world have become increasingly popular as an alternative way to finance infrastructure projects. Some of these projects also have growth opportunities and may be in a better position to be developed as a multi-staged project. Economic valuation methods such as discounted cash flow (DCF) ignore strategic values such as expansion options embedded in these projects, which result in undervalued project. Real options, commonly used with decision trees analysis (DTA), were introduced to address this problem of the DCF. However, The DTA assumes the time to make a decision is predetermined. This assumption is invalid when the decisions can be made any time within a project life. This paper presents a flexible point decision tree analysis (FP-DTA) and the Branching and Folding Back (BFB) method, upon which FP-DTA, real options, and Monte Carlo simulation are based. This approach helps facilitate project economic evaluation under uncertainty.

Key Words: Decision-making, Real options, Decision tree analysis (DTA), Monte Carlo simulation

1. INTRODUCTION

Infrastructure plays a critical role in economic and social development. It enables the delivery of goods and services that help promote essential economic growth and ensure the people’s quality of life. A program called Public Private Partnership, or PPP, has received particular interest in recent years as an alternative way for financing infrastructure projects. Very often, PPP projects are structured using a mechanism called Build/Operate/Transfer (BOT), in which a public entity is involved in contracting a private entity to design, finance, construct, and operate a facility for a number of years (i.e., concession period), and transfer the facility to the public entity at the end of the concession period.
The main advantages of employing BOT for financing infrastructure are two-fold. First, the government can tap private funds for financing its infrastructure. This is especially important if the government is facing a pressing fiscal problem, and raising tax is an unattractive option. Second, the government may benefit from the expertise and experience of a private entity to operate and manage a facility in a more cost-efficient way. In addition, the private entity may also have several similar projects in its portfolio; as a result, it may enjoy the additional benefits of the economies of scale.

However, major characteristics of BOT infrastructure projects create the challenges in implementing them successfully. First, BOT projects have a complex setting that involves multiple players with diverging goals. This multi-lateral party setting may impede a fair risk allocation among the parties involved. Second, BOT projects typically have a long concession period, i.e., 25 to 50 years. With such a long concession period, it is difficult for BOT analysts to predict and measure the factors that might adversely affect the project profitability. Finally, BOT projects typically involve a new undertaking with no historical data available (i.e., Greenfield project). This characteristic makes it more difficult for BOT analysts to quantify the risks in these projects accurately.

The last two characteristics of BOT projects manifest the importance of effective management of uncertainty and risk. One possible method that can be employed by the management to cope with uncertainty surrounding BOT projects is to identify “inherent” flexibility and “designed” flexibility and then implements this collection of flexibilities into the projects.

Commonly found flexibility in BOT infrastructure projects is the ability of project managers to expand the capacity of the facilities in light of heavy demand. Valuing these projects embedded with growth opportunities by traditional discounted cash flow (DCF) using expected cash flows may result in undervalued projects, thereby increasing the chance of rejecting high potential projects based on their misleading and negative net present values (NPV). This is because the DCF presumes passive management style and therefore ignores strategic decisions that may be of value under certain circumstances.

Decision tree analysis (DTA) can be used in conjunction with the DCF method to determine optimal decision about the expansion opportunities. However, conventional DTA assumes that decision points (time to make decisions) are predetermined in order to calculate the expected project values associated with each decision before discounting them back to the present time. In practice, time to make such decisions is difficult to predict. Moreover, determining these decision points tends to be exacerbated in BOT projects with a long concession period.

Strategic values embedded in projects, such as opportunities to expand the capacity, can be captured through the use of real options. Real options analysis is not a substitute but a complement to the DCF. However, because of difficulty to forecast traffic demand accurately in these projects, particular in those having a long concession period, exact time to make this decision tends to be unknown.

Simulation such as Monte Carlo method has increasingly been employed to handle uncertainty inherent in projects. Essentially, it involves randomly forward projections of underlying risk variables assumed to follow certain types of probability distribution. Usually the underlying risk variables are also assumed to behave consistently throughout its life span. However, in certain situations they may be affected critically by decisions made during the
course of a project. For example, one of the underlying risk variables in BOT highway projects is realized traffic volume. Capped by the current maximum capacity, this variable will evolve differently after the decision point made about expansion at some point of time during the course of the projects. That is, if the decision made is to expand the capacity, then realized traffic volume will evolve under a new condition and be capped under a new maximum capacity. If the decision made is instead not to expand, then realized traffic volume still evolves under the same condition set as before.

This characteristic of sequential decision makings makes options to expand the capacity compounded. In other words, sequential decision making creates a path-dependent problem. Unfortunately, backward algorithm as is employed in DTA and dynamic programming cannot be used in conjunction with Monte Carlo simulation to value these compounded options (path-dependent options) since optimal decisions at each decision point can alter the succeeding values of their associated paths. Accordingly, the optimal decision may no longer be an optimal decision. One of the goals of this paper is to address this problem and propose a new method for dealing with compounded or path-dependent expansion options.

In this paper we present (1) a modified decision tree model called flexible point decision tree and (2) a new method called the Branching and Folding Back (BFB) method, which has a foundation on flexible point decision tree, real options, and Monte Carlo simulation. This new integrated method is proposed to be used for valuing projects with growth opportunities and flexible management decision timing.

The remainder of the paper is organized as follows. In Section 2, we provide a background to tools and techniques currently employed to evaluate project economics. Section 3 summarizes the characteristics of a hypothetical project. Basic assumptions and behavior of decision makers are given in Section 4. Next, we present a new model and method for valuing projects with growth opportunities in Section 5. Section 6 shows base case analysis, and Section 7 provides the results from applying the new method. They are discussed in Section 8. Finally, Section 9 concludes the paper.

2. BACKGROUND

This section provides a background of decision making under uncertainty, analytical techniques for valuing projects under uncertainty, namely, discounted cash flow (DCF), decision tree analysis (DTA), and real options (RO).

2.1 Discounted Cash Flow (DCF)

Prior to the development of real options, corporate managers and strategists were grappling intuitively with the elusive elements of managerial operating flexibility. Many recognized that the de facto discounted cash flow criteria often undervalued investment opportunities because they ignored important strategic considerations (Hayes and Abernathy, 1980; Hayes and Garvin, 1982).

It is now widely recognized that traditional DCF approaches to the appraisal of capital investment projects, such as the standard net present value (NPV) rule, cannot properly capture management’s flexibility to adapt and revise later decisions in response to changing market conditions. Traditional DCF approaches make implicit assumptions concerning an expected scenario of cash flows and presume management’s passive commitment to a certain
static operating strategy (e.g., to initiate a capital project immediately, and to operate it continuously at base scale until the end of its predetermined expected useful life).

However, in the actual operating environment, which is characterized by change, uncertainty and competitive interactions, the realization of cash flows will probably differ from what management expected at the outset. This is especially true for infrastructure investments, which are typically characterized by a long operation period, thereby making the forecasting of future demand and cash flows increasingly difficult. As new information arrives and uncertainty about market conditions and future cash flows is gradually unfolded through the passage of time, management may have valuable flexibility to alter its initial operating strategy in order to capitalize on favorable future opportunities or to react in a manner that to mitigate losses. For example, management may defer, expand, scale down, abandon, or alter a project at various stages throughout its operating life.

2.2 Conventional Decision Tree Analysis (DTA)

The “decision tree” is an analytical tool that helps business managers resolve uncertainties in making investment decisions (Magee, 1964). The tree is made up of a series of nodes and branches. Each branch represents an alternative course of action or decision. At the end of each branch is another node representing a chance event. Associated with each branch is a payoff, denoted by \( \Pi \) and shown at the end of the terminal branch of the course (see Figure 1).

![Decision Tree Diagram](image)

**Figure 1** Using decision tree analysis (DTA) to determine optimal decision and the course of actions to be made to obtain optimality

The optimal decision of a problem structured using a decision tree can be obtained by a technique called *average out and folding back* (Raiffa, 1968), which involves the following sequence of calculations in a backward fashion. Start from terminal chance nodes, their expected values are the sum of the product between the payoffs (\( \Pi \)) and its associated probability. Next, the expected values associated with decisions being made at each decision point are compared and selected according to optimization objectives (e.g., value maximization). These selected values are then rolled back to the previous decision nodes. Repeat these steps until the value of the first decision node is calculated. Figure 1 illustrates the aforementioned calculations to determine the optimal decision.
Traditional DTA has its major drawbacks in that: (i) time to make a decision may not be predetermined; therefore, traditional DTA cannot capture the value of flexibility in choosing only the right time to do an analysis and then make a well-informed judgment; (ii) probability for each alternative may be difficult to estimate; (iii) expected value (a point estimate) is calculated from expected values with its associated probability, thereby ignoring the risk associated with each outcome.

The first drawback is graphically presented in Figure 2. Occurring at time \( t = 0 \), decision Point 1 is known with absolute certainty. Decision Point 2, on the other hand, is conditional on the arriving of relevant information, and it can be postponed until a set of investment criteria is met. The Decision Point 2, therefore, can be called flexible decision point. The expected value at a decision node is denoted by \( E_{i|I_i} [V|I_i] \), which is the conditional expectation of the project value \( V \) at flexible decision point \( 2 \), occurring at time \( i \), based on the information available up to this time \( I = I_1 \ldots I_i \).

![Figure 2 Flexible decision point occurring within the useful life of a project](image)

### 2.3 Real Options

In option theory, an option gives the holder the right, but not the obligation, to buy (call option) or to sell (put option) a designated asset at a predetermined price (strike or exercise price). The designated asset that can be bought or sold is called the underlying asset. This can be either a financial asset (stock, bond, Treasury bond, forward contract, currency, stock index, etc.) or a real asset (a raw material or mining asset, for example). Options on financial assets are called financial options, whereas those on real assets are termed real options.

Of the exercise features of options, “European” and “American” style features are the most common ones. An American-style option gives the holder the right to exercise the option at any time no later than its maturity or expired date. Because of this capability to be exercised before the maturity date, American options are also grouped into “early exercise” options. A European-style option, on the other hand, can be exercised only at maturity. It is therefore intuitive that, all things being equal, American-type options are more valuable than European-
type ones because they have more flexibility in terms of timing to exercise. It is this flexibility that increases the value of American-style options.

Real options, a term coined by Myers (1977), is commonly defined as managerial flexibility to make decisions on real assets. Examples of such managerial flexibility that can be modeled as real options are flexibility to expand the scale of operation, to adapt, to switch, and to abandon a project. Most real options are also American type. For example, expansion opportunities can be exploited at any time prior to the end of a project. Thus, they are typically modeled as American options. A good introduction to real options can be found in Dixit and Pindyck (1994), Trigeorgis (1996), Luehrman (1998), Amram and Kulatilaka (1999), and Copeland and Antikarov (2001).

Real options on projects can be identified throughout project life, as exhibited in Figure 3. Generally, most of these real options occur naturally (e.g., to defer and to abandon), while others may be implemented with extra costs (e.g., to expand and to switch operation).

![Figure 3 Types of real options found in projects over a project’s useful life](image)

### 3. HYPOTHETICAL PROJECT

A hypothetical project is assumed to be a multi-stage BOT highway project with three-development stage, initially having 4 lanes and called Stage 1. The project can be fully developed into an 8-lane project. For the sake of simplicity, we assume that the project can be expanded only stage by stage. That is, the project can only be developed from Stage 1 to Stage 2 (i.e., two more lanes added), but not from Stage 1 to Stage 3. Figure 4 exhibits one scenario in which the realized traffic volume \(Z_t\) for the first time (at time \(t = i\)) reaches its maximum capacity of Stage 1. Thus, the decision about whether or not to expand the capacity has to be made at this time step. If the decision is not to expand, then the project retains status quo. If the decision is made otherwise, the project is developed into the next stage by adding 2 more lanes. This can be seen in Figure 4.

It is important to distinguish between the “traffic demand” variable \(Y_t\) and the “realized traffic volume” \(Z_t\). The amount of traffic demand captured by the operator is called realized traffic volume \(Z_t\), which can be represented as a function of traffic demand and maximum capacity of the facility as follow

\[
Z_t = \min(Y_t, \Psi) ,
\]

where \(Y_t\) = demand variable and \(\Psi\) = maximum capacity of the facility.
4. BASIC ASSUMPTIONS AND DECISION-MAKING BEHAVIORS

For the sake of simplicity and illustrative purpose, additional assumptions and decision-making behavior are assumed as follows:

1) The concession of the hypothetical project granted by a public client to a consortium for $T$ years, where construction phase takes $T_c$ years and the operation phase lasts $T_p$ years. Consider discrete time with a yearly frequency both in the construction period, i.e., year $i$ belongs to a finite set $[0, i, ..., T_c]$, and in the operation period, i.e., year $j$ belongs to a finite set $[T_c + 1, j, ..., T]$.

2) If yearly traffic volume reaches, for the first time, its maximum capacity at any given stage, the decision makers will have two options at hand, i.e., either to expand the capacity or to retain the status quo.

3) The decision to expand the capacity is made based on value maximization rules. That is, the decision makers will expand the capacity only if the expected benefits exceed the expected costs incurred due to the expansion, otherwise they will simply choose not to expand.

4) Decisions are made on a yearly basis, i.e., expansion decision is a discrete-time decision making.

5. PROPOSED MODEL AND METHOD

This section presents a modified decision tree analysis called flexible point decision tree and a new method for valuing compounded options called Branching and Folding Back (BFB) method.

5.1 Flexible Point Decision Tree Model (FP-DTA)

Unlike traditional decision tree model, flexible point decision trees can capture decision makers’ flexibility to wait and see before making a decision. This leads to a more correct estimated value of a project under uncertainty and the management is not passive to the outcomes of uncertainty. The comparison between the traditional DTA and flexible point decision tree (FP-DTA) is exhibited in Figure 5.
Alternatives or Options (Decision flexibility)

Decision Point (No flexibility)

Alternatives or Options (Decision flexibility)

Decision Point (Time flexibility)

Conventional Decision Tree (One dimensional flexibility)

Flexible Point Decision Tree (Two dimensional flexibility)

Figure 5 Comparison between conventional decision tree and flexible point decision tree

5.2 Modeling Flexibility to Expand Scale of Operation

This section provides an “integrated method” of flexible point decision tree analysis (FP-DTA) and real options approaches for modeling flexibility to expand the scale of operation. One of the main goals in this section is to demonstrate how a “development strategy” of the project can be determined. A development strategy herein is defined as a sequence of decision makings that maximizes the return on investment.

The first tool used to structure the sequences of decision makings about the expansion is a flexible point decision tree. In our hypothetical project, the results from using this technique are presented in Figure 6. In the figure, there are three possible sequences of decision makings. The first sequence is not to expand at times \( t = i \) and \( t = j \). The second one is to expand at time \( t = i \) but not at time \( t = j \), and the third one is to expand both at times \( t = i \) and \( t = j \) (see Figure 6).

Realized traffic volume \( Z_t \)

Note:

- \( A(t=i) \) = the expected project value if the decision made at time \( t = i \) is to expand,
- \( B(t=i) \) = the expected project value if the decision made at time \( t = i \) is not to expand,
- \( q \) = development stage, \( \psi \) = sequences of system evolution, \( \psi_q \) = maximum capacity of development at stage \( q \), \( Y_t \) = demand variable, and \( Z_t \) = realized traffic volume.

Figure 6 Modeling flexibility to expand the scale of operation by an integrated method of flexible point decision tree analysis (FP-DTA) and real options approaches
At each decision node (see Figure 6), presented by a small square, two project values are estimated: \( A \) for the expected project value if the decision made is to expand; and \( B \) for the expected project value if the decision made is otherwise not to expand (see Figure 6 for detail).

Instead of developing the formulae specifically for this hypothetical project, we develop this technique in a more generalized approach so that it can be easily extended and applied to other types of projects.

Denote \( A(t=k,q+1) \) as the “net project value” discounted to time \( t=k \) (\( disc_{t=k} \)) if the decision is to expand from stage \( q \) to stage \( q+1 \), and denote \( B(t=k,q) \) as the “net project value” discounted to time \( t=k \) (\( disc_{t=k} \)) if the decision is otherwise not to expand (see Figure 6). The values of \( A(t=k,q+1) \) and \( B(t=k,q) \) can be computed by

\[
A(t=k,q+1) = disc_{t=k} \left[ PV = f \left( Z_t; P_t, C_t, DS_t \right) \right],
\]

\[
B(t=k,q) = K_{t,q} = disc_{t=k} \left[ PV = f \left( Z_t; P_t, C_t, DS_t \right) \right]
\]

where: \( k \) is the time at which the project at stage \( q \) reaches for the first time its maximum capacity; \( PV \) is project value, defined as a function of

1. realized traffic volume \( Z_t = \min(Y_t, \Psi_q) \), where \( Y_t \) = yearly demand variable and \( \Psi_q \) = maximum capacity of development stage \( q \);
2. three vectors of parameters, which are toll price \( P_t \), operating cost \( C_t \), and the debt service \( DS_t \), all relative to year \( t \).

It can be noticed that the decision rules used to select the choices between expansion and status quo are akin to those employed in financial options. Here, \( A(t=k,q+1) \) can be treated as the underlying risk variable, whereas \( B(t=k,q) \) can be viewed as the exercise price of the opportunity to expand the capacity (i.e., \( B(t=k,q) = K_{t,q} \)). However, the exercise price \( K_{t,q} \) is not constant and evolves over time \( (i) \) and depends on development stage \( (q) \). That is, the exercise price \( K_{t,q} \) is a two-state, discrete-time variable.

Clearly, the project should be expanded only if \( A \) is greater than \( B \). Accordingly, the payoff function \( (\Pi_{\text{FEX}}) \) of opportunities to expand can be computed as in Eq.(4):

\[
\Pi_{\text{FEX}}(t=k,q) = (A_{t,q+1} - K_{t,q}, 0)^+. 
\]

Thus, it follows that:

1. If \( \Pi_{\text{FEX}}(t=k,q) > 0 \), then the decision to be made at time \( t=k \) is to expand. As a result, the project is developed into the next stage \( (q=q+1) \) and \( t_k = k \).
2. If \( \Pi_{\text{FEX}}(t=k,q) = 0 \), then the decision to be made at time \( t=k \) is not to expand. As a result, the project is retained the current stage \( (q=q) \) and \( t_k = 0 \).

Finally, the development strategy of the hypothetical project, in which there are at most two opportunities to expand the capacity, can be determined by using dynamic programming. This strategy can also be represented by \( \Phi = \{t_i, t_j\} \), which characterizes the evolution of the development.
For example, if the development strategy of a simulated demand path $\omega_1$ is computed as $\Phi^{\omega_1} = \{6, 0\}$, it implies that the project should be expanded its capacity at year 6, after which there will be no further expansion, i.e., $\psi^{\omega_1} = \{1, 2, 2\}$. Note that because the availability of the second flexibility to expand is conditional on the previous decision made on the first one, this flexibility or option is therefore “compounded.” As a result, this constraint has to be taken into account in determining the development strategy, $\Phi = \{t_i, t_j\}$. In our hypothetical project, the rule constrained by this situation simply follows that if $t_i = 0$, then $t_j = 0$.

5.3 Branching and Folding Back (BFB) Method
Options can be employed to model flexibility to expand the capacity. The decisions whether or not to expand the capacity are similar to options in that if the net present value of expected future benefits exceeds the immediate costs of expansion, the system should be expanded the capacity. Otherwise the capacity of the system should not be expanded. However, modeling this flexibility as options is more complex than it appears in most literature, mainly because it involves a series of decision making, which make the options “compounded.” Compounded options create a difficulty in valuing their value known as path dependency. From another perspective, a system with flexibility to expand can also be viewed as a multi-stage development, in which the success of one stage of the development creates another opportunity to expand the system into a higher stage.

This section provides a tractable method called the Branching and Folding Back method to solve compounded options created by flexibility to expand the capacity. A multi-stage highway project is used as a case example to illustrate the concept of the valuation methods of flexibility to expand the capacity.

5.3.1 Forward Process of the BFB Method
In the forward process, two main steps are explained as follows:

1. Determine all possible sequences of system evolutions, and then model, for each sequence, a new risk variable from the risk variable that creates path dependency. This can be called a “branching” process. The goal of this step is to purge the variable from being a path-dependent variable.
2. Determine the probability associated with each sequence of system evolutions. This probability indicates the likelihood that the project will be developed into that sequence of system evolutions. This step is called probability estimation.

5.3.1.1 Branching Process
The following steps detail the branching process using the BOT highway project in which traffic demand $Y$, which is the underlying risk variable that creates path dependency, is branched into realized traffic volume variable $Z$, a new risk variable.

1. Simulate yearly traffic demand for the service during operation period ($T_p$) for $N$ paths, i.e., $\{Y^{\omega_i} | t = 0, 1, ..., T_p; i = 1, 2, ..., N\}$, where $\omega_i$ is the simulated path $i$.
2. For each simulated path, determine the decision point at which the decision about expansion has to be made. For example, in the hypothetical project with two expansion opportunities, the decision points are denoted by $t = i$ and $t = j$. Accordingly, the matrix of decision points, which indicate the decision whether or not to expand the capacity of the project for all simulated paths, is given by:
3. Determine all possible system evolutions of a multi-stage development. Flexible point decision trees can be employed to structure the sequence of system evolution.

4. Once the sequences of system evolution are established, corresponding expansion cost ($C_{i,\psi_q}(\Delta l)$) and a package of traffic volumes ($Z_{i,\psi_q}$) associated with each sequence $q$ of the system evolutions ($\psi_q$) can be determined accordingly. Each package of realized traffic volumes are now considered path-independent. For the case project comprising three sequences of system evolutions, they can be represented as follows:

$$
\begin{align*}
\{Z_{i,\psi_1}\}_{i=1,2,\ldots,T, j=1,\ldots,N} &= \min(Y_{i,\psi_1}, \Psi_1) \\
\{Z_{i,\psi_2}\}_{i=1,2,\ldots,T, j=1,\ldots,N} &= \min(Y_{i,\psi_2}, \Psi_2) \\
\{Z_{i,\psi_3}\}_{i=1,2,\ldots,T, j=1,\ldots,N} &= \min(Y_{i,\psi_3}, \Psi_3)
\end{align*}
$$

where $\Psi_q$ is the maximum capacity of the system at development stage $q = \{1, 2, 3\}$ and $T$ is the number of operating years.

5. The expansion cost function associated with each of the sequences, $C_{\psi_q}(t = k)$, can be defined as:

$$
C_{\psi_q}(t = k) = I_{t=k} \cdot (\Delta l = 2)(C_e + C_L)(1 + r_f)^{T_e - k},
$$

where

$$
I_t = \begin{cases} 
1 & \text{if } t > 0 \\
0 & \text{otherwise}
\end{cases}
$$

$C_e$ is the estimated cost per lane of expansion at the beginning of construction period, $C_L$ is cost of land acquisition or right of way, $\Delta l$ is the number of lanes expanded, $r_f$ is average inflation rate, and $T_e$ is expected construction period.

5.3.1.2 Probability Estimation

In this step, the probability that the project will be developed into each sequence of system evolutions (e.g., for the case project, the probabilities are $\Pr\{\psi_1\}, \Pr\{\psi_2\}, \Pr\{\psi_3\}$) is estimated. For our case project, the development strategy matrix $\Phi$ of $N$ simulated paths can be presented by:

$$
\Phi = \begin{bmatrix}
\{t_{i,1}, t_{j,1}\} \\
\{t_{i,2}, t_{j,2}\} \\
\vdots \\
\{t_{i,N}, t_{j,N}\}
\end{bmatrix}
$$
Accordingly, the probability associated with each of the sequence of system evolutions of the case project can be calculated by:

\[
P\{\psi_1\} = \left[\frac{n\{t_{i_0}, t_{j_0}\}}{N}\right]/N
\]

\[
P\{\psi_2\} = \left[\frac{n\{t_{i_0}, t_{j_0}\}}{N}\right]/N
\]

\[
P\{\psi_3\} = \left[\frac{n\{t_{i_0}, t_{j_0}\}}{N}\right]/N
\]

(8)

5.3.2 Backward Process of the BFB Method

Once the forward process is completed, the backward (folding back) process can be started. The second process of the BFB involves simple “backward” calculations of project value. This algorithm is presented in Figure 7.

<table>
<thead>
<tr>
<th>Probabilities</th>
<th>Project values</th>
<th>Resulting traffic volumes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr {\psi_1}</td>
<td>ECF,_{i \psi_1}, i = 1, \ldots, N</td>
<td>A package of traffic volumes, Z_{i \psi_1}, i = 1, \ldots, N</td>
</tr>
<tr>
<td>Pr {\psi_2}</td>
<td>ECF,_{i \psi_2}, i = 1, \ldots, N</td>
<td>A package of traffic volumes, Z_{i \psi_2}, i = 1, \ldots, N</td>
</tr>
<tr>
<td>Pr {\psi_3}</td>
<td>ECF,_{i \psi_3}, i = 1, \ldots, N</td>
<td>A package of traffic volumes, Z_{i \psi_3}, i = 1, \ldots, N</td>
</tr>
</tbody>
</table>

Figure 7 Algorithm of the backward process by BFB method

6. BASE CASE ANALYSIS

The value of the project is measured in terms of the return on equity or equity cash flow (ECF), given by (Esty, 1999)

\[
ECF_i = X_i - C_i - Tax_i - DS_i
\]

\[
ECF_i = Z_i \times P_i - C_i - Tax_i - DS_i,
\]

where \(X_i\) is yearly revenue, \(P_i\) is average toll price, \(C_i\) is O&M costs, \(Tax_i\) is tax, and \(DS_i\) is debt service, all relative to year \(t\). The net present value of the project is expected to be

\[
E\{NPV( ECF)\} = \sum_{i=0}^{T} \frac{E\{ECF_i\}}{(1 + r)^i}
\]

Table 1 and Table 2 summarize major characteristics of the project and the capital structure and the costs of capital employed in the hypothetical project, respectively. We assume that the project can be expanded at most two times during the operation years due to the constraint on available right of way. It is also assumed that each expansion involves adding two more lanes into the road. During the construction, the cost of construction payments made to the Contractor is $500 million; fees and interest from the construction loan are $78.50 million, as shown in Table 3. Therefore, based on a 24-month project schedule, total cost to the Sponsors is expected to be $578.50 million. During the operation period, expected annual average daily
traffic \((AADT)\) and average toll rates \((P)\) are presented in Table 4. The net present value of equity cash flow is expected to be a positive of $7.93 million, i.e., \(NPV = 7.93\) million.

Table 1 Main characteristics of the project

<p>| | |</p>
<table>
<thead>
<tr>
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<th></th>
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</thead>
<tbody>
<tr>
<td>Expected construction period ((T_C))</td>
<td>months</td>
</tr>
<tr>
<td>Operation period ((T_P))</td>
<td>years</td>
</tr>
<tr>
<td>Expected construction cost of 4 lanes</td>
<td>US M</td>
</tr>
<tr>
<td>Debt</td>
<td>US M</td>
</tr>
<tr>
<td>Equity</td>
<td>US M</td>
</tr>
<tr>
<td>Cost of debt (Before refinancing)</td>
<td>per year</td>
</tr>
<tr>
<td>Upfront fee of debt</td>
<td>per year</td>
</tr>
<tr>
<td>Cost of debt (After refinancing)</td>
<td>per year</td>
</tr>
<tr>
<td>Cost of equity</td>
<td>per year</td>
</tr>
<tr>
<td>Grace period</td>
<td>yrs</td>
</tr>
<tr>
<td>Amortization period of loan</td>
<td>yrs</td>
</tr>
<tr>
<td>Maximum capacity per lane</td>
<td>vehicles per day</td>
</tr>
<tr>
<td>Cost of expansion per lane</td>
<td>US M</td>
</tr>
<tr>
<td>Expected time for expansion of 2 lanes</td>
<td>years</td>
</tr>
<tr>
<td>Maximum number of lanes</td>
<td>lanes</td>
</tr>
</tbody>
</table>

Table 2 Capital structure and costs of capital

<table>
<thead>
<tr>
<th></th>
<th>US M</th>
<th>478.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity</td>
<td></td>
<td>100.00</td>
</tr>
<tr>
<td>Cost of debt (before refinancing)</td>
<td>% p.a.</td>
<td>15.0%</td>
</tr>
<tr>
<td>Cost of debt (after refinancing)</td>
<td>% p.a.</td>
<td>10.0%</td>
</tr>
<tr>
<td>Cost of equity ((r_e))</td>
<td>% p.a.</td>
<td>15.0%</td>
</tr>
<tr>
<td>Tax rate</td>
<td>% p.a.</td>
<td>30.0%</td>
</tr>
</tbody>
</table>

Table 3 Base case schedule of equity and debt funding for construction of development stage 1 (4-lane toll road)

<table>
<thead>
<tr>
<th>Capital Expenditure</th>
<th>24-month schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction Costs</td>
<td>US M 500.00</td>
</tr>
<tr>
<td>Disbursement Profile</td>
<td>%</td>
</tr>
<tr>
<td>6 months</td>
<td>25%  25%  25%  25%</td>
</tr>
<tr>
<td></td>
<td>125  125  125  125</td>
</tr>
<tr>
<td>Construction: Funding</td>
<td></td>
</tr>
<tr>
<td>Construction Cost</td>
<td>US M 500.00</td>
</tr>
<tr>
<td></td>
<td>125.00 125.00 125.00 125.00 125.00</td>
</tr>
<tr>
<td>Equity: Funding</td>
<td>US M 100.00</td>
</tr>
<tr>
<td></td>
<td>25.00  25.00  25.00  25.00  25.00</td>
</tr>
<tr>
<td>Debt: Funding</td>
<td>US M 400.00</td>
</tr>
<tr>
<td></td>
<td>100.00 100.00 100.00 100.00 100.00</td>
</tr>
<tr>
<td>Debt: Interest During Construction</td>
<td>US M 72.50</td>
</tr>
<tr>
<td></td>
<td>7.18  14.60 21.53 29.20</td>
</tr>
<tr>
<td>Debt: Upfront Fee (1.5%)</td>
<td>US M 6.00</td>
</tr>
<tr>
<td></td>
<td>6.00  0.00  0.00  0.00  0.00</td>
</tr>
<tr>
<td>Total Debt Financing</td>
<td>478.50</td>
</tr>
<tr>
<td></td>
<td>113.18 114.60 121.53 129.20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Construction: Debt</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Disbursement to Fund Project Costs</td>
<td>US M 400</td>
</tr>
<tr>
<td></td>
<td>100    100     100    100</td>
</tr>
<tr>
<td>Balance</td>
<td>US M</td>
</tr>
<tr>
<td></td>
<td>100    200    300    400</td>
</tr>
<tr>
<td>Refinance @ Completion</td>
<td>US M (478.50)</td>
</tr>
<tr>
<td></td>
<td>0       0      0      0</td>
</tr>
<tr>
<td>Interest</td>
<td></td>
</tr>
<tr>
<td>All-in-Rate</td>
<td>% p.a.</td>
</tr>
<tr>
<td></td>
<td>15.00% 15.00% 15.00% 15.00%</td>
</tr>
<tr>
<td>All-in-Rate (6 months)</td>
<td>% p.p.</td>
</tr>
<tr>
<td></td>
<td>7.18%   7.30% 7.18%   7.30%</td>
</tr>
<tr>
<td>Interest During Construction</td>
<td>US M 72.50</td>
</tr>
<tr>
<td></td>
<td>7.18   14.60  21.53  29.20</td>
</tr>
</tbody>
</table>
Table 4 Expected annual average daily traffic (AADT) and average toll rates

<table>
<thead>
<tr>
<th>Expected annual average daily traffic (AADT)</th>
<th>$\text{AADT}<em>t = (1 + g_t)\text{AADT}</em>{t-1}; \text{AADT}_{t-1} = 30,000, g_t = \text{growth rate} = 5%$ for $t \leq 10$ and $3%$ for $t &gt; 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average toll rates ($P_t$)</td>
<td>$P_t = 6.0, t = [1, 5]; P_t = 6.5, t = [6, 10]; P_t = 7, t = [10, 15]; P_t = 8, t = [16, 20]; P_t = 9, t = [21, 25]; P_t = 10, t = [26, 30]; P_t = 12, t = [31, 35];</td>
</tr>
</tbody>
</table>

7. NUMERICAL RESULTS

We adopted the Variance Model (VM) proposed by Chiara and Garvin (2008) to model demand risk variable $Y_t$. The original VM was modified by adding jumps into the model, i.e., the VM with jumps (Kokkaew, 2010). It is the VM with jumps that will be used in this paper. In the VM with jumps, demand risk $Y_t$ can be modeled as

$$Y_t = Y_0 + \sum_{k=1}^{t} \Delta Y_k + \text{jump}$$

$$Y_t = Y_0 + \sum_{k=1}^{t} \Delta Y_k + \Delta d$$

where $\Delta d$ is a jump process assumed to follow a Poisson process and be independent of the stochastic process $\Delta Y_t$. Additionally, jump-size distribution ($\phi$) is lognormal distributed, with the arrival rate of $\lambda$. When modeling the revenue risk using the VM, the jump process should be considered only if the project revenues are assumed to be meaningfully affected by economic cycles (boom and bust), which determine the arrival rate of the jump process accordingly. The size of the jumps can be determined using a statistical analysis of historical data collected from similar projects.

Additional assumptions and parameters used in the simulation are as follows:

1. Expected AADT and average toll rates are similar to those used in base case analysis, as shown in Table 4.
2. Inflation rate is 2.5%, and the cost of land acquisition is about $10 million per lane.
3. The random part of the stochastic process is assumed to follow beta probability distribution, with a mean of zero and a unit variance, i.e.,

   $$\varepsilon_t = \text{beta} (\mu = 0, \sigma^2 = 1, \alpha = 3, \beta = 3).$$

4. The standard deviation of the first year demand is estimated to be 2,000,000 vehicles per year, with the variance reduction of 0.5.
5. The jump process is assumed to follow the Poisson distribution, with the arrival rate of $1/30$ ($\lambda = 1/30$) for both jump-up and jump-down. The jump-size distribution follows lognormal distribution. The means of the jump-size are 100,000 vehicles for the jump-up process, and 300,000 vehicles for the jump-down process.

Note that, for the sake of illustrative purpose, we assume that the above parameter values are obtained from project analysts and traffic engineers. In real BOT projects, however, these parameters can be calculated using historical data from the projects of a similar type.
For simulated path $\omega(i)$, the net present value of the project is

\[
NPV(ECF^{\omega(i)}) = \sum_{t=0}^{T} \frac{ECF^{\omega(i)}_t}{(1 + r_i)^t}
\]

Then, for $N$ simulations, expected NPV to the Sponsors, $E\{NPV(ECF)\}$, can be estimated by

\[
E\{NPV(ECF)\} \approx \frac{1}{N} \sum_{i=1}^{N} NPV(ECF^{\omega(i)})
\]

The results from 10,000 simulations are presented in Figure 8.

\[\text{Figure 8 Expected NPV of ECF for three development schemes}\]

Estimated probability obtained from the simulation is 0.02, 0.97, and 0.01, for development scheme 1, 2, and 3, respectively. Accordingly, expected ECF from applying the BFB method is $18.68$ million. Furthermore, probability that the project value will be negative is 0.42, 0.37, and 0.46, for development scheme 1, 2, and 3, respectively.

8. DISCUSSION

In the case of no expansion, the expected NPV of ECF from the base case analysis is higher than simulation’s. This is because, in simulation, we assume traffic demand to follow a stochastic process with jumps. Also, the magnitude of jumping downs, typically triggered by economic downturn, is greater than that of jumping up. Therefore, it can be expected that under this scenario the ECF should be lower than previously estimated.

The results from applying the BFB method show that the project has a high growth potential. The value of such growth is quite obvious when compared between the project with no expansion (Figure 8(a)) and with one expansion opportunity (Figure 8(b)), jumping from $5.95$ million to $19.10$ million. However, this value decreases substantially to just $3.16$ million when the project is expanded twice (Figure 8(c)). This reduction in the project value is because the additional costs of the second expansion outweigh the increased revenues from this expansion. Accordingly, based on the results from the analysis, we would advise the Sponsors to consider expansion only once, from stage 1 (4 lanes) to stage 2 (6 lanes), but no expansion into stage 3 (8 lanes). The effectiveness and reliability from applying the BFB method to real BOT projects in large part depends on the accuracy of project information elicited from people involved in the project, which may not be readily available and therefore needs to be carefully estimated.
9. CONCLUSION

Public Private Partnerships or PPPs are increasingly welcome by the public entity who seeks to tap private resources and expertise in delivery physical and social infrastructure. Because most infrastructure projects are capital intensive, and they face significant uncertainty such as demand for the service, risk allocation and mitigation is the key to the project success.

We have developed a technique called Flexible Point Decision Tree Analysis (FP-DTA) and Branching and Folding Back (BFB) to address the problem of traditional methods such as DCF and DTA in determining project value as these methods do not take into account the flexibility to expand which results in project being undervalue. The FP-DTA and BFB model flexibility to expand the scale of operation as compounded options. Projects with growth opportunities can be evaluated using the proposed technique. We have illustrated in our numerical simulation that the FP-DTA and BFB is powerful yet simple enough for analysts to employ. The only drawback of this technique is that it requires substantial information of the project in order to use the technique successfully. With a little modification, the BFB method can be extended to value flexibility that can be modeled as compounded options or path-dependent options embedded in other operations.

REFERENCES

Raiffa, H. (1968) Decision analysis: Introductory lectures on choices under uncertainty, Addison-Wesley, MA.