Bi-level Transit Subsidy Optimization Models under Ecological Footprint Constraints

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Abstract: To maximize social welfare under transportation sustainability, bi-level budget allocation models are proposed, where the upper level maximizes the social welfare of trip makers by allocating budget to subsidize bus fare discount and bus frequency and to acquire green land for accommodating excess footprint. Two models are developed: the single generation (SG) model and the across generation (AG) model. The SG model assumes these decisions are made under the consideration of the contemporary generation alone, while the AG model compromises these decisions with the consequent generation. A case study on an exemplified network is conducted. Results show that the measures of bus fare discount and bus frequency increase can attract remarkable bus patronage and achieve almost the same social welfare. The optimal decision of the contemporary generation compromises the total utility of the contemporary generation with that of the next generation by leaving partial budget to the next generation. In contrast to traditional transit network design model, the proposed model further considers the trade-off and decisions across generations.

Keywords: Budget allocation, Bi-level programming model, Ecological footprint.

1. INTRODUCTION

Transportation significantly impacts social systems and ecosystems. Sustainable transport has received increasing attention worldwide under sustainable development policies and initiatives initiated by the Brundtland Commission, which has formally been recognized as the World Commission on Environment and Development (WCED) since 1987. The OECD (1996) defines environmentally sustainable transport as transportation posing no threat to public health or ecosystems and fulfilling mobility needs that comply with use of renewable resources at below their rates of regeneration and use of non-renewable resources at below the rates of development of renewable substitutes. Pertaining to this concept, several works have addressed implications of the scope of transportation sustainability and directions and indicators of sustainable transportation. For instance, in addition to developing criteria to evaluate CO2, NOx, particles emission and land use, Geurs and van Wee (2000) evaluated the sustainability of transportation by using three scenarios. Steg and Gifford (2005) presented behavioral and technological strategies that not only differ as to how they may improve different sustainability aspects, but also may to how they affect the quality of life of citizens. Amekudzi et al. (2009) developed a sustainability footprint model to assess the quality of life for society, as well as how transportation systems impact natural environments. Wilhelm and Posch (2003) evaluated the impacts of mobility management projects in thirteen European countries. Shiftan et al. (2003) adopted a scenario building scheme to plan a sustainable transport system. Richardson (2005) developed analytical frameworks to illustrate how indicators of transport sustainability interact with each other. Lee et al. (2008) devised a
sustainability evaluation system to facilitate infrastructure investment decisions by applying the “System of Sustainable Development Indicators for Taiwan.” May et al. (2008) described a comprehensive approach to the identification and treatment of barriers to effective decision-making in sustainable transport. Kepaptsoglou and Karlaftis (2009) conducted a very comprehensive review of transit route design problems. Jeon et al. (2013) used multiple criteria decision making (MCDM) methods, in conjunction with four sustainability indexes of system effectiveness, environmental, economic, and social impacts to identify clearly superior alternatives and consider trade-offs that are presented by competing plan alternatives.

Few of the above studies have adopted mathematical modeling approaches to elucidate how different parties interact with each other under sustainable transport contexts. Users generally select optimal transport services according to their own interests, such as how to maximize their utility or minimize travel costs. Regulators, i.e. governmental agencies, attempt to maximize the social welfare of all trip makers, i.e. utility, but retain transportation footprints under an ecological limit by encouraging the patronage of public transportation and discouraging the usage of private vehicles. Chiou et al. (2013) subsequently proposed two bi-level programming models, i.e. an operational model and a planning model, for intercity passenger transport systems. That study assumed that government attempts to provide transportation infrastructures and regulate fares or tolls to achieve four overall sustainability indices, i.e. energy consumption, air pollution, safety, and travel time. The carriers simply determine the frequency of services, with the fares regulated by the government, to maximize profits. The passengers choose available transport modes to maximize their utilities. Although Chiou et al. successfully modeled the intertwining behaviors of users, carriers and the government and produced optimal decisions for construction horizon and fare/toll rates transportation systems, two gaps still remain. First, the upper level focuses on a sustainability index defined by a weighted sum of four indices. However, government focuses on maximizing social welfare not sustainability because such a goal can be easily achieved by prohibiting the use of all fossil fueled and low occupancy vehicles. Second, sustainable development is most often defined as the development that meets the needs of the present generation without preventing future generations to meet their own needs (United Nations, 1987). This definition implies that the decisions made by the contemporary generation should compromise with the needs of future generations. To remedy these gaps, this study develops and compares two models, i.e. single generation (SG) model and across generation (AG) model. The SG model assumes decisions are made while considering the contemporary generation alone, while the AG model compromises these decisions with the next generation. The rest of this paper is organized as follows. Section 2 presents the formulations of the SG and AG models. Section 3 then briefly introduces the proposed solution algorithms for both models. Next, Section 4 describes a case study of an exemplified network. Section 5 discusses sensitive analysis with policy implications. Conclusions are finally drawn in Section 6, along with recommendations for future research.

2. MODEL FORMULATION

This paper compares budget allocation results determined solely based on the best interest of the contemporary generation or on the consideration across generations by formulating two bi-level programming models, i.e. SG model and AG model. For clarity, some postulations are stated first before presenting the model formulations.

2.1 Postulations
As is well known, regulators, carriers and users should be considered in sustainable transportation. However, carriers operate worldwide under a highly regulated environment, including their operating networks, service frequencies and fare rates. To avoid the financial deficit of carriers and ensure user affordability, the average cost pricing method (zero economic profit pricing), such as cost-plus pricing and rate-of-return pricing, is commonly adopted to regulate a fare rate of public transportation. In contrast, the financial deficit of carriers should be subsidized if the regulator attempts either to further lower the fare rate or to increase the service frequency of public transportation in order to attract more customers. For instance, the Taiwan government subsidizes more than half of the operating cost of bus transportation. Two subsidies, i.e. user-side and provider-side, are commonly adopted. User-side subsidy provides a fare subsidy, i.e. fare discount, which attempts to maintain a more competitive fare rate of public transportation than the travel cost of private vehicles. Provider-side subsidy subsidizes the financial deficit of carriers incurred by operating unprofitable routes or frequencies that are requested by the regulator. Since the behaviors of carriers are regulated, this work models only how regulators and users interact with each other.

Additionally, as is generally assumed, the regulator attempts to maximize social welfare (SW) of all trip makers under an environmentally allowable level of ecological footprint by allocating an adequate budget. The total utility of trip makers is a good proxy variable for estimating SW consisting of consumer surplus and producer surplus, since the financial deficit or surplus of carriers is subsidized and regulated.

Obviously, to comply with the environmentally allowable level requirements of transportation footprint, another budgeting approach purchases additional green land, i.e. the treeplanting land, to accommodate the excess transportation footprint once too many private vehicles are used. Notably, strictly regulating the use of private vehicles is also an effective means of curtailing the footprint. However, such a management strategy is ineffective in most democratic countries and even politically infeasible due to strong user opposition. Therefore, this work considers only the measures to increase the attractiveness of public transportation.

The lower level of the proposed model determines the mode and route choice behaviors of users in response to decisions of the upper level. The mode choice behavior is modeled by a logit model, and the route choice behavior is determined based on user equilibrium principle.

### 2.2 Proposed Framework

The proposed bi-level programming model can be represented as a leader-follower, or Stackelberg, game where the regulator is the leader and the users are the followers.

Fig. 1 illustrates the framework of the proposed model. The upper level is a budget allocation model, while the lower level contains mode choice and route choice models. The model maximizes total utility under budgetary constraints, transport system capacity and available land area in terms of an ecological footprint.
Upper level: Budget allocation model

Objective:
Max Total utility

Constraints:
1. Budget
2. Capacity of transport system
3. Available land area (ecological footprint)

Decision variables:
1. Bus fare discount ($x$)
2. Bus frequency increase ($y$)
3. Green land acquisition ($z$)

Lower level: Mode choice model & Route choice model

Mode choice model (Logit model)

Objective: Max Individual utility

Decision: mode choice

Route choice model (User equilibrium)

Objective: Min Individual travel cost

Decision: route choice

Demands of public and private transportation

Figure 1. Framework of the proposed model

According to Fig. 1, three sub-models are the budget allocation model, mode choice and route choice models. Although the upper level optimally determines bus fare discount ($x$), bus frequency ($y$), and green land acquisition ($z$), its objective function is an aggregated sum of individual mode choice utility affected by the lower level. Conversely, the lower level determines the mode and route choice behaviors based on given values of $x$, $y$, and $z$. Obviously, both levels cannot be solved separately.

2.3. SG Model

The SG model determines the optimal decisions associated with Fig. 1 based on the best interest of the contemporary generation, which can be expressed as:

[Upper level]

$$\text{Max } U^T = \sum_{rs \in N} q^{rs} P_r^{rs} U_c^{rs} + \sum_{rs \in N} q^{rs} \frac{P_r^{rs}}{U_b^{rs}}$$  \hspace{1cm} (1)

Subject to

$$( \sum_{rs \in N} q^{rs} P_r^{rs} )x + SD + (GL)z \leq B$$  \hspace{1cm} (2)

$$EF \leq A + z$$  \hspace{1cm} (3)

$$SD = \begin{cases} ( \sum_{rs \in N} q^{rs} P_r^{rs} )FC - ( \sum_{rs \in N} q^{rs} P_r^{rs} )TC_b, & \text{if } ( \sum_{rs \in N} q^{rs} P_r^{rs} )FC \geq ( \sum_{rs \in N} q^{rs} P_r^{rs} )TC_b \\ 0, & \text{else} \end{cases}$$  \hspace{1cm} (4)

$$EF = \left( \sum_{a \in L} d_y a + \sum_{k \in K} r_k v \right) \frac{FC}{F_c} + \frac{EC}{EL}$$  \hspace{1cm} (5)

[Lower level]

$$\text{Min } \sum_{a \in L} \int_0^{v_a} \left[ 1.0 + 0.15 \left( \frac{v_a}{C_a} \right) \right]^4 dv_a$$  \hspace{1cm} (6)

Subject to
\[
\sum_{k \in K_{rs}} f_k = \frac{q^{rs} Pr^{rs}}{l_c} \quad \text{for } r, s \in N 
\]

\[
v_a = \sum_{r,s \in N} \sum_{k \in K'} f_k \delta_{ak} \quad \text{(8)}
\]

\[q^{rs} Pr^{rs} \leq l_b y \quad \text{for } r, s \in N \text{ and } r \neq s \quad \text{(9)}
\]

\[Pr^{rs}_j = \frac{e^{U^{rs}_j}}{e^{U^{rs}_j} + e^{U^{rs}_c}} \quad \text{for } j = b, c \quad \text{(10)}
\]

\[U^{rs}_j = \alpha IT^{rs}_j + \beta OT^{rs}_j + \gamma TC^{rs}_j \quad \text{for } r, s \in N \text{ and } j = b, c \quad \text{(11)}
\]

\[OT^{rs}_b = \frac{30}{y} \quad \text{(12)}
\]

\[f_k \geq 0, \text{ for } k \in K^{rs} \quad \text{(13)}
\]

where, \(U^T\) denotes the total utility which sums up individual utility of all trip makers; \(q^{rs}\) denotes the trip demand from origin \(r\) to destination \(s\) (i.e. the O-D pair \(rs\)); \(Pr^{rs}_b\) denotes the market share of public transportation (hereinafter referred to as bus) of O-D pair \(rs\); \(Pr^{rs}_c\) denotes the market share of private transportation (hereinafter referred to as car) of O-D pair \(rs\); \(U^{rs}_c\) denotes the utility of a trip maker selecting cars of O-D pair \(rs\); \(U^{rs}_b\) denotes the utility of a trip maker selecting buses of O-D pair \(rs\); \(SD\) represents the financial deficit of the bus operator defined by Eq. (4); \(GL\) represents the acquisition cost of a hectare of green land area (NT dollars; 30 NT dollars is approximately equivalent to $1 USD), including land purchase or rent cost and treeplanting cost; \(B\) is the governmental budget for the generation (NT dollars); \(N\) is a set of the network nodes; \(L\) is a set of network links; \(A\) is the original green land area in terms of ecological footprints (ha) of the study network; \(EF\) is the ecological footprints produced by cars and buses defined by Eq. (5); \(FC\) is bus operating cost (NT dollars/bus-km); \(TC_b\) and \(TC_c\) denote the travel costs of selecting a bus (i.e. fare) and car (i.e. fuel cost and depreciation cost); \(R_b\) denotes a set of bus routes; \(d_a\) denotes the distance of link \(a\) (km); \(v_a\) denotes the traffic volume of link \(a\) (pcu/hr); \(r_k\) denotes the distance of bus route \(k\) (km). \(F_c\) and \(F_b\) are fuel efficiency of cars and buses, respectively (km/l); \(EC\) denotes the \(CO_2\) emission coefficient of fossil fuel (ton/l); and \(EL\) denotes an energy-to-land ratio, which is used to convert emitted \(CO_2\) into land area (ton/ha). The three decision variables are \(x\), \(y\), and \(z\). Where \(x\) denotes the fare discount for each bus passenger (NT dollars); \(y\) denotes the bus frequency (bus journey/hour); and \(z\) denotes the amount of green land acquisition (ha).

In the lower level, \(t_a\) denotes the free-flow travel time of link \(a\); \(C_a\) denotes the link capacity \(a\); \(l_c\) denotes the load factor of cars (persons/car); \(l_b\) denotes the capacity of buses (persons/bus); \(\delta_{a,k^{rs}}\) denotes an indicator with the value of 1 representing that route \(k\) contains link \(a\) and 0 else of O-D pair \(rs\); \(K^{rs}\) denotes a set of routes connecting O-D pair \(rs\); \(f_k\) denotes the traffic volume of route \(k\); \(IT_j\) and \(OT_j\) denote in-vehicle travel time and out-of-vehicle travel time, respectively; and \(j = b, c\) (b represents bus and c represents car). \(\alpha\), \(\beta\) and \(\gamma\) denote...
the coefficients of the mode choice model (Logit model) associated with in-vehicle travel time, out-of-vehicle travel time and travel cost.

Eq. (2) expresses the budget constraint. Three terms on the left hand side of the equation are the total amount of fare subsidy, frequency subsidy and green land acquisition cost. Eq. (3) stipulates that the total transportation footprints should be less than the original green land (A) and additionally acquired green land (z). Eqs. (4) and (5) are definitional equations used to determine SD and EF. In Eq. (4), the financial deficit of the bus company equals the total operating cost minus total fare box revenue. SD is set as zero in case that there is a positive profit (no need to provide deficit subsidy). Eqs. (6)–(8) constitute a user equilibrium model. Eq. (9) represents the capacity of bus system. A logit mode choice model is expressed by Eqs. (10)–(11). Eq. (12) assumes that the waiting time of bus passengers equals half of the bus inter-arrival time (=1/2×60/y=30/y).

2.4 AG Model

As for the above SG model, the regulator determines an optimal budget allocation plan for the contemporary generation. Without considering the interests of the next generation, the contemporary generation tends to spend all of the allocated budget and leave as much as footprint to the next generation. The underlying logic is obviously against the sustainability concept. Therefore, the proposed AG model also considers the opportunity cost of budget from the perspective of the next generation, which can be expressed as follows:

\[ \{ \text{Contemporary generation} \} \]

\[ \{ \text{Upper level} \} \]

\[
Max \ f = U^T - s_B (B_g - \Delta B_g) = \sum_{rscN} q^{rs} P_{c}^{rs} U_{c}^{rs} + \sum_{rscN} q^{rs} P_{b}^{rs} U_{b}^{rs} - s_B (B_g - \Delta B_g) \]  

(14)

Subject to

\[
(\sum_{rscN} q^{rs} P_{b}^{rs})x + SD + (GL)z \leq B_g \]  

(15)

\[ EF \leq A + z \]  

(16)

\[
SD = \begin{cases} 
(\sum_{k \in R_b} d_{k}y)FC - (\sum_{rscN} q^{rs} P_{b}^{rs})TC_b, & \text{if } (\sum_{k \in R_b} d_{k}y)FC \geq (\sum_{rscN} q^{rs} P_{b}^{rs})TC_b \\
0, & \text{else}
\end{cases} \]  

(17)

\[
EF = \left( \frac{\sum_{a \in L} d_{a}y}{F_c} + \frac{\sum_{k \in R_b} r_{k}y}{F_b} \right) \frac{EC}{EL} \]  

(18)

\[
\Delta B_g = B_g - \left( \sum_{rscN} q^{rs} P_{b}^{rs})x + SD + (GL)z \right) \]  

(19)

[Lower level]

Eqs. (8)–(15).

\[ \{ \text{Next generation} \} \]

[Upper level]
Max \( U^T = \sum_{rs} q^{rs} Pr^{rs} U_c^{rs} + \sum_{rs} q^{rs} Pr^{rs} U_b^{rs} \) \hspace{1cm} (20)

Subject to

\[
\left( \sum_{rs} q^{rs} Pr_b^{rs} \right) \cdot x + SD + (GL)z \leq B_{g+1} + \Delta B_g
\]

\( EF \leq A + z \) \hspace{1cm} (21)

\[
SD = \begin{cases} 
(\sum_{k \in K_b} d_k y) FC - (\sum_{rs} q^{rs} Pr_b^{rs}) TC_b, & \text{if } (\sum_{k \in K_b} d_k y) FC \geq (\sum_{rs} q^{rs} Pr_b^{rs}) TC_b \\
0, & \text{else}
\end{cases}
\] \hspace{1cm} (22)

\[
EF = \left( \frac{\sum_{a \in L} d_{a y}}{F_c} + \frac{\sum_{k \in K_b} r_{k y}}{F_b} \right) \frac{EC}{EL}
\] \hspace{1cm} (24)

[Lower level] Eqs. (6)~(13).

where, \( s_B \) denotes the shadow price of the budget of the next generation, implying that the each budget dollar left to the next generation increases the amount of total utility of the next generation. According to Eq. (14), a larger budget used in the contemporary generation implies a greater increase in the total utility of the contemporary generation \( (U^T) \), yet reduces the total utility of the next generation \( [s_B(B - \Delta B)] \). Notably, value \( s_B \) of the budget used varies, depending on how much budget remains for the next generation. An increasing shadow price implies a larger remaining budget \( (\Delta B) \), and vice versa.

3. SOLUTION ALGORITHMS

3.1 Solution Algorithm of the SG Model

Bi-level programming models can be converted into one-level optimization problems by at least three approaches: Implicit function theorems can be applied to derive a local description of function, the lower level problem can be replaced by its KKT conditions by a variational inequality, and the lower level objective can be replaced by an additional non-differentiable equation which is called Lipschitz continuous function. However, the proposed lower level model comprising user equilibrium route choice model and mode choice logit model, is too complex to be converted into the upper level. Thus, this study derives the SG model by using genetic algorithms (GAs). The decision variables of \( x, y, z \) are directly encoded by three consecutive genes with a value ranging from 0~9, implying that the values of the three decision variables range from 0~999. A penalty is subtracted from the total utility once the solution violates constraints. The max-min-arithmetic crossover and the non-uniform mutation in Chiu and Lan (2005) are used, as described briefly in the following:

(1) Max-min-arithmetic crossover

Let \( G_w = \{g_{w1}, \ldots, g_{wk}, \ldots, g_{wK} \} \) and \( G_i = \{g'_{i1}, \ldots, g'_{ik}, \ldots, g'_{iK} \} \) be two chromosomes selected for crossover, the following four offsprings will be generated:
\[ G_i^{t+1} = a G_i^t + (1-a) G_v^t \] (25)

\[ G_2^{t+1} = a G_v^t + (1-a) G_i^t \] (26)

\[ G_3^{t+1} \text{ with } g_{3k}^{t+1} = \min \{ g_{uk}^t, g_{vk}^t \} \] (27)

\[ G_4^{t+1} \text{ with } g_{4k}^{t+1} = \min \{ g_{uk}^t, g_{vk}^t \} \] (28)

where, \( a \) is a parameter \((0 < a < 1)\) and \( t \) is the number of generations.

(2) Non-uniform mutation

Let \( G = \{ g_1', \ldots, g_k', \ldots, g_k' \} \) be a chromosome and the gene \( g_k' \) be selected for mutation (the domain of \( g_k' \) is \([ g_k', g_k' ]\)), the value of \( g_k'^{t+1} \) after mutation can be computed as follows:

\[ g_k'^{t+1} = \begin{cases} g_k' + \Delta(t, g_k' - g_k') & \text{if } b = 0 \\ g_k' - \Delta(t, g_k' - g_k') & \text{if } b = 1 \end{cases} \] (29)

where \( b \) randomly takes a binary value of 0 or 1. The function \( \Delta(t, V) \) returns a value in the range of \([0, V]\) such that the probability of \( \Delta(t, V) \) approaches to 0 as \( t \) increases:

\[ \Delta(t, V) = V(1 - r^{(1-t/T)^h}) \] (30)

where \( r \) is a random number in the interval \([0, 1]\), \( T \) is the maximum number of generations and \( h \) is a given constant. In Eq. (30), the value returned by \( \Delta(t, V) \) will gradually decrease as the evolution progresses.

The solution algorithm of the AG model is stated as follows:

Step 0: Initialization. Generate an initial population with \( p \) chromosomes. Each gene randomly takes one integer from \([0, 9]\). Let’s \( IT_{ji}^n \) is the in-vehicle travel time of the shortest path for all O-D pairs under free-flow traffic condition. \( i = 1 \).

Step 1: Fitness calculation. For each of chromosomes, calculate its fitness value by the following sub-steps:

Step 1-1: Mode choice. Based on the values of \( x, y, z \) given by the chromosome and \( IT_{ji}^n \), use of the Logit model in Eqs. (10) and (11) to compute the choice probabilities of cars and buses of all O-D pairs (i.e. \( Pr_{b}^{n} \) and \( Pr_{c}^{n} \)).

Step 1-2: Route choice. Based on the choice probabilities of cars, use of traffic assignment algorithm of Chiou and Lai (2008) to determine the link flow and to updated the travel time \( IT_{ji}^{n+1} \).

Step 1-3: Convergence test. If \( \left| IT_{ji}^{n+1} - IT_{ji}^{n} \right| \leq \varepsilon \) for all O-D pairs, then compute the total utility by Eq. (1) and set it as the fitness value of the chromosome. Otherwise, let \( i = i+1 \) and go back to Step 1-1.

Step 2: Selection.

Step 3: Crossover.

Step 4: Mutation.

Step 5: Stop condition test. If the stop condition is satisfied, the incumbent solution is the optimal solution. If not, go back to Step 3.
3.2 Solution Algorithm of the AG Model

The value of shadow price of budget \((S_B)\) for the next generation varies, depending on how much budget that the contemporary generation leaves to the next one. An iterative solution algorithm is thus required to solve the budget allocation problems of the contemporary and next generations iteratively until the shadow price \((S_B)\) remains unchanged, as illustrated in Fig.2.

![Figure 2. Iterative solution algorithm of the AG model](image)

The solution algorithm is stated below:

{Contemporary generation}

Step 0: Let \(S_B^k = 0\) and \(k=1\).

Step 1-0: **Initialization.**
Step 1-1: **Fitness calculation.** For each of chromosomes, calculate its fitness value by solving the following sub-steps:
  - Step 1-1-1: **Mode choice.**
  - Step 1-2-1: **Route choice.**
  - Step 1-3-1: **Convergence test.** If yes, then compute the fitness value by Eq. (14) (instead of Eq. (1)). Otherwise, let \(i = i+1\) and go back to Step 1-1-1.

Step 1-2: **Selection.**
Step 1-3: **Crossover.**
Step 1-4: **Mutation.**
Step 1-5: **Stop condition test.** If the stop condition is satisfied, compute \(\Delta B\), go to Step 6.
Otherwise, go back to Step 1-3.

{Next generation}

Step 2-0: **Initialization.**
Step 2-1: **Fitness calculation.** For each of chromosomes, calculate its fitness value by solving the following sub-steps:
  - Step 2-1-1: **Mode choice.**
  - Step 2-1-2: **Route choice.**
  - Step 2-1-3: **Convergence test.** If yes, then compute the fitness value by Eq. (20) (the same as Eq. (1)). Otherwise, let \(i\)=\(i+1\) and go back to Step 2-1-1.

Step 2-2: **Selection.**
Step 2-3: **Crossover.**
Step 2-4: **Mutation.**
Step 2-5: **Stop condition test.** If the stop condition is satisfied, compute the shadow price of budget ($S_{B}^{k+1}$). If not, go back to Step 2-3.

Step 3: **Convergence test.** If $|S_{B}^{k+1} - S_{B}^{k}| \leq \varepsilon$, then terminate. The incumbent solutions of the contemporary and next generations are optimal. Otherwise, let $k = k+1$ and go back to Step 1-0.

4. CASE STUDY

This work presents a case study on an exemplified network introduced in Yang and Lam (1996) to investigate the applicability of the proposed models and analyze the effects of parameters. Details of the network and results of the budget allocation are described below.

4.1. Parameter settings

The network contains six nodes and seven links, as (shown OR illustrated) in Fig. 3. Table 1 lists the free-flow travel time, capacity and distance of the links. The trip demands of four OD pairs of 1, 3 and 2, 4 are both set as 200 trip/hr. Assume that only one public transportation service is available: bus and one private vehicle type: car. Four bus routes are operated: Bus route 1 (BR1) from Node 1 to Node 3 through Link 1, BR2 from Node 1 to Node 3 through Links 3, 4, and 6, BR 3 from Node 2 to Node 4 through Links 5, 4, 7 and BR 4 from Node 2 to Node 4 through Link 2. BR1 and BR 2 connect OD pair of 1, 3 and BR3 and BR 4 connect OD pair of 2, 4.

![Figure 3. Network of the exemplified example (Yang and Lam, 1996)](image)

<table>
<thead>
<tr>
<th>Link</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_u$ (min)</td>
<td>8</td>
<td>9</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$C_a$ (veh/hr)</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>40</td>
<td>20</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>$d_u$ (km)</td>
<td>8</td>
<td>9</td>
<td>2</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Tables 2 and 3 give the parameter settings of the models and of bus and car users, respectively.

4.2 Results of the SG Model

According to the formulation of the SG model, three policy variables are bus fare discount ($x$),
bus frequency increase \((y)\), and green land acquisition \((z)\). Exactly how different combinations of these policy decisions affect the objective values is analyzed by proposing and comparing four strategies:

Strategy 0 (S0): Do nothing. (given \(x=0, y=5\) and \(z=0\))

Strategy 1 (S1): Provide bus fare discount \((x)\) + green land acquisition \((z)\) (given \(y=5\)).

Strategy 2 (S2): Provide bus frequency increase \((y)\) + green land acquisition \((z)\) (given \(x=0\)).

Strategy 3 (S3): Provide bus fare discount \((x)\) + bus frequency increase \((y)\) + green land acquisition \((z)\).

Table 2. Parameter settings of the models

<table>
<thead>
<tr>
<th>Item</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget (million NT dollars)</td>
<td>(B)</td>
<td>100</td>
</tr>
<tr>
<td>Original green land area (ha)</td>
<td>(A)</td>
<td>50</td>
</tr>
<tr>
<td>Energy-to-land ratio (ton/ha)</td>
<td>(EL)</td>
<td>6.6</td>
</tr>
<tr>
<td>(CO_2) emission coefficient</td>
<td>(EC)</td>
<td>2.24</td>
</tr>
<tr>
<td>Green land cost (NT dollars/ha)</td>
<td>(GL)</td>
<td>60000</td>
</tr>
<tr>
<td>Parameter of (IT_j)</td>
<td>(\alpha)</td>
<td>0.2</td>
</tr>
<tr>
<td>Parameter of (OT_j)</td>
<td>(\beta)</td>
<td>0.216</td>
</tr>
<tr>
<td>Parameter of (TC_j)</td>
<td>(\gamma)</td>
<td>0.0803</td>
</tr>
</tbody>
</table>

Sources: \(^a\) Lin et al. (2001); \(^b\) Wackernagel et al. (1999); \(^c\) Chiou et al. (2009).

Table 3. Parameter settings associated with bus and car

<table>
<thead>
<tr>
<th>Item</th>
<th>Parameter</th>
<th>Car</th>
<th>Bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out-of-vehicle time (min)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Walking time</td>
<td>(l_j)</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>Parking time</td>
<td>(F_j)</td>
<td>11</td>
<td>2.88</td>
</tr>
<tr>
<td>Waiting time</td>
<td>(TC_j)</td>
<td>30</td>
<td>45</td>
</tr>
</tbody>
</table>

Sources: \(^a\) Su et al. (2005); \(^b\) Feng et al. (2001).

Table 4 compares the performances of the four strategies. According to Table 4, in contrast with the do nothing strategy (S0), the total utility and ecological footprint are largely curtailed. Interestingly, although both S1 and S2 can achieve nearly the same total utility and mode choice share, S1 can produce a significantly lesser footprint than S2, because S1 provides bus fare discount of 34 NT dollars for each bus passenger, accounting for 76% of the full fare of 45 NT dollars to increase bus patronage. Meanwhile, S2 largely increases the bus frequency (from 5 bus journeys per hour to 12 bus journeys) to attract bus passengers but results in a significantly heavier footprint due to the high emission characteristic of buses. The most flexible strategy (S3) can achieve the highest total utility and sustain the footprint at a relatively low level.

Table 4. Comparisons of four strategies

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Decision variable</th>
<th>Budget allocation</th>
<th>Market share</th>
<th>EF</th>
<th>(U^f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>S1</td>
<td>Do nothing.</td>
<td>5</td>
<td>0</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>S2</td>
<td>Provide bus fare</td>
<td>5</td>
<td>5</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>
4.3 Results of the AG Model

To further consider the welfare of the next generation, the AG model introduces the shadow price of budget into the objective function. Table 5 summarizes the results of the AG model, indicating that the total utility of the contemporary generation is lowered more than that of the SG model. This is owing to that 24,104,730 NT dollars is left to the next generation with consideration of the shadow price of 74.829 (for one million NT dollars). Although the total utility of the next generation surpasses that of the contemporary generation, the total utility is lowered once the next generation considers the welfare of the following generation. Notably, the budget allocated to bus frequency (deficit subsidy) for the contemporary generation equals zero since the fare box revenue exceeds the operating cost. Furthermore, most of the budget still goes to the fare discount subsidy, similar to the results of the SG model.

Table 5. Results of the AG model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Contemporary generation</th>
<th>Next generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>Percentage</td>
<td>Value</td>
</tr>
<tr>
<td>$x$</td>
<td>22</td>
<td>47%</td>
</tr>
<tr>
<td>$y$</td>
<td>10</td>
<td>17%</td>
</tr>
<tr>
<td>$z$</td>
<td>202</td>
<td>12%</td>
</tr>
<tr>
<td>$B-\Delta B$ (NT dollars)</td>
<td>75,895,270</td>
<td>75.9%</td>
</tr>
<tr>
<td>$\Delta B$ (NT dollars)</td>
<td>24,104,730</td>
<td>24.1%</td>
</tr>
<tr>
<td>$U^T$</td>
<td>-3,904</td>
<td>-2,456</td>
</tr>
<tr>
<td>$S_B$</td>
<td></td>
<td>74.829</td>
</tr>
</tbody>
</table>

5. CONCLUDING REMARKS

This work develops two bi-level programming models, i.e. the SG model and the AG model, to maximize total utility (a proxy of social welfare) under constraints of budget, capacity of transport system and ecological footprint. Three policy measures are also considered: bus fare discount, bus frequency increase, and green land acquisition. The former two measures attempt to increase the attractiveness of bus, while the last one is simply accommodates the excess footprint. A case study on an exemplified network demonstrates the applicability of the proposed models. Results of the SG model indicate that the measure of bus fare discount and the measure of bus frequency increase can both attract remarkable percentage of bus usage, as well as achieve nearly the same amount of social welfare benefits. However, the latter generates a significantly larger footprint than that of bus fare discount due to the high emission characteristic of buses. Results of the AG model demonstrate that the optimal decision of the contemporary generation compromise its total utility with that of the next generation by intentionally leaving part of budget to the next generation.

Despite its contributions, this work has several limitations, pointing the way for future research. For simplicity, this work considers only one public transportation type, i.e. bus and
one private vehicle type, i.e. car. Other modes, such as air transportation, railway, motorcycle, and bicycle, should be considered. Additionally, other governmental measures, such as transportation infrastructure construction plan, congestion toll, parking fee, should be introduced. Moreover, the proposed models should be applied to a field case so that more fruitful and practical policy suggestions can be made. Finally, due to the computation constraints, this work considers only interactions (in terms of shadow price) between two consecutive generations. More future generations should thus be incorporated. However, doing so would require a more complex solution algorithm.

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