BRIDGING THE GAP BETWEEN SOCIAL AND PRIVATE
OPTIMAL TIMING OF INVESTMENT:
DEVELOPING COUNTRIES’ PERSPECTIVE

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Abstract: In this paper, the optimal timing of investment, defined as the time at which the net
benefit of the project is maximized, from the public and private perspectives are derived. Because of the differing views of these sectors, public and private optimal timing may not coincide, thus creating an optimal timing gap. However, coordination between these sectors is important considering the increasing convergence for more private sector participation in providing public goods such as transportation and to achieve maximum social benefits even under this condition. Policies used in developing countries for encouraging private sector participation are examined for effectiveness in narrowing this timing gap. These include tax holiday, increased debt ratio, longer loan repayment period, special interest rate used in the computation of the loan annual amortization, and lump-sum financial subsidy. Criteria for perfect synchronization as well as acceptability of lump-sum financial subsidy are formulated.

Key words: Public optimal timing, private optimal timing, optimal timing gap, perfect synchronization, acceptability of lump-sum financial subsidy

1. INTRODUCTION:

Traditionally, government has been charged with the responsibility of providing public goods
such as infrastructure. Yet, the increased need for transport infrastructure brought about by a
rapidly expanding urban population has over-extended government capacities, especially in
developing countries. Thus, with the increasing need for mobility on one hand, and a very
limited government infrastructure allocation on the other hand, an increasing convergence for
more private sector participation is noted in Asia (Allport, 1998). With the implementation of
the Privatization policy, the government must now consider the private sector’s point of view
on project appraisal to guide it in its dealings with the latter (UNIDO, 1996).

One of the key issues in the efficient delivery of basic public services is its timeliness. In this
paper, this net benefit is expressed as the Net Present Value (NPV). Optimal timing of
investment is therefore defined as the time at which the NPV of the project is maximized.
However, considering the differences in views on project appraisal between the government
and private sector, what the government perceives to be the optimal timing of investment may
not coincide with that of the private sector.

This paper focuses on the gap between the optimal timing of investment from the government
(hereinafter termed ‘social optimal timing’) and private (private optimal timing) perspectives
and its policy implications in the implementation of infrastructure projects. In as much as this
gap may lead to loss of social benefits, government must therefore strive to narrow this gap
using various policy instruments. Here, project implementation is treated as a pure timing
process only, without consideration of risk. This is done to isolate and demonstrate the impact
of time on the Net Present Value and thus the effectiveness of policies over time.
2. SOCIAL OPTIMAL TIMING:

In the paradigm of the classical Discounted Cash Flow (DCF) method, a project that yields a positive NPV should be implemented immediately. On the other hand, that which yields a negative NPV should be abandoned outright. This paradigm is reinforced by the strong emotional appeal of the ‘do something’ mentality. Oftentimes, impatience with bureaucracy and the seemingly long process of planning result in a strong predisposition to implement ‘build-now’ decision (Chu, et. al, 1998). However, Marglin (1963) suggested that under certain conditions a negative NPV could change to a positive NPV if the implementation is delayed. In this case, there is value in waiting. Likewise, even when the project’s benefits exceeds its cost does not necessarily mean that it is best to implement the project immediately. There exists an optimal timing for implementation when the NPV of the project is maximized.

Following the classical framework of Marglin (1963) and Morisugi (1977) for optimal timing of investment, the following assumptions are used in the formulation of the models: 1) Investment or construction is considered as a mass point investment, implemented only at a single point of time thus, optimal timing of investment is considered synonymous with optimal timing for opening of service, 2) Constant growth rate of annual net benefit and specified cases of such are considered, and 3) A single and independent project is to be implemented.

2.1. Net Present Value As A Function of Timing

Optimal timing is defined as the point in time at which the Net Present Value (NPV) is maximized. In symbol,

\[ T^* = \arg \max V(T) \]  

Net Present Value of the project \( V(T) \) may be expressed as:

\[ V(T) = -I \exp(-\rho T) + \int_T^\infty (b(t) - c(t)) \exp(-\rho t) dt \]  

where \( V(T) \)-net present value of the project, \( I \)-investment cost, \( b(t) \)-annual benefit, \( c(t) \)-annual running cost, \( \rho \)-social discount rate, \( T \)-Timing of opening of service and \( T^* \)-Optimal Timing.

2.2 Net Present Value In The Specified Cases

The annual growth of net benefit, incurred only after the investment has been implemented, may be expressed as:

\[ b(t) - c(t) = (\bar{b} - \bar{c}) \exp(\omega t) \]  

where \( \bar{b} - \bar{c} \) is the initial value of the annual net benefit at \( t=0 \). Substituting this to equation (1) yields:

\[ V(T) = -I \exp(-\rho T) + (\bar{b} - \bar{c}) \int_T^\infty \exp\{(\omega - \rho) t\} dt \]  

or

\[ V(T) = -I \exp(-\rho T) + (\bar{b} - \bar{c})/(\rho - \omega) \exp\{(\omega - \rho) T\} \]  

2.3 Optimal Timing When \( \omega = 0 \)

From equation (4.a),

\[ V(T) = -I \exp(-\rho T) + (\bar{b} - \bar{c}) \int_T^\infty \exp(-\rho t) dt \]
or
\[
V(T) = -I + \left[ \frac{\bar{b} - \bar{c}}{\rho} \right] \exp(-\rho T)
\]  \hspace{1cm} (5b)

For this particular case, NPV is a monotonously decreasing function of \( T \) if \( V(0) > 0 \) but never equals zero at any given time. When \( V(0) < 0 \), NPV is a monotonously increasing function but never equals zero or becomes positive. Therefore, for zero growth rate of net benefit \( (\omega = 0) \), the optimal timing is now when \( V(0) > 0 \). The choice in project implementation is ‘now or never’. Otherwise, when \( V(0) = -I + \left[ \frac{\bar{b} - \bar{c}}{\rho} \right] \), the project should never be implemented.

2.4 Optimal Timing When \( \omega > 0 \)
When the growth rate is positive, the NPV is maximized at \( \bar{T} \) where
\[
d\bar{T} /dT = 0.
\]  \hspace{1cm} \begin{array}{r}
\frac{dV}{dT} > 0 & \text{for } T < \bar{T} \\
\frac{dV}{dT} < 0 & \text{for } T > \bar{T}
\end{array}
\]  \hspace{1cm} (6)

The maximum at \( \bar{T} \) means that when \( T < \bar{T} \), NPV is still increasing. However, when \( T > \bar{T} \), NPV will start to decline.

The implication is that the investment must be at optimal timing \( T^* \), otherwise loss is incurred.

When \( V(0) < 0 \), there exists \( \bar{T} < \bar{T} \), hereinafter referred to as the minimum project maturity, at which NPV which was previously negative becomes zero. Using Equation (4.b), the equation for \( \bar{T} \) may be derived as:
\[
\bar{T} = (1/\omega) \ln \left[ \frac{1}{\rho I/\left( \bar{b} - \bar{c} \right)} \right]
\]  \hspace{1cm} (7)

2.5 Optimal Timing When \( \omega < 0 \)
Under a negative growth rate, \( dV/dT < 0 \) and \( dV/dT > 0 \) for \( T > \bar{T} \) where \( \bar{T} \) gives the minimum of NPV, a condition that is in contrast with that when the growth rate is positive. Thus, it can be concluded that the best timing is \( T^* = 0 \) or “do it now!”. This implies that any timing \( T > \bar{T} \) results in a negative NPV. When \( V(0) < 0 \), \( V(T) \) never becomes positive, thus the project should never be implemented.

When \( V(0) > 0 \), there exists \( 0 < \hat{T} < \bar{T} \) such that \( V(\hat{T}) = 0 \), where \( \hat{T} \) can be termed as the maximum tolerance of delay. Again using Equation 4a but using \( \hat{T} \) and \( \infty \) as lower and upper limits, respectively, an expression for \( \hat{T} \) is derived as:
\[
\hat{T} = \frac{1}{\omega} \ln \left[ \frac{1}{\rho I/\left( \bar{b} - \bar{c} \right)} \right]
\]  \hspace{1cm} (8)
Since $\hat{T} < \hat{T}$, when $T > \hat{T}$, then $V(T)$ will continue to be negative. The implication is that the project should not be implemented beyond the maximum tolerance of delay.

3. LOSS FROM OPTIMAL TIMING

From definition, the optimal timing of opening of service maximizes the net benefit of the project. Thus, when the project is not implemented at $T^*$, then loss is incurred. This loss may be defined as:

$$L = V(T^* + h) - V(T^*)$$  (9)

The loss is dependent on two factors: the optimal timing $T^*$ and the displacement in time from the optimal $h$. By definition, $L$ is always non-positive, and $L$ is Too-Early-Loss if $h > 0$ and Too-Late-Loss if $h < 0$. Thus utilizing the equation (4.b), loss $L$ can be expressed as:

$$L = L[\exp(-\rho T^*) - \exp(-\rho(T^* + h))] - \frac{b-c}{\rho-\omega}[\exp((\omega-\rho)T^*) - \exp((\omega-\rho)(T^* + h))]$$  (10)

To determine the consequences of loss under various cases, the partial derivative of $L$ with respect to $h$ is taken:

$$\frac{\partial L}{\partial h} = \frac{dV(T^* + h)}{dh} = \rho L \exp(-\rho(T^* + h)) - (b-c)\exp((\omega-\rho)(T^* + h))$$  (11)

3.1 The Loss In The Case When $\omega > 0$

Based on Equation 11, for $\omega > 0$ and constant $\rho$, $I$, and $b-c$, $\partial L/\partial h < 0$ for $h > 0$, and $\partial L/\partial h > 0 > 0$ for $h < 0$, indicating that the best timing is $T = T^*$. Figure 1 shows the loss pattern using equation 10. Under these conditions, the NPV is positive even when Too-Late-Loss is incurred, while the NPV can be negative when the Too-Early-Loss occurs. The Too-Late-Loss may be more tolerable than the Too–Early-Loss. The NPV is positive even if the opening is delayed. This, therefore, negates the general notion that when the project has been delayed, it should be abandoned. With respect to the optimal timing under the condition when $\omega > 0$, the delay may mean a more acceptable level of loss.

![Figure 1](image)

3.2 The Loss In The Case When $\omega < 0$

In contrast, for the case of $\omega < 0$, $L = V(0 + h) - V(0)$, no clear characteristics like (14a) and (14b) can be derived. However, it has been established in subsection 2.5 that when $T^* = 0$, if the delay is beyond $\hat{T}$ or $h > \hat{T}$, the NPV is negative ($V(h) < 0$). This implies that that if the project is not implemented immediately, loss will continue to increase.

3.3 Implications Of Loss

A summary of the various implications of early implementation or delay in project implementation is shown in Table 1.
TABLE 1. Implications of Too-Early and Too-Late Loss on Project Implementation

| $V(0)>0, \rho I/(\bar{b} - \bar{c}) > 1$ | \\hline Too Early Loss ($T < T^*$) | Too-Late Loss ($T > T^*$) \\
| $\omega=0$ | Implement | Implement \\
| $\omega>0$ | Implement | Implement \\
| $\omega<0$ | Implement | Implement \\

$V(0)>0, \rho I/(\bar{b} - \bar{c}) < 1$

| $\omega=0$ | Implement | Abandon \\
| $\omega>0$ | Implement | Implement \\
| $\omega<0$ | Implement | Abandon \\

$V(0)<0, \rho I/(\bar{b} - \bar{c}) > 1$

| $\omega=0$ | Abandon | Abandon \\
| $\omega>0$ | Implement | Implement \\
| $\omega<0$ | Abandon | Abandon \\

$V(0)<0, \rho I/(\bar{b} - \bar{c}) < 1$

| $\omega=0$ | Implement | Implement \\
| $\omega>0$ | Implement | Implement \\
| $\omega<0$ | Implement | Implement \\

4. OPTIMAL TIMING FROM THE PRIVATE SECTOR’S PERSPECTIVE

4.1 Differences In Project Appraisal Between Government And Private Concessionaire:

With the implementation of the Privatization Policy, project appraisal from the private sector point of view is now one of the main considerations in infrastructure project financing. In this section, the Net Present Value from the Private Concessionaire’s point of view (hereinafter referred to as Financial Net Present Value, FNPV) is formulated. In the formulation, three key differences are taken into account (Table 2). Whereas the government considers the economic benefit of the project by considering all the stakeholders, the private sector is more interested in the financial viability of the project.

TABLE 2. Project Evaluation from Government and Private Concessionaire Perspectives

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Government</th>
<th>Private Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appraisal</td>
<td>Economic</td>
<td>Financial</td>
</tr>
<tr>
<td>Benefits considered</td>
<td>Non-monetary such as environmental and health impacts</td>
<td>Monetary</td>
</tr>
<tr>
<td>Discount rate</td>
<td>Social Discount Rate (lower than market discount rate)</td>
<td>Market Discount Rate (Higher than social rate to reflect opportunity cost of capital)</td>
</tr>
<tr>
<td>Cashflow calculation</td>
<td>Before tax cash flow</td>
<td>After-tax cash flow</td>
</tr>
</tbody>
</table>


4.2 Financial Net Present Value (FNPV)

Taking these differences into consideration, the $FNPV$ for private concessionaires may be derived using equation (4.b) as basis. For private concessionaires, whose capital outlay consist of debt as well as equity, financial present value of investment $I$ may be expressed as:

$$(1-u)I \exp(-\eta T)$$

(12a)

where $u$ is the debt ratio for the project and $\eta$ denotes the market interest rate used for discounting of time value of $I$ and is taken as the effective rate of interest to include effect of inflation. The integral of the term for discounting of the initial net income, denoted as
\((\bar{R} - M)\) for initial revenue \(R\) and maintenance cost \(M\), may be expressed as:
\[
\frac{(\bar{R} - M)}{(\eta - \omega)} \exp(\omega - \eta)T
\]  
(12b)

Finally, a term for discounting of the repayment of debt, \(uI\), must be included:
\[
\int_{T}^{T+G} A \exp(-\eta t)dt
\]  
(12c)

An assumption is made here that repayment of debt will start after the opening of service. Therefore, the net present value for private company may be expressed as:
\[
V_p(T) = -(1 - u)I \exp(-\eta T) + \theta \left[ \frac{(\bar{R} - M)}{(\eta - \omega)} \exp(\omega - \eta)T - \int_{T}^{T+G} A \exp(-\eta t)dt \right]
\]  
(12d)

where \(\theta = \) factor applied to represent income after taxes. However, for consistency, annuity should be expressed in continuous compounding form. Thus, Financial Net Present Value becomes:
\[
V_p(T) = \exp(-\eta T) \left[ \frac{\theta(\bar{R} - M)}{(\eta - \omega)} \exp(\omega T) - (1 - u)I - \frac{\theta I u}{\eta} \right]
\]  
(13)

### 4.3 Private Optimal Timing When \(\omega=0\):
When \(\omega=0\), Financial Net Present Value will be reduced to:
\[
V_p(T) = \exp(-\eta T) \left[ \frac{\theta(\bar{R} - M)}{(\eta - \omega)} - (1 - u)I - \frac{\theta I u}{\eta} \right]
\]  
(14)

Under this condition, optimal timing \(T_p^*\) is now when \(V_p(0)>0\) and never when \(V_p(0)<0\). The trend of FNPV is similar to that of the social NPV.

### 4.4 Private Optimal Timing When \(\omega>0\):
When the growth rate is positive, FNPV is maximized at \(\tilde{T}_p\) such that \(dV_p(\tilde{T})/dT = 0\)

\[
\frac{dV_p(T)}{dT} = \exp(-\eta T) \left[ -\theta(\bar{R} - M) \exp(\omega T) + I\eta(1-u) + \theta I u \exp(\eta - 1) \right] = 0
\]  
(15)

Therefore,
\[
\tilde{T}_p = \frac{1}{\omega} \ln \left[ \frac{I}{\theta(\bar{R} - M)} \right] \left\{ \eta (1-u) + \theta I u (\exp\eta - 1) \right\}
\]  
(16)

Similar to the trend of social optimal timing under the same growth condition, the maximum at \(\tilde{T}_p\) means that
\[
\frac{dV_p(T)}{dT} > 0 \text{ when } T < \tilde{T}_p \text{ and } \frac{dV_p(T)}{dT} < 0 \text{ when } T > \tilde{T}_p
\]

If \(\frac{I}{\theta(\bar{R} - M)} \{ \eta (1-u) + \theta I u (\exp\eta - 1) \} > 1\) then \(\tilde{T}_p = \tilde{T}_p > 0\)

Minimum project maturity from the private perspective, \(\overline{T}_p\), may be computed using the formula:
\[
\overline{T}_p = \frac{1}{\omega} \ln \left[ \frac{(\eta - \omega)I}{\theta(\bar{R} - M)} \right] \left\{ (1-u) + \frac{\theta I u}{\eta} (\exp\eta - 1) \right\}
\]  
(17)
4.5 Private Optimal Timing When \( \omega < 0 \):

Under a negative growth rate, when FNPV at time 0 is positive, when \( T < \tilde{T}_p \), NPV is decreasing and a reverse trend is noted when \( T > \tilde{T}_p \). Under this condition, the optimal timing for implementation is ‘now’.

The private maximum tolerance of delay is given by:

\[
\hat{T}_p = \frac{1}{\omega} \ln \left( \frac{(\eta - \omega) M}{\theta(\tilde{R} - \tilde{M})} \left( (1 - u) + \frac{\theta u}{\eta} (\exp \eta - 1) \right) \right)
\]

\[\text{(18)}\]

5. IMPPLICATION OF THE DIFFERENCE IN TIMING BETWEEN GOVERNMENT AND PRIVATE CONCESSIONAIRE

In this paper, the social optimal timing, \( T^* \), is considered to be the efficient timing for the opening of service of the project to maximize benefits to the public. This is premised on the assumption that the government takes into consideration all the stakeholders of the project. However, due to the difference in project evaluation from the public and private perspectives, a gap between public and private optimal timing for opening of service occurs. Hence, the objective of the government should be to narrow this gap in optimal timing thereby minimizing loss.

5.1 Gap Between Social And Private Optimal Timing

From equations (3) and (12b), the growth of the net benefit and net income over time is dependent on the growth rate of annual net benefit. For transport infrastructure projects, this is expressed as the growth rate of demand. Since the social discount rate is always less than the market interest rate (\( \rho < \eta \)) and that for environmentally sound projects, (\( \tilde{b} - \tilde{c} > (\tilde{R} - \tilde{M}) \)), it can be deduced that at any time, \( T \), \( b(T) - c(T) > R(T) - M(T) \). Based on characteristics of optimal timing, the following possible scenarios for gap may be surmised:

1) When \( \omega = 0 \), in the instance when \( V(0) < 0, T = \tilde{T}_p = \infty \). Under these conditions, the project must be abandoned. However, when \( V(0) > 0 \) and \( T = 0 \), private optimal timing may be now \( (T_p = 0) \), or never \( (T_p = \infty) \). The value of the private optimal timing will depend on the project characteristics.

2) When \( \omega > 0 \), for the case when \( \rho l/(\tilde{b} - \tilde{c}) > 1 \), \( T^* = \tilde{T} \) and private optimal timing is later than social optimal timing \( (T^* < T_p) \). Moreover, when \( V(0) < 0 \), the private minimum project maturity is later than the social minimum project maturity \( (\tilde{T} < \tilde{T}_p) \). Szymanski (1991) stated that under this condition, the private sector may opt to invest at \( \tilde{T}_p \) rather than at \( T^* \) due to competition. Private optimal timing is also later than social optimal timing even when \( \rho l/(\tilde{b} - \tilde{c}) < 1 \) and \( V(0) > 0 \). It must be noted that under these conditions, the social optimal timing is now \( (T^* = 0) \).

3) For \( \omega < 0 \), the project is deemed viable from the government perspective when \( \rho l/(\tilde{b} - \tilde{c}) < 1 \) and \( V(0) > 0 \). Under these conditions \( T_p = \infty \), even when the discount rates and the initial net benefit and initial net income are taken to be equal. This is attributed to the fact that the private sector takes into account after-tax cash flows. However, when \( V(0) < 0 \), both the social and private optimal timing will be infinite, indicating that it will never implement the project without any change in the project characteristics. Table 3 summarizes the trends that have been described thus far.
TABLE 3: Gap Between Social and Private Optimal Timing

| $\omega>0$ | $\omega<0$ |
| $\rho I/(\bar{b}-c)>1$ | $\rho I/(\bar{b}-c)<1$ |
| $\omega=0$ | $\omega>0$ | $\omega<0$ |
| $T^*=T_0^*; T^*<T_p^*$ | $T^*=0; T_p^*=\infty$ or $T_p^*=0$ | $T^*=0; T_p^*=\infty$ or $T_p^*=0$ |
| $\rho I/(\bar{b}-c)>1$ | $\rho I/(\bar{b}-c)<1$ |
| $\omega=0$ | $\omega>0$ | $\omega<0$ |
| $T^*=\infty$; $T_p^*=\infty$ | **Never Implement** | **Never Implement** |

5.2 Synchronization Of Optimal Timing

In the Philippines, various policy instruments are used to encourage private sector participation in infrastructure financing. These policies may also be used to effect the revision of private optimal timing to approximate the social optimal timing. Table 4 lists some of these policies and the corresponding pertinent perimeters that can be manipulated to simulate their respective impacts on private optimal timing.

TABLE 4. Pertinent Parameters for Policy Evaluation

<table>
<thead>
<tr>
<th>POLICY/PROJECT SCHEME</th>
<th>Pertinent Parameter/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax Holiday</td>
<td>Increase $\theta$</td>
</tr>
<tr>
<td>Debt Ratio</td>
<td>Increase $u$</td>
</tr>
<tr>
<td>Longer Loan Tenure</td>
<td>Lengthen $G$</td>
</tr>
<tr>
<td>Special Interest Rate</td>
<td>Lower $\beta$</td>
</tr>
<tr>
<td>Public-Private Partnership</td>
<td>Decrease $I$</td>
</tr>
<tr>
<td></td>
<td>Lower $\beta$</td>
</tr>
<tr>
<td></td>
<td>Lengthen $G$</td>
</tr>
</tbody>
</table>

When $T^* < T_p^*$, the government’s objective is to hasten the private timing $T_p^*$. Therefore it must offer incentives that will decrease $I$ such as bearing the cost of land acquisition and right-of-way, decrease annual maintenance cost $M$ by subsidizing maintenance cost as well as granting tax holidays for a certain duration of time, and increase annual revenue $R$. Examples of the latter include the minimum ridership guarantee used in urban rail projects under Private Financing Initiative (PFI) in developing countries such as the Philippines and granting the concessionaire the right to develop the depots into commercial establishments.

Another scenario is when optimal timing of private sector as well as of the Government is now ($T_p^*=T^*=0$). In this case, the critical synchronization parameter will be the maximum tolerance of delay $\hat{T}_p$. Government must encourage the private concessionaire, through the various policy instruments, to implement the project as soon as possible before $\hat{T}_p$ described above.

When tax holiday is granted, $\theta$ is increased to 1 and to decrease the equity required from the private sector, the debt ratio $u$ is increased. It must be pointed out that under the
Public-Private Partnership project scheme, where the Government is part of the Concession Company, lower special interest rates and longer loan tenures are granted. Moreover, the Government may grant financial subsidies such as bearing the cost of land acquisition or infrastructure.

5.3 Synchronization Under Financial Subsidy
The easiest form of incentive offered by the government for optimal timing synchronization is the lump sum (LS) financial subsidy. When financial subsidy is granted, investment cost \( I \) will be reduced to \( I' \) such that:

\[ I' = \kappa I \]  

(19a)

where \( \kappa \) is the rate of private finance. Conversely, \( (1-\kappa) \) is the rate of public subsidy. Thus the enhanced Net Present Value due to financial subsidy, \( V_p^K(T) \), can be written as:

\[ V_p^K(T) = \left( \frac{\theta(R-M)}{(\eta - \omega)} \exp(\omega T) - (1-u)\kappa I - \frac{\kappa \theta u}{\eta} (\exp(\eta T) - 1) \right) \exp(-\eta T) \]  

(19b)

5.3.1 Perfect Synchronization When \( \omega > 0 \):
Using equation (19), when \( \omega > 0 \), revised private optimal timing due to financial subsidy \( \tilde{T}_p^* \) can be expressed as:

\[ \tilde{T}_p^* = \frac{1}{\omega} \ln \left( \frac{\kappa I}{\theta(R-M)} \right) \left( \eta(1-u) + \theta u(\exp(\eta T) - 1) \right) \]  

(20a)

When no financial subsidy is given, then \( \kappa = 1 \) transforming the above equation to that defined in Equation (19). For revised optimal timing of project under PFI:

\[ T_p^* = \frac{1}{\omega} \ln \left( \frac{\kappa I}{\theta(R-M)} \left( \eta(1-u) + \theta u(\exp(\beta - 1) - 1) \right) \right) \]  

(20b)

When the growth rate of benefit is positive, there are two possible social optimal timing, depending on the project characteristics. The first case is when \( T^* = \tilde{T} \). Under this condition, the rate of private finance for perfect synchronization, \( \kappa_p \), can be derived by equating the revised private optimal timing to the social optimal timing as shown below:

\[ \frac{1}{\omega} \ln \left( \frac{\kappa I}{\theta(R-M)} \left( \eta(1-u) + \theta u(\exp(\eta T) - 1) \right) \right) = \frac{1}{\omega} \ln \left( \frac{\rho I}{(b-c)} \right) \]  

(21a)

Simplifying, the rate of private finance for perfect synchronization can be expressed as:

\[ \kappa_p = \frac{\rho \theta(R-M)}{(b-c) \left( \eta(1-u) + \theta u(\exp(\eta T) - 1) \right) \left( 1 - \exp(-\eta G) \right)} \]  

(21b)

Or under the special conditions for PFI:

\[ \kappa_p = \frac{\theta(R-M)}{(b-c) \left( \eta(1-u) + \theta u(\exp(\beta T) - 1) \right) \left( 1 - \exp(-\beta G) \right)} \]  

(21c)

The second possible social optimal timing is when \( T^* = 0 \). Setting \( T_p^* = \tilde{T}_p \), then perfect synchronization occurs when:

\[ \kappa_p = \frac{\theta(R-M)}{\eta I(1-u) + \theta u I(\exp(\eta T) - 1)} \]  

(22a)

Under the special conditions of PFI, it transforms to:

\[ \kappa_p = \frac{\theta(R-M)}{\eta I(1-u) + \theta u I(\exp(\beta T) - 1) \left( 1 - \exp(-\beta G) \right)} \]  

(22b)
In the first instance of social optimal timing, the optimal timing gap can be expressed in terms of $\kappa$:

$$
\bar{T} - \bar{T}_p = \left\{ \frac{1}{\omega} \ln \left( \frac{\rho L}{(b-c)} \right) \right\} - \left\{ \frac{1}{\omega} \frac{\kappa L}{\theta(R-M)} \right\} \left[ \eta(1-u) + \theta u(\exp \eta - 1) \right]
$$

(23a)

To determine the direction of the underlying relationship, the partial derivative of the square of the gap in optimal timing with respect to rate of the rate of private finance is taken:

$$
\frac{\partial}{\partial \kappa} \left( \bar{T} - \bar{T}_p \right)^2 = 2(\bar{T} - \bar{T}_p) \frac{\partial}{\partial \kappa} \left( \bar{T} - \bar{T}_p \right)
$$

(23b)

$$
= 2(\bar{T} - \bar{T}_p) \left( -\frac{1}{\omega \kappa} \right)
$$

(23c)

If $\bar{T} - \bar{T}_p \geq 0$, then $\frac{\partial}{\partial \kappa} \left( \bar{T} - \bar{T}_p \right)^2 \leq 0$

(23d)

From Equation (23c), as $\kappa$ decreases (indicating an increasing rate of public subsidy, 1-$\kappa$), the timing gap increases. Since $\bar{T}$ is later, granting of financial subsidy will only increase the optimal timing gap since as a corollary since lower investment cost will encourage the private sector to implement earlier. Therefore, when the social optimal timing $\bar{T}$ is later, the rate of public subsidy must be decreased. On the other hand,

If $\bar{T} - \bar{T}_p \leq 0$, then $\frac{\partial}{\partial \kappa} \left( \bar{T} - \bar{T}_p \right)^2 / \partial \kappa \geq 0$

Equation (23d) suggests that when the private optimal timing $\bar{T}_p$ is later, a decrease in the value of $\kappa$ (indicating an increase in the rate of public subsidy 1-$\kappa$) will bring about a corresponding decrease in the optimal timing gap. Thus for this case, rate of public subsidy must be increased to achieve synchronization. Figures 2 and 3 show the respective trends.

5.3.2 Synchronization When $\omega<0$:

Under a negative growth rate of net benefit, the social optimal timing is *now or never*. However, based on the discussion of optimal timing gap, when the social optimal timing is now $T^* = 0$, $T^*_p$ can be equal to $\infty$. This suggests that $V_p(0) < 0$. However, when $V_p(0)$ becomes positive thru the granting of the financial subsidy, then $T_p$ equals zero, synchronizing with $T^*$. Thus, the rate of private finance may be derived as:
Thus, the rate of private finance for perfect synchronization, considering special interest rate under the Private Financing Initiatives (PFI), can be expressed as:

$$V^*_p(0) = \left[ -(1-u)\kappa_p I - \frac{\kappa_p \theta I u}{\eta} (\exp \eta - 1) + \frac{\theta(\bar{R} - \bar{M})}{(\eta - \omega)} \right] \geq 0$$

(24a)

Under a negative growth rate, rate for private finance that will effect perfect synchronization is not a single number but a range of values for which $V^*_p(0) \geq 0$. Figure 4 shows the impact of rate of private finance $\kappa$ on the optimal timing gap, under the same assumption for the relationships of the parameters $\eta$, $\rho$, $(\bar{R} - \bar{M})$, and $(\bar{b} - \bar{c})$ as in the previous section. It must be emphasized that until $V^*_p(0)$ becomes positive, private optimal timing remains equal to infinity ($T_p^* = \infty$).

$$\kappa_p \leq \frac{\theta(\bar{R} - \bar{M})}{(\eta - \omega)} \left\{ \left[ \frac{\eta}{(1-u)\eta I + \theta I u (\exp \beta - 1)} \right] \frac{1 - \exp(-\eta G)}{1 - \exp(-\beta G)} \right\}$$

(24b)

When the private optimal timing is equal to $\infty$, the trend of the optimal timing gap in relation to the financial subsidy factor $\kappa$ is similar to that depicted in Figure 4.

5.3.3 Synchronization When $\omega = 0$:

Similar to the condition when growth rate is negative, under a zero growth rate of net benefit, the choice in project implementation is now or never. Thus, the rate of private finance for perfect synchronization can be expressed as:

$$\kappa_p \leq \frac{\theta(\bar{R} - \bar{M})}{\eta I (1-u) + \theta I u (\exp \eta - 1)}$$

(25a)

Similarly, under PFI conditions,

$$\kappa_p \leq \left\{ \left[ \frac{\theta(\bar{R} - \bar{M})}{(1-u)\eta I + \theta I u (\exp \beta - 1)} \right] \frac{1 - \exp(-\eta G)}{1 - \exp(-\beta G)} \right\}$$

(25b)

Under all conditions, rate of private finance should be between 0 and 1, $0 \leq \kappa \leq 1$. A summary of the rate of public subsidy for perfect synchronization of social and private optimal timing under various conditions are summarized in Table 5.

Fig. 4. Trend of Timing Gap when $\omega < 0$
TABLE 5. Rate of Public Subsidy for Perfect Synchronization

<table>
<thead>
<tr>
<th>$T^* - T_p^* &gt; 0$</th>
<th>$T^* - T_p^* &lt; 0$</th>
</tr>
</thead>
</table>
| $\omega = 0$    | When $T^* = 0$ and $T_p^* = \infty$, then  
| ---             | $(1-\kappa)_p \geq 1 - \frac{\theta(R - \bar{M})}{\eta I(1-u) + \theta u I(\exp \eta - 1)}$ |

**$\omega > 0$**

For both cases,  
$T^* = \bar{T}; T_p^* = \bar{T}_p$  
and  
$T^* = \bar{T}; T_p^* = 0$,  
$(1-\kappa)_p = \infty$  
Decrease Financial Subsidy

| $\omega < 0$    | When $T^* = 0$ and $T_p^* = \infty$, then  
| ---             | $(1-\kappa)_p \leq 1 - \frac{\theta(R - \bar{M})}{\eta I(1-u) + \theta u I(\exp \eta - 1)}$ |

5.4 Acceptability Of Lump-Sum Financial Subsidy

Although it is the objective of the government to provide public goods to the populace as efficiently and as sufficiently as possible, the amount of financial subsidy is not without limit, especially for Lump-sum financial subsidy which requires actual transfer of money from the government coffers to the private sector. The grant must be justified by the increase in social surplus brought about by the change in private optimal timing resulting from the subsidy granted. As has been discussed in a previous subsection, when the project is not implemented at the social optimal timing (assumed to maximize benefits of all stakeholders), loss is incurred. Thus, the acceptability of granting LS financial subsidy may be expressed as:

$$\left(-\left[V(T^*_p) - V(T^*)\right] - \left[V(T^*_p^*) - V(T^*)\right]\right) \geq (1-\kappa)I$$  \hspace{1cm} (26a)

Since loss has been defined as non-positive, the negative sign transforms the left-hand side of the equation to gain. Simplifying,

$$V(T^*_p^*) - V(T^*_p) \geq (1-\kappa)I$$  \hspace{1cm} (26b)

For transport projects, direct benefits consist of decrease in vehicle operating costs, travel time, and accidents, as well as increased comfort, convenience, and reliability of service. Of the four mentioned, the easiest to quantify would be savings in vehicle operating costs and travel time. Since investment cost, $I$, and social discount rate, $\rho$, are assumed to be constant values, then only the annual growth of net benefit determines the constraint of the financial subsidy.

The initial net benefit may be made proportional to the decrease in travel time as benefit and decrease in generalized costs:

$$\left(\bar{b} - \bar{c}\right) \propto \left(\tau \Delta N - \Delta \rho \right)$$  \hspace{1cm} (27a)

where $\tau$ is the travel time cost and $\rho$ denotes the generalized costs (including VOC and others). Thus, the financial support offered should be dependent on the quantity of travel time savings, decrease in generalized cost, and demand for the facility under evaluation. Demand for the facility over time may also be expressed as:

$$Q(t) \propto \exp(\omega t)$$  \hspace{1cm} (27b)
5.4.1 Acceptability of Lump Sum Financial Subsidy When \( \omega > 0 \):
From Equation (4.b),
\[
V(T) = -I \exp(-\rho T) + \frac{[b - c]}{(\rho - \omega)} \exp((\omega - \rho)T)
\]
Therefore, \( V(T_p^*) = -I \exp(-\rho T_p^*) + \frac{[b - c]}{(\rho - \omega)} \exp((\omega - \rho)T_p^*) \) (28a)
And \( V(T_{p}^{\kappa}) = -I \exp(-\rho T_{p}^{\kappa}) + \frac{[b - c]}{(\rho - \omega)} \exp((\omega - \rho)T_{p}^{\kappa}) \) (28b)
When \( T_{p}^{\kappa} = T_{p}^{*} \), then using Equations (16) and (20a) for the appropriate expressions of private optimal timing and revised private optimal timing, respectively, Equation 28b is transformed to the inequality found below, with consideration of the special conditions of PFI.

\[
\frac{\theta(\bar{R} - M)}{\eta I (1 - u) + \theta u (\exp \beta - 1)} \frac{1 - \exp(-\eta G)}{1 - \exp(-\beta G)} \geq 0
\]

(29)
It is interesting to note that when the rate of private finance \( \kappa \) is made equal to zero (corresponding to a rate of public subsidy of 1), the inequality expressed in equation (34) is not satisfied. It can be therefore be surmised that under usual conditions, full LS financial subsidy may not acceptable. The patterns of the left-hand side of Equation (25b), which hereinafter be referred to as \( F(\kappa) \), with respect to the values of the rate of private finance \( \kappa \) and the rate of public subsidy \( (1 - \kappa) \) are shown below.

It is interesting to note that when \( \rho I / (b - c) > 1 \), the trend of \( F(\kappa) \) indicates that financial subsidy
is not acceptable as a tool for synchronizing optimal timing. This holds true for conditions when the cost-benefit term for private optimal timing \[ I/\theta(R - M)\eta[1-u] + \theta u[\exp \eta - 1] \] is greater or less than 1. However, when \( \rho l/(b-c) < 1 \), a range of values for \( \kappa \) and \( (1-\kappa) \), such that \( 0 < \kappa < 1 \) and \( 0 < (1-\kappa) < 1 \), respectively. These are indicated as the minimum acceptable rate of private finance, \( \kappa_L^* \), maximum acceptable rate of private finance, \( \kappa_H^* \), maximum optimal acceptable rate of public subsidy, \( (1-\kappa)_H^* \), and minimum optimal acceptable rate of public subsidy, \( (1-\kappa)_H^* \).

### 5.4.2 Acceptability When \( \omega < 0 \):

Under a negative growth rate of net benefit, when \( T^*_P = \infty \), the loss to the public sector is equal to \( V(0) \), loss of benefit due to the non-implementation of the project. Therefore, to minimize this loss, the government can offer financial subsidy to change \( T^*_P = \infty \) to \( T^*_P = 0 \).

An important assumption here is that unlike the behavior of the private sector under a positive growth rate of benefit, (i.e., implement as soon as \( V_r(T) = 0 \) due to competition), the private sector would opt to implement at \( T = 0 \) when \( V_r(T) \) is greatest. In symbol, acceptability of granting of financial subsidy when \( \omega < 0 \) is:

\[
V(0) \geq (1-\kappa)I
\]

From Equation 4.b, at time \( T = 0 \), Social Net Present Value is transformed to:

\[
V(0) = -I + \left[\frac{\bar{b} - \bar{c}}{\rho - \omega}\right]
\]

Thus, the granting of financial subsidy under a negative growth rate is deemed acceptable when the inequality found below is satisfied:

\[
\kappa \geq 2 - \frac{1}{I}[\bar{b} - \bar{c}]/(\rho - \omega) \]

where \( 1/I[\bar{b} - \bar{c}]/(\rho - \omega) \geq 1 \) for projects under a negative growth rate considered feasible from the government’s point of view, as described in a previous sub-section. The trends of \( F(\kappa) \) with respect to the rate of private finance and rate of public subsidy are shown in Figure 7. The trends show that there exists an optimal rate of \( \kappa \) and \( (1-\kappa) \) at which LS financial subsidy is an acceptable synchronization mechanism.

\[
\text{Fig. 7. Trend of } F(\kappa) \text{ when } \omega < 0 \text{ and } \rho I/[\bar{b} - \bar{c}] < 1
\]

### 5.4.3 Acceptability When \( \omega = 0 \):

When the growth rate of benefit is zero, the social optimal timing is likewise now \( T^* = 0 \) or never \( T^* = \infty \). Thus, the condition for acceptability becomes:

\[
\kappa \geq 2 - \frac{1}{I}[\bar{b} - \bar{c}]/(\rho) \]

where \( 1/I[\bar{b} - \bar{c}]/(\rho - \omega) \geq 1 \) for projects deemed suitable for implementation under a zero growth rate of net benefit. Figure 8 shows that under a zero growth rate of net benefit, the trends of \( F(\kappa) \) with respect to \( \kappa \) and \( (1-\kappa) \) are similar to the case when \( \omega < 0 \). There exists an
optimal rate of private finance/rate of public subsidy at which financial subsidy is an acceptable synchronization mechanism.

\[
F(\kappa) = \kappa(1-\kappa)\frac{1}{1-(1-\kappa)}
\]

![Fig. 8. Trends of F(\(\kappa\)) when \(\omega=0\) and \(\rho I/(b-c) < 1\)](image)

Table 6 shows the implication of public acceptability of financial subsidy on synchronizing social and private optimal timing. It can be deduced that when the \(\kappa\) and \((1-\kappa)\) do not coincide with those required for perfect synchronization, then imperfect revision of the private optimal timing is the next best alternative.

<table>
<thead>
<tr>
<th>(\omega=0)</th>
<th>(\rho I/(b-c) &gt; 1)</th>
<th>(\rho I/(b-c) &lt; 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>When (T^* = 0; T_p^* = \infty), (1-\kappa) (\neq) (1-\kappa), then perfect synchronization Otherwise use ((1-\kappa))' for imperfect synchronization</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(\omega&gt;0)</th>
<th>(\rho I/(b-c) &gt; 1)</th>
<th>(\rho I/(b-c) &lt; 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega&lt;0)</td>
<td>(\rho I/(b-c) &gt; 1)</td>
<td>(\rho I/(b-c) &lt; 1)</td>
</tr>
<tr>
<td>(\omega=0)</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>(\omega&gt;0)</td>
<td>(\rho I/(b-c) &gt; 1)</td>
<td>(\rho I/(b-c) &lt; 1)</td>
</tr>
<tr>
<td>(\omega&lt;0)</td>
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</tbody>
</table>

6. CONCLUSION

The Privatization Policy has been conceived to ease the financial burden of the government without sacrificing the delivery of necessary services to the public. However, it is faced with the challenge of contending with the private sector’s perspective in project evaluation and
thereby designing its policies accordingly for a more effective and efficient implementation of the project.

In this paper a framework for evaluation of policies based on the concept of optimal timing was set forth. Since government is believed to be ‘altruistic’ in the sense that it is not profit-oriented and considers all the stakeholders of the project, its optimal timing for the opening of service is considered to be more efficient. However, considering the differing views of government and private sector, government must constantly seek to ‘persuade’ the private sector to implement the project at $T^*$ using the various policy instruments. The following can be deduced from this paper:

1) Evaluation of the social net present value and optimal timing under various constant growth rates showed that under all the specified cases of constant growth rates, social optimal timing $T^*$ is ‘now’ when $\rho I/(\delta - \gamma) < 1$ and $V(0) > 0$ and $T^* = \tilde{T}$ under a positive constant growth rate of demand ($\omega > 0$). In all other conditions, the project must be abandoned.

2) When the opening of service is not done at the optimal timing, loss is incurred. Investigation of the pattern of loss revealed that Too-Early Loss is more severe than Too-Late Loss, especially under a positive constant growth rate of net benefit. This affirms the value of delaying of the opening of service for projects implemented under this condition.

3) To effect a revision of the private optimal timing, various policy instruments are used in the Philippines. However, the most commonly used and the easily tractable investment is the Lump-sum financial subsidy. However, it is acceptable only when the social optimal timing is now. When $T^* = \tilde{T}$, LS financial subsidy is not acceptable from the viewpoint of equity. Moreover, when $T^* = 0$, depending on project characteristics, synchronization may be perfect or imperfect.

REFERENCES