A Heuristic Solution Algorithm for the Combined Model of the Four Travel Choices with Variable Demand

Huey-Kuo CHEN

Graduate School of Civil Engineering, National Central University, Taoyuan, 32001, Taiwan
E-mail: ncutone@ncu.edu.tw

Abstract: In this paper, the problem that consists of the four travel choices (i.e., trip generation, trip distribution, modal split, and traffic assignment) with variable demand is deliberately formulated as a non-linear mathematical model. The associated optimality conditions are presented. A heuristic solution algorithm that adopts combined model concept was proposed. By way of supernetwork techniques, each combined model can be addressed as an “extended” traffic assignment problem and solved using any available traffic assignment solution algorithm. As compared with the traditional four-step travel demand forecasting procedure, this heuristic has advantage of considering the travelers’ travel behavior throughout the solution procedure and hence more suitable to be applied to the transportation planning applications in the real world.

Keywords: Traffic Assignment, Combined Model, Supernetwork, Variable Demand

1. INTRODUCTION

Travel demand forecasting that consists four steps, i.e., trip generation, trip distribution, modal split, and traffic assignment, is essentially the core module and hence critical to the success of a transportation planning project. In the past, these four steps have been performed using a top-down sequential procedure. The drawback associated with this sequential procedure is mainly due to its internal inconsistency between the aforementioned four modules. For example, the costs used to determine origin-destination demands are generally different from those generated from the later traffic assignment. To remedy this drawback, “feedback” operation is introduced into the traditional sequential travel demand forecasting procedure and different combinations of weights between two consecutive iterations were deliberately experimented (Boyce and Bar-Gera, 2003, 2004; Boyce, Lupa and Zhang, 1994; Boyce, O’Neill and Scherr, 2007). The results obtained from this “feedback” sequential procedure are better than those from the traditionally sequential procedure, the problem of internal inconsistencies, though to a lesser extent, remains to be further improved.

It is observed that in the sequential travel demand forecasting procedure, the key factors, mainly travel costs, were used to determine the travel patterns in the first three steps, i.e., trip generation, trip distribution and modal split. Unfortunately, these travel costs are externally provided and kept unchanged in each iteration. Indeed this treatment fails to reflect real situations in which travel costs are subject to change depending on the traffic situations. In other words, in each module, traffic demands and travel costs are interacted until an equilibration state (Wardrop, 1952) has been reached. This idea of solving each module using internally generated travel costs naturally leads us to develop an improved version of the traditional sequential travel demand forecasting procedure which is called heuristic solution algorithm hereafter for convenience.
This paper is organized as follows. Section 2 presents a nonlinear mathematical model for the four travel choices (i.e., trip generation, trip distribution, modal split, and traffic assignment) with variable demand. Optimality conditions are also derived. Section 3 proposes the heuristic solution algorithm and extensively elaborated step by step. Section 4 concludes the paper with few remarks.

2. MODEL FORMULATION

In travel demand forecasting, all of four travel choices, i.e., trip generation, trip distribution, model split and traffic assignment must be considered (Safwat and Magnanti, 1988; Sheffi and Daganzo, 1980; Zhou, Chen, Wong, 2009). In certain circumstances, additional dimension due to variable traffic demand, which can be expressed as a function of minimum travel costs, must be also taken into account (Chen, 2011, 2013, 2014). Putting all of these together, the most complicated travel demand model, called the combined model of the four travel choices with variable demand, results. This problem can be formulated as a nonlinear mathematical model as follows.

\[
\min_{\{x_{m,l}, q^m, q^r, q^s\}} z = \sum_r \sum_{m,l} g^m_{rs} c_{ml}(\omega) d\omega + \sum_r \sum_{m,l} g^r_{rs} c^r_{ml}(\omega) d\omega
\]

\[
+ \sum_r \sum_{m,l} q^s_{rs} c^s_{ml}(\omega) d\omega + \sum_r \sum_{m,l} q^r_{rs} c^r_{ml}(\omega) d\omega - \int_0^1 D^{-1}(\omega) d\omega
\]

The associated feasible region \( \Omega \) is defined by the following constraints.

Flow conservation constraints:
\[
\sum_p f^{rs}_{mp} = q^s_{mr} \quad \forall r,s,m \quad (\mu_m^s)
\]

\[
\sum_m q^s_{mr} = q^r \quad \forall r,s \quad (\mu^r)
\]

\[
\sum_r q^r = q \quad (\mu)
\]

\[
q + e = q_{\text{max}}
\]

Non-negativity constraints:
\[
f^{rs}_{mp} \geq 0 \quad \forall r,s,m,p
\]

Definitional constraints:
\[
x_{ml} = \sum_r \sum_{m,l} f^{rs}_{mp} \delta_{mlp} \quad \forall m,l
\]

\[
c^{rs}_{mp} = \sum_q c_{ml} \delta_{mlp} \quad \forall r,s,m,p
\]

where,
\( c_{ml} \) : travel cost function associated with link \( l \) and mode \( m \)
\( c^r \) : dummy travel cost function for trips generating from origin \( r \)
\( c^s \) : dummy travel cost function for trips between origin \( r \) and destination \( s \)
\( c^s_{ml} \) : dummy travel cost function for trips between O-D pair \( rs \) by mode \( m \)
\( D^{-1}(q) \) : inverse demand function; equivalent to the excess demand function \( E(e) \)
\[ e \] : excess flow (non-travelers’ demand) associated with the entire area,
\[ e = \bar{q}_{\text{max}} - q \]

\[ f_{mp}^{rs} \] : flow on path \( p \) by mode \( m \) between O-D pair

\[ q \] : total OD trip rate from the entire area

\[ \bar{q}_{\text{max}} \] : upper limit of total OD trip rate from the entire area

\[ q' \] : total OD trip rate from origin \( r \)

\[ q'^{rs} \] : trip rate between origin \( r \) and destination \( s \)

\[ q''^{rs} \] : total flow by mode \( m \) between origin \( r \) and destination \( s \)

\[ x_{ml} \] : flow on link \( l \) by mode \( m \)

\[ \delta_{mlp}^{rs} \] : 1, if link \( l \) associated with mode \( m \) is on path \( p \) between O-D pair \( rs \); 0, otherwise.

\[ \mu \] : dual variable associated with Eq. (1e)

\[ \mu' \] : dual variable associated with Eq. (1d)

\[ \mu'^{rs} \] : dual variable associated with Eq. (1c)

\[ \mu''_{m} \] : dual variable associated with Eq. (1b)

(1a) minimizes the objective function \( z \) which is defined as the sum of integrals of “real” cost functions for physical links, and “virtual” cost functions for dummy links that are associated with modal choice, trip distribution, trip origin as well as variable demand, respectively. (1b) conserves flows by mode \( m \) between O-D pair \( rs \). (1c) conserves flows between O-D pair \( rs \). (1d) conserves flows generated at origin \( r \). (1e) conserves total flows for the entire area. (1f) sets the upper limit \( \bar{q}_{\text{max}} \) for total flows for the entire area. The excess demand \( e \) is defined as non-travelers. (1g) requires flows \( f_{mp}^{rs} \) in path \( p \) by mode \( m \) between O-D pair \( rs \) be negative. (1h)-(1i) define the relationships between link and path variables through the incidence variable \( \delta_{mlp}^{rs} \).

In general, route choice is made more based on deterministic traffic information. However, the other travel choices, i.e., trip generation, trip distribution and model split are assumed in compliance with stochastic traffic information by which a logit formula is usually adopted for making decisions. To this end, the virtual cost functions that are associated with these three travel choices can be hypothesized, respectively, as follows.

\[ c'^{r} = \frac{1}{\eta} \ln q'^{r} - M'^{r} \]  
(2a)

\[ c'^{rs} = \frac{1}{\gamma} \ln q'^{rs} - M'^{rs} \]  
(2b)

\[ c''_{m}^{rs} = \frac{1}{\theta} \ln q''_{m}^{rs} - M''_{m}^{rs} \]  
(2c)

where,

\( M'^{r} \) : trip production capability associated with origin \( r \)

\( M'^{rs} \) : attractive interaction between O-D pair \( rs \)
\( M^r_m \): preference measure associated with mode \( m \) between O-D pair \( rs \)

\( \theta, \gamma, \eta \): dispersion parameters associated with modal choice, trip distribution and trip generation, respectively, in the logit function

With the virtual travel costs assumed in (2a)-(2c), the optimality conditions corresponding to the combined model (1a) can be derived as follows.

\[
D^1(q) = \mu \text{ if } q > 0 \\
\leq \mu \text{ if } q = 0
\]

(3a)

\[
q' = q \cdot e^{-\eta(\mu' - M')}
\]

(3b)

\[
q'' = q' \cdot e^{-\gamma(\mu'' - M'')}
\]

(3c)

\[
q^m'' = q'' \cdot e^{-\gamma(\mu^m'' - M^m'')}
\]

(3d)

\[
e^r_m = f_{mp} > 0 \\
\geq f_{mp} \text{ if } f_{mp} = 0
\]

(3e)

(3a) indicates that for the entire research area, trips can occur only when the “shortest” travel cost \( \mu \) is less than the threshold \( D^1(0) \). (3b)-(3d) state that trip generation, trip distribution and model split are in compliance with logit formulas. (3e) is essentially a reminiscent of Wardrop user principle decomposed by mode \( m \).

### 3. SOLUTION ALGORITHM

The TG/TD/MC/TA/VD combined model (1a) is not easy to be solved because the flow conservation for trip generation, trip distribution, model choice, as well as variable demand constraint must be satisfied simultaneously which is indeed not easy. Here we propose a heuristic solution algorithm which adopts the traditional four-step travel demand forecasting procedure but at each step the associated travel costs are internally determined, rather than being externally provided. This can be achieved by solving at each step a single travel choice problem along with the traffic assignment problem, resulting in a two-step combined model (Abdulaal and LeBlanc, 1979; Boyce and Bar-Gera, 2003; Boyce and Bar-Gera, 2004; Florian and Nguyen, 1978; Florian, Nguyen, and Ferland, 1975; Lundgren and Patriksson, 1998; Larsson, 1992; Jörnsten, 1980; LeBlanc and Farhangian, 1981; Sheffi, 1985). We now formally describe the steps of the proposed heuristic solution algorithm as follows:

**Step 0:** input traffic data including upper limit of trips for the entire traffic area, traffic demand function for the entire study area, virtual costs associated with trip generation, trip distribution, and modal split, respectively, as well as link data such as free flow link travel costs, link capacities and link cost functions.

**Step 1:** solve the area-wide trip generation problem with variable demand (VD/TA), resulting in the trip rates \( q \) for the entire area.
Step 2: solve the trip generation and assignment (TG/TA), resulting in trip rates for each origin \( \{q^r\} \).

Step 3: solve the trip distribution and assignment (TD/TA), resulting in O-D trip rates \( \{q^{rs}\} \).

Step 4: solve the modal split and assignment (MC/TA), resulting in the O-D trip rates by mode \( \{x^m_{rq}\} \) and mode-specific link flows by \( \{x_{ml}\} \).

Step 5: stop.

For each of Steps 1-3, one two-step combined model must be constructed and solved. In the following sections, we will formulate each of these three combined model in sequel. Noted that by way of supernetwork representation technique, these three combined models can be addressed as an “extended” traffic assignment problem and easily solved by any suitable solution algorithm such as Frank-Wolfe method (Frank and Wolfe, 1956).

### 3.1 Mathematical Formulation and Supernetwork Representation for the VD/TA Combined Model

The VD/TA combined model which solves for the entire area demand \( q \) and link flows \( \{x_l\} \) can be formulated as follows.

\[
\min_{\{x_l, q\} \in \Omega_4} z = \sum_{q=1}^q c_i(\omega)d\omega - \int_0^q D^{-1}(\omega)d\omega
\]  

(4a)

The feasible region \( \Omega_4 \) is defined by the following constraints.

Flow conservation constraints:
\[
\sum_{p} f_{OD}^{\tilde{p}} = q
\]  

(4b)

\[
qu + e = \tilde{a}_{\max}
\]  

(4c)

Non-negativity constraints:
\[
f_{OD}^{\tilde{p}} \geq 0 \quad \forall \tilde{p}
\]  

(4d)

Definitional constraints:
\[
\begin{align*}
x_l &= \sum_{p} f_{OD}^{\tilde{p}} \delta_{OD}^{\tilde{p}} \quad \forall l \\
\hat{c}_{OD}^{\tilde{p}} &= \sum_{l} c_l \delta_{OD}^{\tilde{p}} \quad \forall \tilde{p}
\end{align*}
\]  

(4e)  

(4f)

where, \( c_l \) : cost function for link \( l \)

\( \hat{c}_{OD}^{\tilde{p}} \) : cost for path \( \tilde{p} \) between superorigin \( O \) and superdestination \( D \)

\( D^{-1}(q) \) : inverse demand function for the entire study area

\( f_{OD}^{\tilde{p}} \) : flow in path \( \tilde{p} \) between superorigin \( O \) and superdestination \( D \)

\( \tilde{p} \) : path designation in the associated supernetwork

\( q \) : total trips generated for the entire study area

\( x_l \) : flow on link \( l \)
\[ \delta_{p}^{OD} : \begin{cases} 1, & \text{if link } l \text{ is on path } \tilde{p} \text{ between superorigin } O \text{ and superdestination } D; \\ 0, & \text{otherwise} \end{cases} \]

The supernetwork representation associated with model (4a) is presented in Figure 1.

\text{TA: traffic assignment; VD: variable demand}

\text{O: superorigin; D: superdestination}

\text{r: origin; s: destination}

\text{Figure 1. Supernetwork representation for the VD/TA combined model}

The virtual link cost functions used in the supernetwork (Figure 1) are assumed as follows:

\[ c^{OD} : \text{cost function for excess flow link } OD, \quad c^{OD} = D^{-1}(q) \]
\[ c_{O \rightarrow r} : \text{cost function for link } O \rightarrow r, \quad c_{O \rightarrow r} = 0 \]
\[ c_{s \rightarrow D} : \text{cost function for link } s \rightarrow D, \quad c_{s \rightarrow D} = 0 \]

3.2 Mathematical Formulation and Supernetwork Representation for the TG/TA Combined Model

The TG/TA combined model which solves for trip generations \( \{q'\} \) and link flows \( \{x_{i}\} \) can be formulated as follows.

\[ \min_{\{x_{i}, q'\} \in \Omega_2} z = \sum_{i} \int_{0}^{\omega_0} c_{i}(\omega) d\omega + \sum_{i} \int_{0}^{\omega_0} c'_{i}(\omega) d\omega \quad (5a) \]

The feasible region \( \Omega_2 \) is defined by the following constraints.

Flow conservation constraints:
\[ \sum_{p} f_{p}^{OD} = \bar{q} \quad (5b) \]

Non-negativity constraints:
\[ f_{p}^{OD} \geq 0 \quad \forall \tilde{p} \quad (5c) \]
Definitional constraints:
\[
x_l = \sum_{\tilde{p}} f_{\tilde{p}}^{OD} \delta_{\tilde{p}}^{OD} \quad \forall \tilde{p}
\]
\[
s_{\tilde{p}}^{OD} = c^r + \sum_{\tilde{p}} c^r \delta_{\tilde{p}}^{OD} \quad \forall r, \tilde{p}, p \in \tilde{p}
\]

where,
\[
c_l : \text{cost function for link } l
\]
\[
c_{O \rightarrow r} : \text{cost function for link } O \rightarrow r, \quad c_{O \rightarrow r} = c^r
\]
\[
c^r : \text{virtual cost function associated with trips generated at origin } r,
\]
\[
c^r = \frac{1}{\eta} \ln q^r - M^r
\]
\[
c_{s \rightarrow D} : \text{cost function for link } s \rightarrow D, \quad c_{s \rightarrow D} = 0
\]
\[
s_{\tilde{p}}^{OD} : \text{cost for path } \tilde{p} \text{ between superorigin } O \text{ and superdestination } D \text{ via origin } r
\]
\[
f_{\tilde{p}}^{OD} : \text{flow in path } \tilde{p} \text{ between superorigin } O \text{ and superdestination } D
\]
\[
M^r : \text{trip production capability associated with origin } r
\]
\[
\tilde{p} : \text{path designation in the associated supernetwork}
\]
\[
q^r : \text{trips generated at origin } r
\]
\[
x_l : \text{flow on link } l
\]
\[
\delta_{\tilde{p}}^{OD} : 1, \text{ if link } l \text{ is on path } \tilde{p} \text{ between superorigin } O \text{ and superdestination } D; 0, \text{ otherwise}
\]

The supernetwork representation associated with model (5a) is presented in Figure 2a. The trips \( q^r \) generated at each origin \( r \) is equal to the flow on dummy link \( O \rightarrow r \). For example, flow on link \( O \rightarrow 1 \) denotes the trips generated at \( r=1 \), i.e., \( q^r=1 \).

\( r \): origin; \( s \): destination; \( O \): superorigin; \( D \): superdestination

TA: traffic assignment; TG: trip generation

Figure 2a. Supernetwork representation for the TG/TA combined model
The virtual link cost functions used in the supernetwork (Figure 2a) are assumed as follows:

\[ c_{O \rightarrow r} : \text{cost function for dummy link } O \rightarrow r, \quad c_{O \rightarrow r} = c' = \frac{1}{\eta} \ln q' - M' \]

\[ c_{s \rightarrow D} : \text{cost function for dummy link } s \rightarrow D, \quad c_{s \rightarrow D} = 0 \]

\[ \eta : \text{dispersion parameter associated with trip generation in the logit model} \]

If the relevant part for trip generation in model (5a) is replaced by the corresponding trip attraction component, the above combined TG/TA model will become combined trip attraction and traffic assignment model (TAT/TA). The corresponding supernetwork represented is drawn in Figure 2b. The trips \( q' \) attracted to each origin \( s \) is equal to the flow on dummy link \( s \rightarrow D \). For example, flow on link \( 1 \rightarrow D \) denotes the trips attracted to at \( s=1 \), i.e., \( q^1 \).

\[ r: \text{origin}; \quad s: \text{destination}; \quad O: \text{superorigin}; \quad D: \text{superdestination} \]

\[ \text{TA: traffic assignment; TAT: trip attraction} \]

The virtual link cost functions used in the supernetwork (Figure 2b) are assumed as follows:

\[ c_{O \rightarrow r} : \text{cost function for dummy link } O \rightarrow r, \quad c_{O \rightarrow r} = 0 \]

\[ c_{s \rightarrow D} : \text{cost function for dummy link } s \rightarrow D, \quad c_{s \rightarrow D} = c' = \frac{1}{\eta} \ln q' - M' \]

\[ M' : \text{attractiveness associated with destination } s \]

\[ \eta' : \text{dispersion parameter associated with trip attraction in the logit model} \]
3.3 Mathematical Formulation and Supernetwork Representation for the TD/TA Combined Model

The TD/TA combined model which solves for O-D trip demands \( \{ q_{rs} \} \) and link flows \( \{ x_i \} \) can be formulated as follows.

\[
\begin{align*}
\min_{\{x_i,q''_{rs}\} \in \Omega_3} z &= \sum_i \int_0^{\gamma_i} c_i(\omega) d\omega + \sum_{s} \sum_{r} \int_0^{\gamma_{rs}} c^{rs}(\omega) d\omega \\
\end{align*}
\]

(6a)

The feasible region \( \Omega_3 \) is defined by the following constraints.

Flow conservation constraints:
\[
\sum_{\bar{p}} f_{\bar{p}}^{rs} = \bar{q}' \quad \forall r
\]

(6b)

Non-negativity constraints:
\[
f_{\bar{p}}^{rs} \geq 0 \quad \forall (r, \bar{r}), \bar{p}
\]

(6c)

Definitional constraints:
\[
x_i = \sum_{(r,\bar{r})} \sum_{\bar{p}} f_{\bar{p}}^{rs} \delta_{\bar{p}}^{rs} \quad \forall l
\]

(6d)

\[
c_{\bar{p}}^{\bar{r}(s)} = \sum c_{i} \delta_{\bar{p}}^{rs} + c^{rs} \quad \forall (r, \bar{r}(s)), s, \bar{p}, p \in \bar{p}
\]

(6e)

where,
\[
c_{\bar{p}}^{\bar{r}(s)} \quad : \text{cost for path } \bar{p} \text{ between origin } r \text{ and superdestination } \bar{r} \text{ via destination } s
\]

\[
c^{rs} \quad : \text{virtual cost function associate with trip demand between O-D pair } rs,
\]

\[
c^{rs} = \frac{1}{\gamma} \ln q^{rs} - M^{rs} \quad \forall r, s
\]

\( M^{rs} \quad : \text{attractive interaction between origin } r \text{ and destination } s
\]

\( \bar{p} \quad : \text{path designation in the associated supernetwork}
\]

\( \bar{q}' \quad : \text{trips generated at origin } r
\]

\( \bar{r} \quad : \text{superdestination designation}
\]

\( \delta_{\bar{p}}^{rs} \quad : 1, \text{ if link } l \text{ is on path } \bar{p} \text{ between origin } r \text{ and destination } s; 0, \text{ otherwise}
\]

\( \delta_{\bar{p}}^{\bar{r}} \quad : 1, \text{ if link } l \text{ is on path } \bar{p} \text{ between origin } r \text{ and superdestination } \bar{r}; 0, \text{ otherwise}
\]

The supernetwork representation associated with model (6) is presented in Figure 3. The trips \( q^{rs} \) between O-D \( rs \) is equal to the flow on dummy link \( s \rightarrow \bar{r} \). For example, flow on link \( 3 \rightarrow \bar{1} \) denotes the trips between origin 1 and destination 3, i.e., \( q^{13} \).
The virtual link cost functions used in the supernetwork (Figure 3) are assumed in the following:

\[ c_{s \rightarrow r} \] : cost function for dummy link \( s \rightarrow r \), \( c_{s \rightarrow r} = \frac{1}{\gamma} \ln q^{rs} - M^{rs} \)

\( \gamma \) : dispersion parameter associated with trip distribution in the logit model

3.4 Mathematical Formulation and Supernetwork Representation for the MC/TA Combined Model

The MC/TA combined model which solves for mode-specific O-D trip demands \( \{q_{rs}\} \) and link flows \( \{x_{ml}\} \) can be formulated as follows.

\[ \min_{\{q_{rs}, x_{ml}\} \in \Omega_4} z = \sum_m \sum_l \int_0^{q_{ml}} c_{ml}(\omega)d\omega + \sum_r \sum_s \sum_{m} \int_0^{q_{rs}} c_{rs}^{rs}(\omega)d\omega \]  (7a)

The associated feasible region \( \Omega_4 \) is defined by the following constraints.

Flow conservation constraints:
\[ \sum_m \sum_p f_{mp}^{rs} = \bar{q}^{rs} \quad \forall r, s \]  (7b)

Non-negativity constraints:
\[ f_{mp}^{rs} \geq 0 \quad \forall r, s, m, p \]  (7c)

Definitional constraints:
\[ x_{ml} = \sum_r \sum_s \sum_p f_{mp}^{rs} \delta_{mlp}^{rs} \quad \forall m, l \]  (7d)
\[ \tilde{c}_{mp} = \sum_l c_{ml} \delta_{mlp}^{rs} + c_{m}^{rs} \quad \forall r, s, m, p \]  (7e)
where,

\( c^{rs}_m \) : virtual cost between O-D pair \( rs \) by mode \( m \), \( c^{rs}_m = \frac{1}{\theta} \ln q^{rs}_m - M^{rs}_m \)

\( c^{rs}_{mp} \) : real cost for path \( p \) between O-D pair \( rs \) by mode \( m \)

\( \bar{c}^{rs}_{mp} \) : perceived cost for path \( p \) between O-D pair \( rs \) by mode \( m \)

\( M^{rs} \) : demand between O-D pair \( rs \)

\( \delta^{rs}_{mpl} \) : 1, if link \( l \) is on path \( \tilde{p} \) between origin \( r \) and destination \( s \) by mode \( m \); 0, otherwise

When only two transportation modes, e.g., automobile (\( m=A \)) and transit (\( m=T \)) exist, the supernetwork representation associated with model (7a) can be drawn in Figure 4. To make mode choice decision, the perceived cost considered between O-D pair \( rs \) must include both the “real” cost and virtual mode choice cost \( c^{rs}_m \). Suppose mode split follows binomial logit formula, then the virtual mode cost for automobile and transit are, respectively,

\( c^{rs}_A = \frac{1}{\theta} \ln q^{rs}_A - M^{rs}_A \) and \( c^{rs}_T = \frac{1}{\theta} \ln q^{rs}_T - M^{rs}_T \). Furthermore, if we take automobile as the reference mode, and the corresponding cost is 0, then the “relative” costs for all the transportation modes can be expressed as follows.

\[
\bar{c}^{rs}_A = 0 \quad \forall r,s
\]

\[
\bar{c}^{rs}_T = \left( \frac{1}{\theta} \ln q^{rs}_T - M^{rs}_T \right) - \left( \frac{1}{\theta} \ln q^{rs}_A - M^{rs}_A \right) = \frac{1}{\theta} \ln \left( \frac{q^{rs}_T}{q^{rs}_A} \right) \left( M^{rs}_T - M^{rs}_A \right) \quad \forall r,s
\]

where,

\( M^{rs}_m \) : attractive interaction between O-D pair \( rs \) by mode \( m \)

\( (M^{rs}_T - M^{rs}_A) \) : transit preference parameter between O-D pair \( rs \)

\( \theta \) : dispersion parameter associated with mode choice in the logit model
The virtual link cost functions used in the supernetwork (Figure 4) are assumed as follows:

\[ \bar{c}_{rs}^{T} = c_{rs}^{T} + \frac{1}{\theta} \ln \frac{q_{rs}^{T}}{q_{\lambda}^{T}} \left( M_{T}^{rs} - M_{\lambda}^{rs} \right) \]

\[ \frac{1}{\theta} \ln \frac{q_{rs}^{T}}{q_{\lambda}^{T}} \left( M_{T}^{rs} - M_{\lambda}^{rs} \right) \] : "relative" virtual preference cost for mode transit between O-D pair \( rs \)

\[ \bar{c}_{r}^{T} \] : real travel cost for transit link connecting origin \( r \) and destination \( s \)

4. NUMERICAL EXAMPLE

We take a simple TD/TA combined model with 2-OD, 6-link, 5-node network (Figure 5a) for testing.

Figure 5. Numerical example of TD/TA combined model
Assume the total trips generated at origin \( r \) is 4 units, i.e., \( q_r = 4 \), attractive interaction between O-D pairs are 0, i.e., \( M_{ij}^r = 0, M_{ij}^r = 0 \), and the dispersion parameter of logit model is \( \gamma = 1.0 \). The cost functions for the six links are assumed as follows.

\[
\begin{align*}
    c_1 &= 0.25 + x_1 \quad \text{(8a)} \\
    c_2 &= 0.5 + x_2 \quad \text{(8b)} \\
    c_3 &= 1 + 2x_3 \quad \text{(8c)} \\
    c_4 &= 2 + x_4 \quad \text{(8d)} \\
    c_5 &= 1 + 0.5x_5 \quad \text{(8e)} \\
    c_6 &= 1 + 0.75x_6 \quad \text{(8f)}
\end{align*}
\]

This TD/TA combined model can be addressed as an extended traffic assignment model by way of the supernetwork representation depicted in Figure (5b). The virtual cost functions for the two dummy links are assumed as \( c_7 = \ln x_7, c_8 = \ln x_8 \), respectively.

We applied the well-known Frank-Wolfe solution algorithm to solve the extended traffic assignment problem shown in Figure 5b. The link and route results are summarized, respectively, in Tables 1 and 2.

### Table 1. Link results

<table>
<thead>
<tr>
<th>Link Number</th>
<th>Link flow</th>
<th>Link cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.109</td>
<td>1.359</td>
</tr>
<tr>
<td>2</td>
<td>1.109</td>
<td>1.609</td>
</tr>
<tr>
<td>3</td>
<td>0.984</td>
<td>2.967</td>
</tr>
<tr>
<td>4</td>
<td>1.060</td>
<td>3.060</td>
</tr>
<tr>
<td>5</td>
<td>0.848</td>
<td>1.424</td>
</tr>
<tr>
<td>6</td>
<td>0.848</td>
<td>1.636</td>
</tr>
<tr>
<td>7</td>
<td>2.092</td>
<td>0.738</td>
</tr>
<tr>
<td>8</td>
<td>1.908</td>
<td>0.646</td>
</tr>
</tbody>
</table>

### Table 2. Route/subroute results

<table>
<thead>
<tr>
<th>Route Number</th>
<th>Route flow</th>
<th>Route cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: ( r \rightarrow 1 \rightarrow s_1 \rightarrow \tilde{r} )</td>
<td>1.109</td>
<td>3.706</td>
</tr>
<tr>
<td>2: ( r \rightarrow s_1 \rightarrow \tilde{r} )</td>
<td>0.984</td>
<td>3.706</td>
</tr>
<tr>
<td>3: ( r \rightarrow s_2 \rightarrow \tilde{r} )</td>
<td>1.060</td>
<td>3.706</td>
</tr>
<tr>
<td>4: ( r \rightarrow 2 \rightarrow s_2 \rightarrow \tilde{r} )</td>
<td>0.848</td>
<td>3.706</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subroute Number</th>
<th>Subroute flow</th>
<th>Subroute cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: ( r \rightarrow 1 \rightarrow s_1 )</td>
<td>1.109</td>
<td>2.967</td>
</tr>
<tr>
<td>2: ( r \rightarrow s_1 )</td>
<td>0.934</td>
<td>2.967</td>
</tr>
<tr>
<td>3: ( r \rightarrow s_2 )</td>
<td>1.060</td>
<td>3.060</td>
</tr>
<tr>
<td>4: ( r \rightarrow 2 \rightarrow s_2 )</td>
<td>0.848</td>
<td>3.060</td>
</tr>
</tbody>
</table>
It is observed that the obtained solution completely complies with the following conditions:

1) Trip distribution: The computed results for trip distribution satisfy the assumed logit formula.

\[
q_{rs}^{r_i} = 4 \times \frac{\exp(-\pi_{r_i}^{rs})}{\exp(-\pi_{r_i}^{rs}) + \exp(-\pi_{r_i}^{rs})} = 4 \times \frac{0.0514}{0.0514 + 0.0469} = 2.092
\]

\[
q_{rs}^{r_s} = 4 \times \frac{\exp(-\pi_{r_s}^{rs})}{\exp(-\pi_{r_s}^{rs}) + \exp(-\pi_{r_s}^{rs})} = 4 \times \frac{0.0469}{0.0514 + 0.0469} = 1.908
\]

2) Traffic assignment: The computed path travel times for the four used paths are exactly the same (3.706 units) which imply a generalized Wardrop equilibrium.

Overall, the results obtained are conformed to our optimality conditions for the combined trip distribution and traffic assignment combined model. It can be easily extended to other travel choice problems. In a more strict sense, the travelers’ behavior that uses a minimum path and adopts logit formula for other travel choices can be fully satisfied with the most complicated combined model of the four travel choices with variable demand.

5. CONCLUSION AND SUGGESTIONS

In this paper, we have presented a novel solution algorithm for the combined model with the four travel choices and variable demand. This novel algorithm solves the four two-step combined submodels in sequel. In detail, we solve the area-wide trip generation problem with variable demand (VD/TA) at first. With the obtained area-wide trip rates, we solve the trip generation and assignment (TG/TA) problem. With the obtained trip rates associated with each origin, we then solve the trip distribution and assignment (TD/TA) problem, resulting in O-D trip rates, which are in turn used to solve the modal split and assignment (MC/TA) problem. This solution procedure is conceptually similar to the traditional four-step sequential demand forecasting procedure but takes equilibrated travel costs into consideration. Hence internal inconsistency problems between any two consecutive steps are largely reduced. However, up to this point, no meaningful comparison between our proposed solution algorithm and traditional sequential travel demand forecasting with feedback using practical problems has been conducted. It definitely needs more effort in this research direction.

It is noticed that each of the above mentioned combined models can be treated as “extended” traffic assignment problem and hence any traffic assignment solution algorithm can be readily adopted for solutions. In view of the amazing development of the quick-precision traffic assignment solution algorithms (Gentile, 2009; Florian, Constantin and Florian, 2009; Dail, 2006) in the last decade, any such solution algorithm like “TAPAS” algorithm (Bar-Gera, 2010; Nie, 2010) can make the proposed solution procedure more effective in terms of both performance efficiency and solution precision. However, how much improvement can be obtained as compared with traditional four-step sequential demand forecasting procedure (with or without feedback) is still in question and needs an immediate future research.
6. ACKNOWLEDGEMENT

This work was supported in part by the Ministry of Science and Technology, Taiwan under grant No. NSC-100-2221-E-008-093-MY3.

REFERENCES


Chen, H.K. (2013) A combined model with the four travel choices and variable demand. Presented at the Sixth International Conference of Chinese Transportation Professionals, Dalian, China.


