Abstract: This paper studies location externality with a simple model. Firstly, the paper shows that Nash (competitive) equilibrium locations can differ from that of social optimum, and that under what conditions Nash equilibrium location is not equal to that of social optimum. Secondly, we demonstrate that, by examining the impacts of transportation cost on locations, how the Nash equilibrium location can be brought to the social optimum location by imposing the toll tax or subsidies.

Key words: Location Externality A, Social Optimum B, Toll tax /Subsidy C,

1. INTRODUCTION

Firms in an urban area interact with other firms in that area. In particular, offices holding headquarters, service and planning functions communicate with other offices with face-to-face communications. Firms located in an urban area to save face-to-face communication costs even though they should pay high rents. Firms need to interact and distance is an impediment to interaction. The trips performed between the firms are gravitational and its intensity is likely to increase with the number of firms’ setup in each location and decrease with the increase in distance between firms. The need to communicate acts as centripetal force, whereas competition for land and labour market (i.e. in the form of increased land rents, increased wage rates etc.) will act as nature of centrifugal force.

However, when a firm located in an urban area, it only considers its own communication cost ignoring the other firms’ communication cost. The location of firm actually affects the communication costs of other firms with which it is interacting. Accordingly, the market-based location of firms does not lead to the socially efficient location pattern. The socially efficient location pattern is more concentrated than the market-based location pattern because it needs to consider the effects of location of a firm on the interacting costs of other firms.

This externality that firms neglect the communication costs of the other firms is called “communication externality” in Fujita and Thisse (2002). The similar externalities have been studied by some other papers. Beckman (1976) shows by assuming that residents in-elastically communicate with all the urban residents in face-to-face; the population distribution is bell-shaped. Kanemoto (1990) shows that if face-to-face communication between firms itself is not any externalities, the transport cost associated with the communication yields an externality.
Kanemoto (1987) calls this externality as a location externality. This paper calls this externality "location externality"

The purpose of this paper is to study the location externality using a linear model and analyse the effect of location externality on transportation cost further to explore how to bring the equilibrium location patterns to social optimum patterns by imposing the transportation toll fee or by giving the subsidy to firms in the form of second-best measures. At the present we developed a linear model and shown by imposing the toll fee or by giving subsidy to the firms the equilibrium location patterns can be brought to social optimum location patterns.

This paper was structured as follows. Section 2 presents literature reviewed and measures available to cope with location externalities. Section 3 describes about the linear model, in which we try to explain the location externalities, and various location combinations of Nash equilibrium and social optimum patterns, furthermore location summary is given for various location patterns. In section 4 we explored how to bring the Nash equilibrium location patterns to social optimum location patterns by imposing the toll fee or by giving the subsidies. In section 5 conclusions are given further discussing the scope of research.

2. LITERATURE REVIEW

Beckman (1976) shows by assuming that residents in-elastically communicate with all the urban residents in face-to-face. More precisely, the utility of an individual depends on the average distance to all individual with whom this person interacts as well as on the amount of land he/she buys on the market. Under such preferences, the city exhibits a bell shaped population density distribution supported by a similar shaped land rent curve. This study is based on inelastic communications within the residents. Fujita and Thisse (2002) use the same model to show that if the communication cost is doubled, the market-based location of the residents can lead to the socially optimal distribution.

Face-to-face communications are needed between firms (in particular offices), rather than between residents. Such locations of firms are studied by Borukov and Hochman (1977), O’Hara (1977), and Fujita and Thisse (2002). They assume in-elastic face-to-face communication between firms (i.e., one trip between firms) to study the location pattern as Beckman (1976) assumes for residents. Accordingly, the essentially same results are obtained. Kanemoto (1990) shows that if face-to-face communication between firms itself is not any externalities, the transport cost associated with the communication yields an externality. Kanemoto (1987) calls this externality as a location externality. Kanemoto (1990) proposes the Pigouvian tax (Pigou(1920)) as the first best policy to internalize this externality. Specifically, the Pigouvian tax is the subsidy to the firm by locating close to the other firms by the amount that the other firms can save the transportation costs.

Ogawa and Fujita (1980), and Fujita and Ogawa (1982) had studied the locations of firms as well as residents in an urban area. These studies show that, the different ‘equilibrium land use patterns’ may emerge based on values of the economic parameters such as commuting cost and transactions cost. Mainly these studies focussed on space configuration, assuming each firm in a city must interact with all other firms in elastically (in Ogawa and Fujita (1980)) and with certain elasticity (in Fujita and Ogawa (1982)). However, they dealt only on the market equilibriums, but not the social optimal location patterns. Furthermore do not explore how to mitigate the communication externalities.
The recent study by Charlot (2004) elaborating on human capital externalities and mentions face-to-face meetings, word-of-mouth communication and direct interactions between skilled workers are a particular subset of human capital externalities, called communication externalities. Attempted to identify the communication externalities and distinguish from the other external effects of human capital and concluded that communication externalities are present in cities and serve as a conduit for a sizeable fraction of agglomeration effects. Shows that upto 22% of agglomeration effects percolate through communication externalities. Furthermore, the studies above do not explore how to internalize or mitigate the communication externalities in real economy.

From the above studies location externalities play a vital role between the firms for the movement of goods/services/business in a spatial structure. It affects the location of firms and consequently affects the urban structure. In the real economy, existence of location externality is inevitable. The proposed measures are unable to cope up with location externality in real economy. Hence there is a need to explore the possible measures to mitigate the location externality in real economy.

3. LINEAR MODEL

3.1 Explanation of Location Externality

‘When a firm moves from one location to other, it will get a benefit from the movement called internal benefit to that firm. By eventual shifting of this firm the other firm also get same amount of benefit, which is called as external benefit to other firm. This is due to existence of location externalities in a spatial structure.

In the presence of location externalities firms tries to locate closely to other firms to save face-to-face interaction costs to maximise their profits there by they can make any number of trips. Which implies by coming closer they are just ignoring the transportation costs. This represents the situation in real world. This ignoring the location impacts on other firms’ interaction costs called location externalities. Location externalities are negative from social stand point of view. In real economy firms locate considering only their own viewpoint, ignoring the effects of location on other firms’ interaction costs. In the presence of location externalities the firms are not densely packed (spatially located apart). When firms consider the location impacts on other firms interaction costs the firms try to locate closely than market-based pattern, there by leads to densely packed pattern (spatially located closely).

Hence, market-based distribution of firms in an urban structure is unlikely to be optimum. Indeed, from the past studies a comparison of market-based firm density and optimum firm density shows that the former is less concentrated than later. This is due to existence of location externalities among the firms resulting in an insufficient concentration of firms Further more location externality is explained in detail by using simple linear model as given below figure 1.
In figure 1 firm ‘A’ located at distance of $x_{1A}$ from CBD, $x_{AB}$ from firm ‘B’ and interacts with CBD1 and firm ‘B’. Similarly, firm ‘B’ located at distance of $x_{2B}$ from CBD2, and interacts with CBD2 and firm ‘A’. The transportation cost of firm ‘A’ is the sum of transportation cost input from CBD1 and intermediate input from firm ‘B’. Similarly, the transportation cost of firm ‘B’ is the sum of transportation cost input from CBD2 and intermediate input from firm ‘A’, this can be written as given in eq. (1) and (2):

Let the transportation cost of firm ‘A’

$$C_A = \alpha x_{1A} X_1 + \alpha x_{AB} X_B$$  \hfill (1)

Transportation cost of firm ‘B’

$$C_A = \alpha x_{2B} X_2 + \alpha x_{AB} X_A$$  \hfill (2)

Where $X_1, X_2$: Inputs from CBD1 and CBD2

$X_B$: Input from firm ‘B’

$X_A$: Input from firm ‘A’

$\alpha$: Unit transportation cost.

From equations 1 and 2, it is observed that the last term represents the intermediate inputs from firms ‘B’ and ‘A’ respectively. When a firm moves closer to another firm $\alpha x_{AB} X_B$ or $\alpha x_{AB} X_A$ gradually reduces and hence the eventual shifting of one firm closer to another firm reduces the transport cost of its own as well as other firm. This can be attributed to the typical urban externality called location externality.

In real situations the firms will choose its location only from individual point of view, ignoring social viewpoint. The Nash equilibrium condition can be stated as:

“The Transportation cost of firm ‘A’ $C_A$ does not decrease, by firm A’s moving, given firm B’s location is fixed, as well as The Transportation cost of firm ‘B’ $C_B$ does not decrease, by firm B’s moving, given firm A’s location is fixed, ”
From the above figure 2, the firm ‘A’ choose to locate at CBD$_1$, because of least transport cost at CBD$_1$.

From the figure 3, given the firm ‘B’ location the firm ‘A’ can locate anywhere from CBD$_1$ to CBD$_2$, however firm ‘A’ choose to locate at CBD$_1$, because of least total transport cost at CBD$_1$, regardless of the location of firm ‘B’. That is, in moving the firm ‘B’ closer to firm ‘A’, Similarly vice versa. A change in the location of a firm, changes the transport cost of the other firms as well as its own. Since an individual firm does not take into account the effects on other firms, this creates a typical urban externality– location externality.

From figure 4, the total transport cost when the firm ‘A’ choose to locate at CBD$_1$, firm ‘B’ choose to locate at CBD$_2$ is $A+B$ in Nash equilibrium.

In social optimum conditions, the combined cost of firms should be minimum and this is obvious from the Figure 4; minimum transport cost is $C$ when the firms A and B choose to locate at CBD$_2$. Similarly it is $D$ when firms A and B choose to locate at CBD$_1$. From social viewpoint, the total transport cost is $C$ or $D$ (depending upon locations of A and B), whereas from Nash equilibrium it is $A+B$ (when the firm A locate at CBD$_1$ and firm B locate at CBD$_2$). Depending on the specific combination of the above total transportation costs are (i.e. $A+B$, $C$ and $D$). It is possible to get different location patterns, such as Nash equilibrium, social optimum, or identical Nash equilibrium and social optimum depending upon different location patterns, these are further discussed in detail in model frame work next section.
3.2 Model framework.
As we seen in section 3.1 to examine the various location combinations of Nash equilibrium (market-based location pattern) and social optimum location pattern, the model has been developed assuming linear structure in the first place and the Nash equilibrium and social optimum location transportation costs are compared for various possible location combinations. A brief analysis has been presented on how to bring the Nash equilibrium location pattern to social optimum location pattern by giving subsidies or imposing taxes. Further, the model has been discussed below in detail in the subsequent paragraphs.

Assuming a linear structure, where firm ‘A’ will interact with firm ‘B’ at a distance of \( x_{AB} \) and CBD\(_1\) at distance of \( x_{1A} \). Similarly firm ‘B’ will interact with firm ‘A’ at distance of \( x_{AB} \) and CBD\(_2\) at distance of \( x_{2B} \) as shown in figure 5.

Notations:
- \( \alpha_i \): Slope of transport cost with respect to distance; input from CBD\(_1\) to Firm ‘A’
- \( \beta_{1A} \): Slope of transport cost with respect to distance; input from Firm ‘A’ to Firm ‘B’
- \( \beta_2 \): Slope of transport cost with respect to distance; input from CBD\(_2\) to Firm ‘B’
- \( \alpha_{2B} \): Slope of transport cost with respect to distance; input from Firm ‘B’ to Firm ‘A’
- \( x_{1A} \): Distance between CBD\(_1\) to Firm ‘A’
- \( x_{1B} = x_{1A} + x_{AB} \): Distance between CBD\(_1\) to Firm ‘B’
- \( x_{AB} \): Distance between Firm ‘A’ to Firm ‘B’
- \( l \): Distance between CBD\(_1\) to CBD\(_2\).

![Diagram](image-url)

Figure 5 Linear structure of firms in an urban spatial structure and the transportation cost.

3.2.1 Analysis of Nash equilibrium and social optimum.
In this section we discuss the possible location combinations for Nash equilibrium as well a social optimum. From the figure 5 suppose location of firm ‘B’ is fixed, as long as \( \delta = \) are equal, the location of \( x_a < x_b \) is superior than \( x_a > x_b \). Because the transportation cost input from CBD1 to firm ‘A’ is minimum. Hence the location of firm A \( x_a < x_b \) always superior than location of \( x_a > x_b \) and the \( x_a > x_b \) is normally does not occur. Further the analysis is focused on relating to \( x_a \leq x_b \).

3.2.2  Nash equilibrium.
In real situations the firms will choose its location only from individual point of view, ignoring social viewpoint. In Nash equilibrium, individual transport cost is considered:

Total transport cost of firm ‘A’
\[
C_A = \alpha_a x_a + \alpha_b |x_b - x_a|
\]  
(3)

Total transport cost of firm ‘B’
\[
C_B = \beta_a (l - x_b) + \beta_b |x_b - x_a|
\]  
(4)

Minimization of \( C_A \) given the location of firm B at \( X_B \), Minimization of \( C_B \) given the location of firm A at \( X_A \). The relevant Kuhn Tucker conditions are as follows.

Kuhn Tucker conditions:

<table>
<thead>
<tr>
<th>Given location Firm ‘B’</th>
<th>Given location Firm ‘A’</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_a = 0; \frac{\partial C_A}{\partial X_A} &gt; 0 )</td>
<td>( X_b = X_A; \frac{\partial C_B}{\partial X_B} &gt; 0 )</td>
</tr>
<tr>
<td>( 0 &lt; X_A &lt; X_B; \frac{\partial C_A}{\partial X_A} = 0 )</td>
<td>( X_A &lt; X_B &lt; l; \frac{\partial C_B}{\partial X_B} = 0 )</td>
</tr>
<tr>
<td>( X_A = X_B; \frac{\partial C_A}{\partial X_A} &lt; 0 )</td>
<td>( X_B = l; \frac{\partial C_B}{\partial X_B} &lt; 0 )</td>
</tr>
</tbody>
</table>

Nash equilibriums:

From the above Kuhn Tucker conditions the various possible location combinations are as given in Table 1.
### Table 1 Possible Nash equilibrium case studies

<table>
<thead>
<tr>
<th>Case</th>
<th>Value of ( \alpha_i ) and ( \alpha_B )</th>
<th>Value of ( \beta_2 ) and ( \beta_A )</th>
<th>Configuration /Total Transportation cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>( \alpha_i &gt; \alpha_B ) 0 ( \beta_2 &gt; \beta_A ) 1</td>
<td>( C_T = \alpha_B l + \beta_A l = l(\alpha_B + \beta_A) )</td>
<td></td>
</tr>
<tr>
<td>ii</td>
<td>( \alpha_i &gt; \alpha_B ) 0 ( \beta_2 &lt; \beta_A ) ( x_B = x_A )</td>
<td>( C_T = \beta_A l )</td>
<td></td>
</tr>
<tr>
<td>iii</td>
<td>( \alpha_i &lt; \alpha_B ) ( x_A = x_B ) ( \beta_2 &gt; \beta_A ) 1</td>
<td>( C_T = \alpha_A l )</td>
<td></td>
</tr>
<tr>
<td>iv</td>
<td>( \alpha_i &lt; \alpha_B ) ( x_A = x_B ) ( \beta_2 &lt; \beta_A ) ( x_B = x_A )</td>
<td>( C_T = \beta_2 l + (\alpha_1 - \beta_2)x_A )</td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>( \alpha_i = \alpha_B ) 0 ( x_A &lt; x_B ) ( \beta_2 = \beta_A ) 0 ( x_B &lt; l )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vi (a)</td>
<td>( \alpha_i = \alpha_B ) 0 ( x_A &lt; x_B ) ( \beta_2 &lt; \beta_A ) ( x_B = x_A )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vi (b)</td>
<td>( \alpha_i = \alpha_B ) 0 ( x_A &lt; x_B ) ( \beta_2 &gt; \beta_A ) ( x_B = l )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vii (a)</td>
<td>( \alpha_i &gt; \alpha_B ) ( x_A = 0 ) ( \beta_2 = \beta_A ) ( x_A &lt; x_B &lt; l )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>vii (b)</td>
<td>( \alpha_i &lt; \alpha_B ) ( x_A = x_B ) ( \beta_2 = \beta_A ) ( x_A &lt; x_B &lt; l )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The probability of occurrence of the cases from (v) to (vii) are very less, hence studied further it is focussed on first four cases only.

#### 3.2.3 Social Optimum: (Total cost minimization)

In the above section we have seen the possible Nash equilibrium cases, in this section we study the possible social optimum cases further to compare the Nash and social optimum transportation costs for various location settings. Unlike in Nash equilibrium, in social optimum the total transportation costs of both firms should be considered.

Total transport cost in social optimum can be written as:

\[
C_s = C_A + C_B
\]

\[
C_s = \alpha_A x_A + \alpha_B \left| x_B - x_A \right| + \beta_2 \left( l - x_B \right) + \beta_A \left| x_B - x_A \right|
\]

Kuhn Tucker conditions:

Precluding the inner solution, it is possible to have the four relevant conditions as given below

\[
X_A = 0; \frac{\partial C_s}{\partial X_A} > 0; X_B = 0; \frac{\partial C_s}{\partial X_B} > 0
\]
\[ \frac{\partial C_A}{\partial X_A} > 0; X_A = 0; \frac{\partial C_A}{\partial X_B} < 0; X_B = l \]  
(7.2)

\[ X_A = X_B; \frac{\partial C_A}{\partial X_A} < 0; X_B = X_A \text{(Anywhere)}; \frac{\partial C_A}{\partial X_B} > 0 \]  
(7.3)

\[ X_A = X_B; \frac{\partial C_A}{\partial X_A} < 0; X_B = X_A = l; \frac{\partial C_A}{\partial X_B} < 0 \]  
(7.4)

Using the eq from (5.1~ 5.6) and from (7.1) to (7.4), Nash equilibrium and social optimum case studies has been plotted as explained in the next section. It is essential to observe under the given location conditions whether the social optimum and Nash equilibrium costs are equal. Subsequently, classifying the results based on the transport cost conditions, it is possible to have total 16 location combinations as discussed below.

3.2.4 Plotting of Nash equilibrium versus social optimum for various location patterns.

As explained above there are total 16 possible locations patterns, where Nash and social optimum patterns can be discussed. Here we have discussed the first case as follows.

**Nash Equilibrium**

From eq (5.1) where \( \alpha_i > \alpha_B \) firm ‘A’ can locate any where between \( x_A = 0 \sim l \), however when \( x_A = 0 \) the total transportation \( C_A \) is minimum, hence firm ‘A’ choose to locate \( x_A = 0 \), at CBD1. Similarly, from eq.(5.6) as long as \( \beta_A > \beta_B \) firm ‘B’ can locate between \( x_B = l \sim 0 \), however when \( x_B = l \) the total transport \( C_B \) is minimum, hence firm ‘B’ choose to locate \( x_B = l \) i.e at CBD2 as shown in Figure 6.

From eq (5.1) and (5.6), total transportation cost in Nash equilibrium

\[ \alpha_i > \alpha_B \Rightarrow x_A = 0; \beta_A > \beta_B \Rightarrow x_A = l \]  
(8.1)

**Social Optimum**

Total transportation cost in social optimum can be written using the eq (7.1), where

\[ \frac{\partial C_A}{\partial X_A} > 0; x_A = 0 \Rightarrow (\alpha_i - \alpha_B - \beta_A) > 0 \Rightarrow (\alpha_A + \beta_A) < \alpha_i \]  
(8.2)

\[ \frac{\partial C_B}{\partial X_B} > 0; x_B = 0 \Rightarrow (\alpha_B + \beta_A - \beta_B) > 0 \Rightarrow (\alpha_A + \beta_A) > \beta_B \]

Figure 6 Nash equilibrium versus social optimum

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Plotting the above Nash equilibrium and social optimum conditions as shown in the figure 6, using the above eq (8.1) and (8.2). From this total transportation cost in Nash \(((\alpha_b + \beta_x)l)\) is not equal to social optimum transportation cost \((\beta_xl)\). Nash is not equal to social optimum.

Similarly, all other cases the Nash and social optimum location patterns are plotted, and shown in figure 7.

3.2.5 Locations summary.
As explained above, all the Nash and social optimum are plotted for all the possible location conditions as shown in figure 7. A close look at the above plots implies that five conditions of Nash and social optimum are same whereas other five conditions of Nash and social optimum are not equal. Further, the plots further reveal that six other of Nash and social optimum are not compatible for the given column and row conditions. From the above results it is clearly indicates that due to presence of communication externality Nash and social optimum are not equal.

4. HOW TO BRING NASH EQUILIBRIUM LOCATIONS TO SOCIAL OPTIMUM.

After plotting the Nash and social optimum for all possible location conditions, where out of total 16 locations conditions only five locations Nash and social optimum are equal, rest six locations Nash and social optimum costs are not equal. Further it is essential to examine that how to bring the locations patterns where Nash and social optimum are not equal because of the presence of communication externality.

There are two ways of bringing the Nash equilibrium locations to social optimum. One way is by giving the subsidy to the firms, and other way is by imposing the taxes. These are worked out as follows.

Nash Equilibrium
Case I) from eq.(5.1) and eq.(5.6)  
\[ \alpha_i > \alpha_y \Rightarrow X_a = 0; \beta_z > \beta_A \Rightarrow X_B = l \]  
\[ (9.1) \]

Social Optimum
Case I) from eq.(7.1)  
\[ \frac{\partial C_x}{\partial X_a} > 0; X_a = 0 \Rightarrow (\alpha_y + \beta_A) < \alpha_i \]  
\[ \frac{\partial C_x}{\partial X_B} > 0; X_B = 0 \Rightarrow (\alpha_y + \beta_A) > \beta_2 \]  
\[ (9.2) \]

Using the above eq (9.1) and (9.2) the Nash and social optimal locations are plotted as shown in figure 8.
Figure 7 Comparison of Nash equilibrium and social optimum patterns

<table>
<thead>
<tr>
<th>Nash</th>
<th>Social Optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_i &gt; a_j \Rightarrow X_i = 0, \beta_i &gt; \beta_j \Rightarrow X_j = 1 )</td>
<td>Under these row and column conditions Nash equilibrium and social optimum are not compatible</td>
</tr>
<tr>
<td>From eq.(7.1) ( \frac{\partial C_s}{\partial a_i} &gt; 0, X_i = 0 ) ( (a_i - \alpha_i - \beta_i) &gt; 0 ) ( (\alpha_i + \beta_i) &lt; \alpha_j ) ( a_i = \beta_i ) ( \beta_i = \beta_j ) Nash ≠ Social</td>
<td></td>
</tr>
<tr>
<td>Under these row and column conditions Nash equilibrium and social optimum are not compatible</td>
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<tr>
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<td>Under these row and column conditions Nash equilibrium and Social Optimum are not compatible</td>
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</tr>
<tr>
<td>From eq.(7.2) ( \frac{\partial C_s}{\partial a_i} &gt; 0, X_i = 0 ) ( (a_i - \alpha_i - \beta_i) &lt; 0 ) ( (\alpha_i + \beta_i) &lt; \beta_j ) Nash = Social optimum</td>
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<tr>
<td>Under these row and column conditions Nash equilibrium and social optimum are not compatible</td>
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<td>Under these row and column conditions Nash equilibrium and social optimum are not compatible</td>
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<td>Under these row and column conditions Nash equilibrium and Social Optimum are not compatible</td>
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<tr>
<td>From eq.(7.3) ( \frac{\partial C_s}{\partial a_i} &lt; 0, X_i = X_j ) ( (a_i - \alpha_i - \beta_i) &lt; 0 ) ( (\alpha_i + \beta_i) &gt; \alpha_j ) ( a_i = \beta_i ) ( \beta_i = \beta_j ) Nash = Social optimum</td>
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<tr>
<td>Under these row and column conditions Nash equilibrium and Social Optimum are not compatible</td>
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</tr>
<tr>
<td>From eq.(7.4) ( \frac{\partial C_s}{\partial a_i} &lt; 0, X_i = X_j ) ( (a_i - \alpha_i - \beta_i) &lt; 0 ) ( (\alpha_i + \beta_i) &gt; \alpha_j ) ( a_i = \beta_i ) ( \beta_i = \beta_j ) Nash = Social optimum</td>
<td></td>
</tr>
<tr>
<td>Under these row and column conditions Nash equilibrium and social optimum are not compatible</td>
<td></td>
</tr>
</tbody>
</table>
By imposing the tax the Nash equilibrium location can be bring to social optimum location pattern as shown in figure 9.

Total transport cost of Firm ‘B’

\[ C_B = \beta_2 (l - x_B) + \beta_A (x_B - x_A) \]  
(9.3)

Minimizing the transport cost of firm ‘B’ by introducing tax ‘S’

\[ \min_{(x_B, S)} C_B = \beta_2 (l - x_B) + \beta_A (x_B - x_A) + S(x_B) \]  
(9.4)

\[ \frac{\partial C_B}{\partial x_B} = -\beta_2 + \beta_A + \frac{\partial S}{\partial x_B} \]  
(9.5)

The relocation of firm ‘B’ from \( x_B = l \) to \( x_B = 0 \) to attain social optimum as shown in figure 9.

\[ X_B = 0, \frac{\partial C_B}{\partial X_B} > 0 \Rightarrow -\beta_2 + \beta_A + \frac{\partial S}{\partial X_B} > 0 \]  
(9.6)

\[ \Rightarrow \frac{\partial S}{\partial X_B} > \beta_2 - \beta_A \]  
(9.7)

In moving the firm ‘B’ location from \( x_B = l \) to \( x_B = 0 \) the tax to be imposed on the firm ‘B’ would be \( \frac{\partial S}{\partial X_B} > \beta_2 - \beta_A \) to attain social optimum

5. CONCLUSIONS

In this paper, the model has been developed assuming linear structure of firms (a linear urban space). The Nash and social optimums has been plotted for all the possible conditions of location patterns. A close look at the above plots implies that five conditions of Nash and social optimum are same, whereas other five conditions of Nash and social optimum are not equal, which could be due to the presence of the location externalities. Further we explored how to bring the Nash equilibrium location patterns where equilibrium transportation cost is not equal to the social optimum transportation cost. This problem can be overcome by imposing taxes for the appropriate firms or by offering subsidies so as to minimize the total transportation cost. However, this model is only suitable for the linear type of structure. But in reality existence of this pattern is rather minimal. Hence the generalization of model by accommodating number firms by taking trip elasticity in to account as a future scope of work to suite the real conditions would be appropriate.

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