Statistical Estimation of Dynamic Travel Time and Traffic Flow Propagation

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Abstract: This paper formulates dynamic travel time models under density-speed equilibrium states through a statistical approach. The dynamic travel time models are formulated by assuming that travel times are determined by the distribution of the traffic stream condition with respect to either location or time. Then, this paper shows that traffic propagations can be explicitly expressed by a function of a dynamic travel time model. In density-speed equilibrium models, such as the LWR model and the proposed dynamic travel time models, the vehicular speed is instantly adjusted to the density, regardless of the speed of the subject vehicle. As a result, vehicular accelerations in density-speed equilibrium models do not always fall within a feasible region of the vehicular performance. In order to formulate the actual traffic behaviors regarding vehicular accelerations, this paper proposes a non-equilibrium traffic flow model (i.e., continuum car-following model).

Key Words: Propagation, Travel time, Equilibrium

1. INTRODUCTION

A number of continuum traffic-flow models have been developed to formulate the dynamics of traffic flow. These models can be categorized into two classes: density-speed equilibrium models and density-speed non-equilibrium models. The first class includes the LWR model proposed by Lighthill and Whitman (1955) and Richards (1956). The second class is known as the continuum car-following model, including the well-known model of Payne (1971), Aw and Rascle (2000), and Zhang (1998).

The main differences between equilibrium and non-equilibrium models involve acceleration, which relates to vehicular performance, and reaction time, which relates to the driver’s ability. In equilibrium models such as the LWR model, the vehicular acceleration is assumed to be unlimited so that speed is instantly adjusted to density. Consequently, vehicular accelerations in equilibrium models do not always fall within a feasible region of the vehicular actual performance. If the vehicular acceleration is unlimited and there exists no drivers’ reaction time, traffic would not be in a non-equilibrium state because speed would be balanced to density in zero time. In non-equilibrium models, drivers’ reaction time is a factor, and the vehicular acceleration is determined by considering the speed of the subject vehicle. This implies that unbalanced states in density-speed relations exist. Thus, non-equilibrium models are dealing with
much more realistic traffic phenomena. This paper formulates dynamic travel time models under density-speed equilibrium states through a statistical approach. The dynamic travel time models are formulated by assuming that travel times are determined by the distribution of the traffic stream condition with respect to either location or time. Then, this paper shows that traffic propagations under equilibrium states can be explicitly expressed by a function of the proposed dynamic travel time model. In density-speed equilibrium models such as the LWR model and the proposed dynamic travel time models, the vehicular speed is instantly adjusted to the density, regardless of the speed of the subject vehicle. As a result, vehicular accelerations in density-speed equilibrium models do not always fall within a feasible region of the vehicular performance. In order to formulate the actual behavior of traffic regarding vehicular accelerations, this paper also proposes a non-equilibrium traffic flow model (i.e., continuum car-following model).

2. LITERATURE REVIEW

2.1 Whole-Link Travel Time Models
The whole-link travel time model introduced by Friesz et al. (1993) has been widely used in mathematical programming models for dynamic traffic assignments that simulate the interaction between a driver’s route choice pattern and network performance. Friesz et al. (1993) employed a linear link travel time model in which the link travel time of a vehicle entering a link is taken as a linear function of the number of vehicles on a link.

The non-linear version of whole-link travel time models has been proposed by Astarita (1995, 1996), Wu et al. (1998), Xu et al. (1999), Zhu and Marcotte (2000), and Carey and McCartney (2002). Non-linear whole-link travel time models have a general form in which the link travel time is a function of the number of vehicles on a link, as given below:

\[ \tau(t) = f(n(t)) \] (1)

Furthermore, whole-link travel time models are expressed as a function of whole-link variables, such as inflows, outflows, or the number of vehicles on a link (Ran et al., 1993). Carey et al. (2003) introduced another whole-link travel time model in which the travel time of a vehicle entering a link is taken as a function of a weighted average of the inflow rate when a user enters a link and the outflow rate when the same vehicle exits a link.

2.2 Traffic Propagation
The functional relation between travel times (or speeds) and the dispersion of traffic platoons, known as diffusion theory, was studied by Pacey (1956). The main point in diffusion theory is that if travel times are distributed normally, traffic flows at the downstream end of a link can be described by the dispersion in travel times. The diffusion model can be explained as follows.

Flow \( q_j(j) \) in interval \( j \) at the downstream end of a link is the sum, over all \( i \), of the product of flow \( q_i(i) \) in interval \( i \) at the upstream point, and the probability \( g(j-i) \) that the travel time is \( (j-i) \) is given as follows:
\[ q_2(f) = \sum_{i} q_i(i) \cdot g(j - i) \]  

(2)

Grace and Potts (1964) modeled diffusion theory, while Herman et al. (1964) and Seddon (1965) validated the theory through experiments.

The behavior of traffic platoon dispersions was further studied by Robertson (1969). Robertson’s platoon dispersion model describes how inflow to a link disperses at the end of the waiting queue or at the stop line. This model has been used by TRANSYT, an off-line computer program for determining and studying optimum fixed-time coordinated traffic signal timing in any network of roads for which the average traffic flows are known. In Robertson’s platoon dispersion model, traffic flow \( q_2(i + \tau') \) at the downstream end of a link at the interval \( i + \tau' \) is a weighted combination of the departure flow \( q_1(i) \) at the upstream entrance of a link in the interval \( i \) and the arrival flow \( q_2(i + \tau' - 1) \) at the downstream end of a link at the previous time step of \( i + \tau' \):

\[ q_2(i + \tau') = F \cdot q_1(i) + (1 - F) \cdot q_2(i + \tau' - 1) \quad \text{with} \quad \tau' = \beta \cdot \tau \quad \text{and} \quad F = 1/(1 + \alpha \cdot \tau) \]  

(3)

where \( \tau \) is the average travel time between the observation points, \( \alpha \) and \( \beta \) are the calibration constants, and \( F \) is the smoothing factor.

The above two models, functioning as the relation between travel times and traffic flows at the downstream end of a link, lack mathematical support. Finally, Astarita (1996) developed the well-known flow propagation equation based on the FIFO discipline given below:

\[ q(x + l, t + \tau(x, t, l)) = \frac{q(x, t)}{1 + d \tau(x, t, l)/dt} \quad \text{for} \quad (d \tau(x, t, l)/dt > -1) \]  

(4)

where \( \tau(x, t, l) \) is the travel time of a vehicle entering the starting point of a travel distance \( l \) at location \( x \) and time \( t \), \( q(x, t) \) is the flow at location \( x \) and time \( t \), and \( q(x + l, t + \tau(x, t, l)) \) is the flow at the end of a travel distance \( x + l \) and time \( t + \tau(x, t, l) \).

3. DYNAMIC TRAVEL TIME

This paper formulates dynamic travel time models under density-speed equilibrium states through a statistical approach. The dynamic travel time models are formulated by assuming that travel times are determined by the distribution of the traffic stream condition with respect to either location or time. The developed models can be applied to estimate travel times for uninterrupted traffic flows on homogenous roads in geometric conditions.
3.1 Space-Based Travel Time

3.1.1 A Space-Based Instantaneous Travel Time Model

The space-based instantaneous travel time (SBITT) of a vehicle is defined as the time it takes to traverse a distance, assuming that the traffic stream condition along a road does not change during its travel. The SBITT under density-speed equilibrium states has the following form:

\[ \tau^{\text{sin}}(x, t, l) = \int_{l-x}^{x+l} \frac{1}{u_v(x_v, t)} \, dx_v \]

where \( \tau^{\text{sin}}(x, t, l) \) is the instantaneous travel time of a vehicle entering the starting point of a travel distance at location \( x \) and time \( t \) in traversing a distance \( l \). \( u_v(x_v, t) \) is the equilibrium speed functioned by the density through an equilibrium relationship, \( x_v \) is the location variable, and \( t \) is the entry time when the subject vehicle enters the starting point \( x \) of a travel distance.

The SBITT can be explained as follows. In probability theory, a probability density function (pdf) of a continuous random variable is a function that describes the relative likelihood for this random variable to occur in a given interval. The probability (expected value) for the random variable to fall within a particular interval is given by the integral of this variable’s density over the interval. In this research, the traffic stream condition is a random variable and the pdf is defined as the degree to which the traffic stream condition affect the travel time of a vehicle traversing a given distance.

The SBITT can be explained as follows. As an expected value is calculated by integrating the product of a random variable and the probability density function (pdf), the SBITT (expected value) is obtained by integrating the product of the traffic stream condition which is represented by a form of travel time \( l/u_v(x_v, t) \) and the pdf \( f^{\text{sin}}(x_v) \), which is defined as the degree to which the differential elements of the traffic stream condition \( l/u_v(x_v, t) \cdot dx_v \) affect the travel time of a vehicle traversing a distance \( l \). In the SBITT model, the pdf \( f^{\text{sin}}(x_v) \) is uniformly distributed from the starting point of a travel distance \( x \) to its end \( x + l \) in the following way:

\[ f^{\text{sin}}(x_v) = \begin{cases} 
1/l & x \leq x_v \leq x + l \\
0 & \text{Otherwise} 
\end{cases} \]

Integrating the product of the traffic stream condition \( l/u_v(x_v, t) \) and the pdf \( f^{\text{sin}}(x_v) \) yields equation 5. The SBITT model is graphically described in Figure 1.
3.1.2 A Space-Based Expected Travel Time Model

Let $\tau^p(x, t, l)$ be the space-based expected travel time (SBETT) of a vehicle traveling at entry location $x$ and entry time $t$ in traversing a travel distance $l$. The travel distance $l$ can also be defined as “expected travel distance” that a vehicle can traverse during a given time $\tau^p(x, t, l)$. The expected travel distance $l$ is estimated as follows. Note that in order to estimate the expected travel distance $l$, the traffic stream condition along a road must be a form of travel distance as $u_s(x, t) \cdot \tau^p(x, t, l)$. In other words, the form of the traffic stream condition should be equivalent to that of the estimated value.

In fact, the travel time is better described by the equilibrium speed than the non-equilibrium speed (actually observed speed), even though the speed is under non-equilibrium states. The reason is that a non-equilibrium speed under uninterrupted traffic flow conditions will be recovered to an equilibrium speed in a short period of time.

Regarding the pdf, a natural assumption is that it differs according to location along a road. This is illustrated in Figure 5, which shows the traffic stream condition with respect to location $x_v$ at entry time $t$. The expected travel distance $l$ of a vehicle during a given time $\tau^p(x, t, l)$ at entry location $x$ and entry time $t$ in Case A is likely to be shorter than in Case B. The reason is as follows. The downward fluctuation of the traffic stream condition in Case A is located closer to entry location $x$ than in Case B. This downward fluctuation in Case A will impact the expected travel distance $l$ of the subject vehicle more than for Case B during a given time $\tau^p(x, t, l)$. Thus,
the expected travel distance \( l \) in Case A must be shorter than in Case B. This indicates that the pdf should differ with respect to location along a road.

In order to consider dynamic traffic flow, this paper employs the non-linear pdf (exponential distribution), even though various types of pdf can be applied. The parameter of the pdf with the exponential distribution should be automatically the inverse of expected travel distance \( 1/l \). Then, the pdf is given as follows:

\[
f^{\tau}(x_v) = \begin{cases} 
\frac{1}{l} e^{-\frac{l}{l}(x_v-x)} & x_v \geq x \\
0 & x_v < x 
\end{cases}
\]  

Integrating the product of the traffic stream condition \( u_e(x_v,t) \cdot \tau^{\tau}(x,t,l) \) and the pdf yields the “expected travel distance \( l \)” as

\[
l = \int_{x_v}^{\infty} u_e(x_v,t) \cdot \tau^{\tau}(x,t,l) \cdot \frac{1}{l} e^{-\frac{l}{l}(x_v-x)} \, dx_v 
\]  

Rearranging equation 8 yields

\[
\tau^{\tau}(x,t,l) = \int_{x_v}^{x} \frac{1}{u_e(x_v,t)} \cdot \frac{1}{l} e^{-\frac{l}{l}(x_v-x)} \, dx_v 
\]

In equation 8, the distribution of the traffic stream condition along a road is defined as \( u_e(x_v,t) \cdot \tau^{\tau}(x,t,l) \). Note that in equation 9, the traffic stream condition has been shifted to \( 1/u_e(x_v,t) \), which is a form of travel time. The SBETT is graphically described in Figure 3.

![Figure 3 Graphical representation of the SBETT model](image-url)
3.2 Time-Based Travel time

3.2.1 A Time-Based Instantaneous Travel Model

The time-based instantaneous travel time (TBITT) of a vehicle $\tau_{\text{in}}(x,t,l)$ is defined as the time it takes to traverse a distance, assuming that the vehicle experiences the same traffic stream condition during its travel of a distance $l$ as the traffic stream condition at the starting point of a travel distance. The TBITT under density-speed equilibrium states is obtained by integrating the product of the traffic stream condition $l/u_e(x,t_v)$ and the pdf $f_{\text{in}}(t_v)$ with respect to time $t_v$ at the starting point of a travel distance $x$. In the TBITT model, the pdf is uniformly distributed. In order to satisfy the basic statistical condition with the pdf of uniform distribution:

$$\int_{\tau - \tau_{\text{in}}(x,t,l)}^{\tau} f_{\text{in}}(t_v)dt_v = 1$$  \hspace{1cm} (10)

the pdf needs to be

$$f_{\text{in}}(t_v) = \begin{cases} 
\frac{1}{\tau_{\text{in}}(x,t,l)} & t - \tau_{\text{in}}(x,t,l) \leq t_v \leq t \\
0 & \text{Otherwise} 
\end{cases}$$ \hspace{1cm} (11)

Integrating the product of the traffic stream condition $l/u_e(x,t_v)$ and the pdf $f_{\text{in}}(t_v)$ yields

$$\tau_{\text{in}}(x,t,l) = \int_{\tau - \tau_{\text{in}}(x,t,l)}^{\tau} \frac{l}{u_e(x,t_v)} \frac{1}{\tau_{\text{in}}(x,t,l)} dt_v$$ \hspace{1cm} (12)

One of the methods used to solve equation 12 is the “trial-and-error method,” which yields a satisfactory result by trying out various means or theories until the error is sufficiently reduced or eliminated.

3.2.2 A Time-Based Travel Expected Time Model

Let $\tau^u(x,t,l)$ be the time-based expected travel time (TBETT) of a vehicle at location $x$ and time $t$ in traversing a distance $l$. In the TBETT model, the traffic stream condition with respect to time $t_v$ at the starting point of a travel distance $x$ should be defined as a form of travel time:

$$\frac{l}{u_e(x,t_v)}$$ \hspace{1cm} (13)

In the TBETT model, the pdf is assumed to be exponentially distributed with respect to the time difference $(t - t_v)$ between the entry time $t$ when the subject vehicle enters the starting point of a travel distance and the time $t_v$ of the traffic stream condition at the starting point of a travel distance. If the pdf is exponentially distributed, the parameter of the pdf should automatically be the inverse of the expected travel time $1/\tau^u(x,t,l)$. As such, the pdf is given as follows:
Integrating the product of the traffic stream condition and the pdf yields

$$\tau^u(x,t,l) = \int_{-\infty}^{t} \frac{l}{u(x,t,v)} \cdot \frac{1}{\tau^u(x,t,l)} \cdot e^{-\frac{1}{\tau^u(x,t,l)}(t-t_v)} dt_v$$

Equation 15 is the final TBETT model, which can be solved with the trial-and-error method. Equation 15 also can be expressed as follows:

$$\tau^u(x,t,l) = \int_{-\infty}^{t} \frac{s^u(x,t,l)}{u(x,t,v)} \cdot e^{-\frac{s^u(x,t,l)}{l}(t-t_v)} dt_v$$

where $s^u(x,t,l)$ is the average speed of a vehicle during the travel of a distance $l$. The proposed model has an interesting characteristic: The travel distance $l$ and the average speed $s^u(x,t,l)$ consist of the parameter of the pdf. As shown in Figure 4, the pdf with a travel distance $l_1$ that is shorter than the travel distance $l_2$ is steeper than the pdf with the travel distance $l_2$. Only the traffic stream condition that is just in the front of the subject vehicle is critical for the travel time of the subject vehicle traversing a short travel distance $l_1$. On the contrary, the traffic stream condition that is far away from the subject vehicle can significantly affect the travel time of the subject vehicle traversing a long travel distance $l_2$. As such, the pdf with two different average speeds is similarly explained.

Figure 4  Understanding the probability density function

From the TBETT model (equation 15), the travel time of a vehicle traveling a distance between any two locations on a road can also be obtained. Let $\tau^u(x,t,l_1 - l_2)$ be the travel time of a vehicle at location $x$ and time $t$ traveling between the ends of two different travel distances $l_1$ and $l_2$. The travel time $\tau^u(x,t,l_1 - l_2)$ can be calculated using the TBETT model.
and \( l_2 \), which begin from location \( x \). The travel time \( \tau^u(x,t,l_1 \sim l_2) \) is simply obtained with the following:

\[
\tau^u(x,t,l_1 \sim l_2) = \tau^u(x,t,l_2) - \tau^u(x,t,l_1)
\]

(17)

4. PROPAGATION OF FLOW, SPEED, ACCELERATION, AND DENSITY

In this section, traffic propagation models which describe how speed, acceleration, flow and density vary over both location and time along a road. Traffic propagation models are explicitly expressed by a function of a developed travel time model.

Let \( \tau(x,t,l \sim l + \Delta l) \) be the travel time of a vehicle traveling at entry location \( x \) and entry time \( t \) in traversing a distance between the ends of two travel distances \( l \) and \( l + \Delta l \), which begin from entry location \( x \). Then, the following condition is held as

\[
\tau(x,t,l \sim l + \Delta l) = \tau(x,t,l + \Delta l) - \tau(x,t,l)
\]

(18)

Let \( u(x + l,t + \tau(x,t,l)) \) be the speed of a vehicle at exit location \( x + l \) and exit time \( t + \tau(x,t,l) \). Based on equation 18, a vehicle entering the starting point of a travel distance \( l \) at entry location \( x \) and entry time \( t \) will have a speed at the end of travel distance (exit location) \( x + l \) and exit time \( t + \tau(x,t,l) \) as

\[
u(x + l,t + \tau(x,t,l)) = \lim_{\Delta l \to 0} \frac{(l + \Delta l) - l}{\tau(x,t,l \sim l + \Delta l)}
\]

\[
= \lim_{\Delta l \to 0} \frac{\Delta l}{\tau(x,t,l + \Delta l) - \tau(x,t,l)}
\]

\[
= \frac{1}{d\tau(x,t,l)/dl}
\]

(19)

An acceleration propagation equation is obtained as follows. A vehicle entering the starting point of a travel distance \( l \) at entry location \( x \) and entry time \( t \) will have acceleration at the end of a travel distance (exit location) \( x + l \) and time \( t + \tau(x,t,l) \) as

\[
a(x + l,t + \tau(x,t,l)) = \lim_{\Delta l \to 0} \frac{u(x + l + \Delta l,t + \tau(x,t,l + \Delta l)) - u(x + l,t + \tau(x,t,l))}{\tau(x,t,l + \Delta l) - \tau(x,t,l)}
\]

\[
= \frac{du(x + l,t + \tau(x,t,l))/dl}{d\tau(x,t,l)/dl}
\]

(20)

Combining equations 19 and 20 yields
The density at exit location \( x + l \) and exit time \( t + \tau(x,t,l) \) is simply obtained by dividing the flow from equation 4 by the speed from equation 19, as

\[
a(x + l, t + \tau(x,t,l)) = \left[ \frac{1}{d\tau(x,t,l)/dl} \right]/dl
d\tau(x,t,l)/dl
\]  

(21)

Density propagations are expressed by a function of a travel time model and flow at entry point. The flow, speed, acceleration, and density propagation equations can be formulated with the developed TBETT model (or space-based travel time) in which travel distance consists of the parameter of the pdf so that flow, speed, acceleration, and density are estimated at any point with respect to both location and time.

The flow, speed, acceleration, and density propagations under equilibrium states can be formulated by equations 4, 19, 21, and 22 with equation 9 or 15, respectively. The proposed traffic propagations are a function of the dynamic travel time model that has travel distance as the parameter of the pdf so that flow, speed, acceleration, and density are estimated at any point with respect to both location and time. The proposed traffic propagation models can overcome the deficiency of the LWR model which has the inheritable discontinuity of traffic characteristics.

5. NON-EQUILIBRIUM TRAFFIC FLOW THEORY

The proposed models (equations 4, 19, 21, and 22 with equation 9 or 15) estimate flow, speed, acceleration, and density propagations under density-speed equilibrium states. Under equilibrium states, a vehicular speed is assumed to be instantly adjusted to a density. As a result, accelerations estimated from equation 21 with equation 9 or 15 do not always fall into a feasible region of actual vehicular performance. In order to formulate the actual behavior of traffic regarding vehicular accelerations, a non-equilibrium traffic flow model (i.e., continuum car-following model) is formulated, and its implementation is discussed in this section.

As discussed, the main differences between equilibrium and non-equilibrium models are in acceleration, which relates to vehicular performance, and reaction time, which relates to the driver’s ability. In equilibrium models such as the proposed traffic propagation models, the vehicular acceleration is assumed to be unlimited so that speed is instantly adjusted to density. In non-equilibrium models describing unbalanced states in density-speed relations, the vehicular acceleration is determined by considering the speed of the subject vehicle. A non-equilibrium traffic flow model is formulated with limited acceleration relating to vehicular performance and reaction time relating to the driver’s ability.
Under non-equilibrium states of traffic, the TBETT model and the SBETT model, which are modified from equation 9 and 15, will be, respectively,

\[ \tau^{\text{ne}}(x,t,l) = \int_{-\infty}^{t} \frac{l}{u(x,t_\nu)} \cdot \frac{1}{\tau^u(x,t_\nu)} e^{-\frac{1}{\tau^u(x,t_\nu)}} dt_\nu \]  
(23)

and

\[ \tau^{\text{eq}}(x,t,l) = \int_{l}^{\infty} \frac{1}{u(x,t_\nu)} \cdot e^{\frac{1}{\tau(x,t_\nu)}} dx_\nu \]  
(24)

where \( u \) is the non-equilibrium speed (actual observed speed). Equation 21, including the driver’s reaction time \( T \), yields the acceleration at location \( x \) and time \( t+T \) as

\[ a(x,t+T) = \lim_{l \to 0} \frac{d}{dl} \left[ \frac{1}{d\tau(x,t,l)/dl} \right] / dl \]  
(25)

where \( \tau(x,t,l) \) can be equation 9 or 15. Note that \( a(x,t+T) \) is the desired acceleration, which has not taken into account the actual vehicular performance. Then, actual acceleration at location \( x \) and time \( t+T \) will be

\[ A(x,t+T) = \begin{cases} a(x,t+T) & \text{if } a_{\min}(u) \leq a \leq a_{\max}(u) \\ a_{\min}(u) \text{ or } a_{\max}(u) & \text{otherwise} \end{cases} \]  
(26)

where \( a_{\min}(u) \) (or \( a_{\max}(u) \)) is the minimum (or maximum) acceleration of which the vehicle is capable, functioned by speed \( u \).

6. A NUMERICAL EXAMPLE

6.1 Scenario 1
In this section, an illustrative numerical example is presented. The TBETT model is more applicable to practitioners than the SBETT model because collecting the traffic stream condition with respect to time at a certain point is easier than along a road. The objective is to demonstrate the properties of the proposed model (Equation 15) and validate its reasonability for simulating dynamics of traffic flow.

Let equilibrium traffic speeds and flows at location \( x = 0 \) (mile) be as follows:

\[ u_e(0,t_\nu) = \begin{cases} 47 \text{ (mile/hr)} & \text{for } -\infty < t_\nu < 0 \text{ (sec)} \\ 36.5 \text{ (mile/hr)} & \text{for } 0 \leq t_\nu < 400 \text{ (sec)} \end{cases} \]
The layout of the numerical example site is shown in Figure 5.

![Diagram of the site layout](image_url)

**Figure 5** Layout of the case study site

### 6.1.1 Travel times

From the TBETT model (equation 15), the travel time of a vehicle entering the starting point of a travel distance \( l = 0.2 \text{(mile)} \) at entry location \( x = 0 \text{(mile)} \) and entry time \( t \) is calculated as

\[
\tau^v(0,t,0.2) = \int_{-\infty}^{t} \frac{0.2}{\tau^v(0,t,0.2)} \cdot \frac{1}{e^{\tau^v(0,t,0.2)(t-t_v)}} dt_v
\]

\[
= \int_{-\infty}^{0} \frac{0.2}{47} \cdot \frac{1}{\tau^v(0,t,0.2)} e^{-\frac{1}{\tau^v(0,t,0.2)(t-t_v)}} dt_v + \int_{0}^{36.5} \frac{0.2}{36.5} \cdot \frac{1}{\tau^v(0,t,0.2)} e^{-\frac{1}{\tau^v(0,t,0.2)(t-t_v)}} dt_v
\]

\[
= \frac{0.2}{47} \cdot e^{-\frac{1}{\tau^v(0,t,0.2)(t-t_v)}} \bigg|_{-\infty}^{0} + \frac{0.2}{36.5} \cdot e^{-\frac{1}{\tau^v(0,t,0.2)(t-t_v)}} \bigg|_{0}^{36.5}
\]

\[
= \frac{0.2}{47} + \frac{0.2}{36.5} \cdot e^{-\frac{1}{\tau^v(0,t,0.2)}} (36.5 - 0)
\]

Equation 27 is solved by the “trial-and-error method.” Figure 6 shows the estimated profile of travel times with various travel distances versus entry time \( t \) when the subject vehicle enters the stop line \( x = 0 \text{(mile)} \). Results from the estimated travel times versus entry time \( t \) show an upward trend in mean travel time as the entry time \( t \) increases (the positions of vehicles in the queue move back) and the travel times stabilize slowly as travel distance increases.
6.1.2 Speed and Flow Propagations

From equations 19 and 4 with equation 15, speed and flow at exit location $x' = x + l$ and exit time $t' = t + \tau''(x, t, l)$ are estimated, respectively:

$$u(x', t') = \frac{1}{d\tau''(0, t, l)/dl} \quad \text{and} \quad q(x', t') = \frac{q(0, t)}{1 + d\tau''(0, t, l)/dt}$$

with $\tau''(0, t, l) = \int_{0}^{t} \frac{l}{u_c(0, t')} \cdot \frac{1}{\tau''(0, t, l)} e^{-\tau''(0, t, l)/(l-u_c)} dt'$. 

Figures 7 and 8 show the estimated speed and flow profiles versus exit time $t' = t + \tau''(x, t, l)$ at various exit locations $x' = x + l$ where $x = 0(\text{mile})$ and $l = 0.2, 0.4, 0.6, 0.8$ and $1.0$ mile, respectively.
Figure 7  Speeds \( u(x',t') \) where \( x' = x + l \) and \( t' = t + \tau''(0,t,l) \)

![Figure 7](image)

Figure 8  Flows \( q(x',t') \) where \( x' = x + l \) and \( t' = t + \tau''(x,t,l) \)

6.2 Scenario 2

One of the main purposes of the numerical example is to ensure that the model (equations 15) holds under ‘first-in-first-out’ (FIFO) discipline. Under the FIFO discipline, vehicles must leave the end of a travel distance in the same order as the order of arrival at the starting point of a travel distance. According to Astarita (1996), the FIFO discipline is violated only if travel times decline rapidly, that is, if \( d\tau''(x,t,l)/dt \leq -1 \). In order to generate a rapid decline in travel times, a rapid increase in speeds \( u(0,t_v) \) from 36.5(km/hr) to 41(km/hr) at \( t_v = 50(\text{sec}) \) is applied as

\[
\begin{align*}
u_v(0,t_v) &= \begin{cases} 
47(\text{mile/hr}) & \text{for } -\infty < t_v < 0(\text{sec}) \\
36.5(\text{mile/hr}) & \text{for } 0 \leq t_v < 50(\text{sec}) \quad \text{and} \\
41(\text{mile/hr}) & \text{for } 50 \leq t_v < 400(\text{sec}) 
\end{cases} \\
q(0,t_v) &= \begin{cases} 
0(\text{veh/hr}) & \text{for } -\infty < t_v < 0(\text{sec}) \\
2000(\text{veh/hr}) & \text{for } 0 \leq t_v < 50(\text{sec}) \\
1200(\text{veh/hr}) & \text{for } 50 \leq t_v < 400(\text{sec}) 
\end{cases}
\end{align*}
\]

From equation 17, the travel times between two locations on a road are obtained as shown in Figure 9. The sharp increase in speeds causes the travel time \( \tau''(0,t,0.0 \sim 0.2) \) at entry time \( t = 50(\text{sec}) \) to start to drop, as shown in Figure 9, but the FIFO discipline is not violated \( (d\tau''(0,t,0.0 \sim 0.2)/dt > -1) \). An interesting feature is that the travel time \( \tau''(0,t,0.8 \sim 1.0) \) at entry time \( t = 50(\text{sec}) \) does not start to drop, as shown in Figure 9. In addition, the resulting profile of the travel time \( \tau''(0,t,0.8 \sim 1.0) \) is smoother than the travel time \( \tau''(0,t,0.0 \sim 0.2) \). This result illustrates that traffic disperses to areas with lower traffic density.
7. CONCLUSIONS

This paper formulated the models for dynamic travel times using a statistical approach. Dynamic travel time models are obtained by assuming that the travel time of a vehicle depends on the distribution of the traffic stream condition with respect to location or time.

Traffic propagations under equilibrium traffic states can be explicitly expressed by a function of the proposed dynamic travel time model. As discussed, equilibrium models do not accurately portray traffic phenomena. In order to simulate actuality in vehicular acceleration behaviors, we extended the study of equilibrium traffic to non-equilibrium traffic (equation 26).

The models (equations 9 and 15) possess several interesting characteristics. The travel distance consists of the parameter of the pdf so that travel times in traversing any travel distance can be estimated. Furthermore, travel times in traveling between any two locations on a road can also be obtained (see equation 17). Second, the models do not have an unknown factor for geometric road conditions. In the models, it is assumed that travel times are determined by the distribution of the traffic stream condition, $l/u_e$, where $u_e$ is the equilibrium speed that is determined by density. This means that the unknown factor for the geometric road conditions is already reflected in equilibration between density and speed. Finally, the models hold under the FIFO discipline. In order to consider dynamic traffic flow, this paper employs a statistical approach to consider traffic distributions, by which the proposed models can hold under this discipline.

REFERENCES

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