Abstract: The terminal area has become the bottleneck of air traffic, so its stability plays a vital role in the steady operation of air traffic flow management system. In this paper, the following theory and stability theory on road traffic are used for reference to build the traffic following model of terminal area. Stability analysis and parametric analysis are then applied to the terminal area traffic management system. Finally, two terminal area traffic management system stability indices are presented.

Key Words: Terminal area, Stability, Following theory

1. INTRODUCTION

In recent years, with the continuous development of air transport, China's air traffic system is facing increasingly serious airspace and airport congestion, and they have caused large economic losses and hidden troubles. According to space, Air Traffic Flow Management (ATFM) can be divided into the following three parts: airport traffic flow management, terminal area traffic flow management (TATFM) and route traffic flow management. As the terminal area is the juncture of airport and route, and the approach and departure aircrafts should pass through the terminal area to land or climb up to the route, the terminal area always become the bottleneck of the entire system.

Many scholars have researched on the terminal area. They mainly concentrated on the algorithm of aircraft queue scheduling and evaluation of terminal area air traffic capacity (Jiang et al. 2003; Yang et al. 2005; Ma et al. 2003), and have achieved some results, eased the air traffic congestion, improved the utilization of the terminal area and reduced the workload of controller. But security and stability are the prerequisites and bases of air traffic management. If traffic is not steady, a small disturbance may narrow the interval between aircraft, or induce air miss and even collision. This not only adds the workload of the controller and aggravates traffic congestion, but also threatens the safety of air traffic.

Aircrafts’ follow-fly in the terminal area is an important phenomenon in the process of flying in queue. For the sake of reducing the wait time of aircraft on the ground and in the air, and improving the utilization of terminal area, controllers should optimize and schedule the aircraft queue. As the interval between scheduled aircraft is so short, and flight in surpass-forbidden air corridor, they are not checkless, aircrafts’ flight state may be changed because of some disturbances; and the changed flight state of any aircraft will ‘transfer’ to the back one in a particular way. This may lead to flight state change of the subsequent aircraft. This phenomenon is known as the Aircraft Follow-flying phenomenon. The following is the
theory which researches on the moving behavior of 'behind' vehicles following the 'front' vehicles when the queue of vehicles moves in a single lane using mathematical models to analyze and illuminate.

In this paper, a terminal area traffic following model based on the road traffic following theory was built. On this basis we analyzed the stability of the terminal area traffic. Finally, two indexes for the stability of terminal area traffic flow management system were proposed.

2. AIRCRAFTS’ FOLLOW-FLYING MODEL

Aircrafts’ follow-fly in the terminal area is an important phenomenon in the process of flying in queue. When an aircraft changes its flight state, the following aircraft can not take immediate measures because of the influence of pilot's reactive ability, which needs some reaction time. Aircraft follow-flying model is derived from the analysis of controllers’ and the pilots’ reactive characteristic. The controllers’ and the pilots’ reactive process can be summarized as the following four phases:

Sensory Phase: Collect relevant information, including the front aircraft's speed and relative velocity, the interval between aircraft, etc;
Cognitive Phase: Interpret the nature and significance of sensory information, and analyze the obtained information;
Decision-making Phase: Decide the fly strategy according to the experience and obtained information;
Implement Phase: Operate the aircraft according to the decision-making and the front aircraft traffic condition.

The time that the controllers or pilots spend in the above four phases is called reaction time. Before modeling aircrafts’ follow-flying, we make the following assumptions:

(1) Aircraft is divided into three categories, large, medium and small, using 1, 2, 3 denote them respectively, and maneuverability just relates to aircraft type;
(2) Flight rules include Visual Flight Rules (VFR) and Instrument Flight Rules (IFR). In VFR conditions, the pilots completely command of perception, cognition, decision-making and control; In IFR conditions, perception, cognition and decision-making are commanded by controllers, control by the pilots;
(3) The flying speed remains invariable during the reaction time;
(4) The reaction time between pilots or controllers is ignored

In the VFR, the reaction time $T_r$ is the sum of pilot’s perception, and decision-making and control time; In the IFR, the reaction time $T_i$ is the sum of controller’s perception, decision-making time and the pilot's control time. After the controller makes his decision, he should send the decision to pilot, and this will require communication time, so the reaction time in VFR is short than in IFR. We use $V$, $I$ denote VFR and IFR respectively, $T_r$ and $T_i$ denote the reaction time in VFR and IFR respectively.

The following model is a "stimulus-reaction" relationship. We derive the aircraft’s reaction from the interval and speed between and this aircraft(A/C) and the front one, thereby determining the aircraft’s flying state(Wang et al. 2004). Figure 1 is the sketch map of aircrafts’ follow-flying model.
Figure 1 Sketch map of aircrafts’ follow-flying model

\[ s(t) = x_n(t) - x_{n+1}(t) \] — A/C head space at times \( t \); \( n \) — the front aircraft; \( n+1 \) — the rear aircraft; \( u \) — the final speed; \( x_n(t) \) — \( n \) A/C position at times \( t \); \( x_{n+1}(t) \) — \( n+1 \) A/C position at times \( t \);

\[ d_1 = T_k u_{n+1}(t) - n+1 \] A/C flied distance during reaction time \( T_k \);

\( T_k \) — reaction time \( k \in \{V, L\} \);

\[ d_2 = n+1 \] A/C flied distance when decelerating to \( u \);

\[ d_3 = n \] A/C flied distance when decelerating to \( u \);

\( L_{ij} \) — the minimize A/C safety interval between type \( i \& j \); \( i \& j \) respectively denote the type of \( n \& n+1 \) A/C, \( i, j \in \{1, 2, 3\} \).

From the map we know:

\[ s(t) = x_n(t) - x_{n+1}(t) = d_1 + d_2 + L_{ij} - d_3 \] (1)

\[ d_1 = u_{n+1}(t)T_k = u_{n+1}(t + T_k)T_k = x_{n+1}'(t + T_k)T_k \] (2)

\[ d_2 = (u_n^2(t) - u^2) / 2a_i \] (3)

\[ d_3 = (u_n^2(t) - u^2) / 2a_j \] (4)

\( a_i \& a_j \) respectively denote aircraft accelerated performance of type \( i \& j \) A/C.

Put formula (2) into (1), and then derivate about \( t \) on both sides:

\[ u_n(t) - u_{n+1}(t) = u_n'(t + T_k)T_k + d_1' - d_3' \] (5)

\[ d_2' - d_3' = u_{n+1}(t)u_n'(t) / a_i - u_n(t)u_n'(t) / a_j \] (6)

\( u_n'(t) \& u_{n+1}'(t) \) respectively denote the acceleration of \( n \& n+1 \) A/C, they are proportional to the accelerated performance, the ratio coefficient is \( \eta \).

From (5) & (6), we know:

\[ (\eta + 1)[u_n(t) - u_{n+1}(t)] = u_{n+1}'(t + T_k)T_k \] (7)

Suppose \( \lambda_k = \eta + 1 / T_k \), we know:

\[ u_{n+1}'(t + T_k) = \lambda_k [u_n(t) - u_{n+1}(t)] \] (8)
In the formula, \( \lambda \) which is related with controllers’ workload and pilots’ motion intensity is named as reaction intensity coefficient. Its dimension is \( s^{-1} \).

In Wang (2002), supposes \( d_2 = d_3 \), braking distances of different vehicles are the same, then build the following model about road traffic. As there is not much difference between vehicles for low-speed road traffic, the braking distances can be considered the same. But for the high-speed air traffic, there is large difference on aircrafts’ performance and the deceleration distance is proportional to the square of aircraft velocity, if we approximately think they are the same, it will cause a greater error. In this paper, the following model suitable for air traffic is deduced, and a following model suitable for road traffic and air traffic is educed.

3. ANALYZE THE STABILITY OF TATFM SYSTEM

When an aircraft changes its flight state, the following aircraft can not take immediate measures because of the existence of reaction time, this will cause unstable following phenomenon and even crash. Therefore, it is necessary to analyze the stability of the following model to determine the threshold of reaction time that can ensure the stability of flight. The stability of TATFM system studies the influence that disturbance causes on the queue of aircraft flying in the terminal area (including approach and departure) when the disturbance disappears. If the disturbance transfers and strengthens in the queue, we say the TATFM system is unstable. Whereas, if the disturbance transfers and gradually disappears or remain in a certain range, we say the TATFM system is stable. The stability of the following model denoted by equation (8) can be divided into two categories: local stability and gradual stability (Wang, 2002). Local stability concerns on the following A/C reaction to the change of front A/C, that is concerning on the local cooperation. Gradual stability concerns on the performance of each A/C’s undulation characteristics in queue that is concerning on the undulation characteristics of the queue.

3.1 Local Stability Of TATFM System

For the local stability, Herman, Chow, Kometani (Jin and Wang, 2001) have proposed the method that uses the Laplace transform and its inversion transform to solve the differential equations of following model, and then discuss its stability at the end of the 1950s in the 20th century. Yang and Li (2002) use root-locus in automatic control theory to analyze the stability of following model, he gained the root-locus curve and its latent root. According to this, he proposed the delay parameter ranges about the following system stability. This method can be used to analyze the local stability of TATFM system, the conclusions are:

1. When \( \lambda_T \in [0, e^{-1}] \), the interval between A/C not oscillates;
2. When \( \lambda_T \in (e^{-1}, \pi / 2) \), the interval between A/C oscillates, but amplitude decreases in exponential;
3. When \( \lambda_T = \pi / 2 \), the interval between A/C oscillates, the amplitude remains the same;
4. When \( \lambda_T > \pi / 2 \), the interval between A/C oscillates, the amplitude increases.

Therefore, in order to allow the A/C queue remains stability we should ensure \( \lambda_T \leq e^{-1} \);
When \( e^{-1} < \lambda_T < \pi / 2 \), the disturbance oscillations but will soon disappear, this may be called stability as well.
3.2 Gradual Stability Of TATFM System

For equation (8), if amplitude increases, no matter what the initial interval between A/C, it bounds to cause dangerous approaches or collisions at a particular position in the rear of the queue. The numerical solution of (8) can determine the location where dangerous approach or collision occurred. The criterion of gradual stability is to determine whether the oscillation increases or decreases. The criterion is illuminated in Jin and Wang (2001) by using Fourier analysis.

\[ \lambda_k T_k < \frac{1}{2} \left[ \lim_{\omega \to 0} (\omega T_k) / \sin(\omega t) \right] \]  

(9)

\( (\omega T_k) / \sin(\omega t) > 1 \), the limit of the right of (9) is 0.5. That means it is gradual stability as long as \( \lambda_k T_k < 0.5 \) in despite of \( \omega \). The criteria divide the region into two parts: the stable (\( \lambda_k T_k < 0.5 \)) and the instability (\( \lambda_k T_k \geq 0.5 \)).

From the conditions of the local and gradual stability we know: The method to improve the stability is to decrease the reaction intensity coefficient \( \lambda \) or improve controllers and the pilots’ reaction ability.

3.3 Analysis The Stability Parameter Of TATFM System

From the conditions of the local and gradual stability we know: \( \lambda_k T_k \leq e^{-1} \) not only ensures local stability but also gradual stability. The shorter the reaction time \( T_k \) is, the larger the range of \( \lambda_k \) is. This is accord with the reality.

If the initial and final velocity of the following A/C is \( u_{n+1} \) & \( u_0 \) in respectively, we know:

\[ \int_0^\infty u_{n+1}'(t + T_k) dt = u_0 - u_{n+1} \]  

(10)

We can gain the variation of interval between A/C from (8):

\[ \int_0^\infty [u_n(t) - u_{n+1}(t)] dt = \Delta s \]  

(11)

That is

\[ \Delta s = \int_0^\infty [u_n(t) - u_{n+1}(t)] dt = \frac{1}{\lambda_k} \int_0^\infty u_{n+1}'(t + T_k) dt = \frac{u_0 - u_{n+1}}{\lambda_k} \]  

(12)

When \( \lambda_k T_k \leq e^{-1} \) happens, the interval between A/C doesn’t change in the fluctuant form. From the (12), we know the following aircraft variation of interval between A/C from the speed \( u_{n+1} \) to \( u_0 \). To ensure the safety of air traffic, considering the utmost situation --the final speed of A/C is the minimize speed of type \( i \) flying in the air that the A/C belongs to. The maximum change of interval is \( (u_{n+1} - u_{n+1}) / \lambda_k \). The minimum interval should be \( (u_{n+1} - u_{n+1}) / \lambda_k + L_y \) to prevent dangerous approach. To the possible extent, from small
workshops, should take the greatest possible value. The bigger the $\lambda_k$ is, the smaller the interval. But we should guarantee the stability of TATFM system, the ideal value is $(eT_k)^{-1}$.

From the analysis of parameter of following model, we can see: there are two ways to increase the terminal capacity on conditions meeting the stability of the TATFM system: increasing the intensity coefficient $\lambda_k$ and decreasing reaction time $T_k$.

4. THE ANALYSIS OF TATFM SYSTEM STABILITY INDEX

The final goal is not to know TATFM system stability. In order to better regulate and control the TATFM system, the operators and controllers need to know the degree of stability of flow management system. The criterions of local and gradual stability are given part 2, if TATFM system is stable. So what the degree of stability? Meanwhile, in order to guarantee the safety of the terminal area air traffic, the terminal area traffic should not be allowed to operate at the critical state, but should reserve some stability margin(He and Wen,2002). Therefore, there are two indexes of TATFM system stability:

(1) The degree of stability: The degree of stability is index for judging the degree of the terminal area traffic stability, and can be achieved by the ratio of actual flow and ultimate flow at the utmost stability (that is, the capacity of the terminal area at critical stability state), that is $\rho = \frac{q}{C_{sl}} \times 100\%$.

(2) Stability margin: The stability margin can be expressed by the coefficient stability reserves. The coefficient stability reserves of TATFM system is $K = \frac{C_{sl} - q}{q} \times 100\%$.

The technical and economic aspects must be considered when determine the stability index. If the stability index is too large, the stability of TATFM system is good, but it will reduce the traffic flow in normal situation, constraint the capacity of the terminal area, worsen the bottlenecks of the terminal area. If the stability index is too small, this will increase the traffic flow in normal situation and improve the capacity of the terminal area, but the operational safety and reliability is very low, A/C flying in the terminal area prone to dangerously close and even collision.

5. SIMULATION EXAMPLE

According to the actual situation, the controller or pilot's reaction time is $T = 2s$ in general. The following queue flled at the speed of $u$ at the beginning. When $t = 0$, the front A/C firstly accelerated for two seconds and then decelerated for two seconds both at the acceleration of $a_0 = 10m/s^2$. The performance of the A/C acceleration is perfect, the acceleration remained the same in the process of simulation.

From (8), we know the acceleration of rear A/C is
(1) When $\lambda T = e^{-1}$, simulate the process by Matlab, we can deduce the curve of the relative velocity and relative distance of the front and rear A/C as Figure 2 and 3 show.

(2) When $\lambda T = 1.6$, simulate the process by Matlab, we can deduce the curve of the relative velocity and relative distance of the front and rear A/C as Figure 4 and 5 show.
6. CONCLUSIONS

In this paper, the following theory and stability theory on road traffic are used as the reference to analyze TATFM system and build the traffic following model of terminal area. The model has better applicability and can be applied to road traffic and air traffic. And then we make stability and parametric analysis to the TATFM system based on the road traffic stability theory. We gain the criterion of TATFM system stability and measures to improve the stability and capacity. Finally, two TATFM system stability indexes are presented. We can deduce the conclusions from the result of the analysis: the measure to improve system stabilities that increase the capacity of the terminal is to improve the reaction ability of pilots and controllers on conditions that ensure the stability of the TATFM system. Furthermore, the reaction abilities of pilots and controllers not only have relation with their own operation quality, but
also closely relate to the intellectualized degree of the air transport system.

REFERENCES