**Abstract:** The objective is to obtain the most appropriate transport demand models which can likely represent the behavior of port-to-port sea freight movements in terms of OD matrices. The model is developed for the purpose of forecasting the Indonesian sea freight movements using the current OD pattern and the forecasted loading and unloading volumes. The paper will report on a family of aggregate models containing a flexible Gravity-Opportunity model for modeling the trip making behavior in which standard forms of the Gravity and Intervening-Opportunity model can be obtained as special cases. Non-Linear-Least-Squares and Maximum-Likelihood estimation methods were then used to calibrate the parameter of the model. The models have been tested using the total domestic sea freight movements in 2003 (Stramindo 2003 Data) for 25 major ports in Indonesia. The models were found to provide a reasonably good fit and the calibrated parameters can then be used for forecasting purposes.

**Key Words:** Freight Movement, Freight Demand Modelling, Gravity-Opportunity Model

**1. INTRODUCTION**

The concept of an “Origin-Destination (OD) matrix” has been adopted by transport planners to represent the most important features of this travel pattern. An OD matrix is a two-dimensional matrix which contains information about the scale of travel between any sub-areas within a predefined area. The row labels usually represent the origins and the column labels represent the destinations, therefore, the entries in the cells of the matrix represent the corresponding OD flows. In this case, the notation $T_{id}$ will represent the total number of trips (vehicles, persons or tonnes of certain commodities) travelling form origin $i$ to destination $d$ over a period of time. The methods of estimating the future OD matrix can be divided into two main methods: growth-factor methods and synthetic methods. In growth-factor methods, growth rates are applied to present-day inter-zonal movements to estimate future OD matrix. In synthetic methods, an attempt is made to model the causal relationship behind patterns of movement. The types of synthetic methods include: gravity (GR) model, opportunity (OP) model and gravity-opportunity (GO) model. The standard forms of the OP and GR models can be obtained as special cases of the GO model.

Calibration process of the existing OD matrix is needed to obtain parameters of the proposed OD matrix which is used to estimate the future OD matrices. Tamin (1988,2003) has developed 2 (two) main groups of estimation methods that can be used to calibrate the parameters of proposed OD matrix. They are: Least-Squares (LS) and Maximum-Likelihood (ML) estimation methods. The main ideas behind these estimation methods is that we try to calibrate the unknown parameters of the postulated model so that to minimize the deviations.
or differences between the OD matrix estimated by the calibrated model and the observed OD matrix. This can be carried out by using the likelihood measure between the estimated and observed OD matrix, e.g. maximum-likelihood or least-squares.

The objective of the study is to obtain the most appropriate transport demand models which can likely represent the behavior of port-to-port sea freight traffic movements in terms of OD matrix. It can be used to estimate the future sea freight traffic demand using the current OD pattern and the forecasted loading and unloading volume. The paper will report on the use of gravity (GR) and gravity-opportunity (GO) model. Non-Linear-Least-Squares (NLLS) and Maximum-Likelihood (ML) estimation methods were used to calibrate the model parameters.

2. TRANSPORT DEMAND MODEL

2.1 Gravity (GR) Model

The Gravity (GR) model is developed by analogy with Newton's law of gravitation. Newton asserted that the force of attraction, \( F_{id} \), between two bodies is proportional to the product of their masses, \( m_i \) and \( m_d \), divided by the square of the distance between them (\( d_{id}^2 \)). The analogous transport gravity model is:

\[
T_{id} = k \frac{O_i O_d}{d_{id}^2}
\]

This model has some sensible properties. It says that the number of trips from zone \( i \) to zone \( d \) is directly proportional to each of \( O_i \) and \( D_d \) and inversely proportional to the square of the distance between them. Hence, if a particular \( O_i \) and a particular \( D_d \) are each doubled, then the number of trips between these zones would quadruple according to equation (1), when one would be expected that they would only double. Therefore, the following constraint equations on \( T_{id} \) should always be required, such constraints are not satisfied by equation (1):

\[
\sum_d T_{id} = O_i \quad \text{and} \quad \sum_i T_{id} = D_d
\]

where \( O_i \) and \( D_d \) directly represent the total number of trips originating and terminating at \( i \) and \( d \) respectively. These constraint equations can be satisfied if sets of constants \( A_i \) and \( B_d \) associated with production zones and attraction zones respectively are introduced. They are sometimes called ‘balancing factors’.

Assuming that there are \( K \) trip purposes or trip commodities traveling between zones within the study area, then the modified gravity model can be expressed as:

\[
T_{id} = \sum_k T_{id}^k
\]

\[
T_{id}^k = b_k O_i^k D_d^k A_i^k B_d^k f_{id}^k
\]

where:

- \( b_k \) = a scaling parameter which enable us to use different unit between \( T_{id} \) and \( O_i, D_d \)
- \( f_{id}^k \) = the deterrence function = \( f(C_{id}) \)
- \( A_i^k, B_d^k \) = the balancing factors which can be obtained by constraining the following terms:

\[
\sum_d T_{id}^k = b_k O_i^k \quad \text{and} \quad \sum_i T_{id}^k = b_k D_d^k
\]

Hence,

\[
A_i^k = \frac{1}{\sum_d (B_d^k D_d^k f_{id}^k)} \quad \text{and} \quad B_d^k = \frac{1}{\sum_i (A_i^k O_i^k f_{id}^k)}
\]
The equations for $A_i^k$ and $B_d^k$ are solved iteratively, and it can be easily checked that they ensure that $T_{id}^k$ given in equation (3) satisfies the constraint equation (4). This process is repeated until the values of $A_i^k$ and $B_d^k$ converge to certain unique values.

So far, there is no reason to think that distance plays the same role in transport. Hence, a general function of time, distance or generalized cost, normally called as ‘deterrence function’, is introduced. There are three types of deterrence functions being used in this study which are also shown in Figure 1, namely:

- $f^k(C_{id}) = C_{id}^{-\alpha_i}$ (‘Power’ function)  
  \[ \text{(6)} \]

- $f^k(C_{id}) = e^{-\beta_i C_{id}}$ (‘Exponential’ function)  
  \[ \text{(7)} \]

- $f^k(C_{id}) = C_{id}^{-\alpha_i} e^{-\beta_i C_{id}}$ (‘Tanner’ function)  
  \[ \text{(8)} \]

![Figure 1 Types of Deterrence Functions](image)

### 2.2 Gravity-Opportunity (GO) Model

#### 2.2.1 Background

Wills (1986) developed a flexible gravity-opportunity (GO) model for trip distribution in which standard forms of the gravity and intervening-opportunity model are obtained as special cases. Hence the question of choice between gravity or intervening-opportunity approaches is decided empirically and statistically by restrictions on parameters which control the global functional form of the trip distribution mechanism.

#### 2.2.2 Definitions

An ordered OD matrix. Let origins and destinations be numbered consecutively in the usual way, such that $i=1,2,..,I$ are origins and $d=1,2,..,J$ are destinations, and let $T_{id}$ be the observed trips from origin $i$ to destination $d$. Define now a transformation $d_{ij}^\theta$ for each origin $i$ such that:

$$d_{ij}^\theta = \begin{cases} 1 & \text{if destination } d \text{ is the } j^{th} \text{ position in ascending order of distance away from } i \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

and then the ordered OD matrix can be obtained by the following transformation as:

$$Z_{ij} = \sum_d (\delta_{jd} T_{id}) \tag{10}$$
Thus, $Z_{ij}$ represents the trips from origin $i$ to the $j^{th}$ destination ranked by distance away from $i$. Note that $j$ is always defined as a function of $i$, so it is perhaps more correctly designated as $j(i)$ but for notational simplicity we omit the $i$ as being understood. While the ordering transformation $\delta_{jd}$ produces an ordered OD matrix, its inverse $\delta_{jd}^{-1}$ allows the observed OD matrix to be recovered by:

$$T_{id} = \sum_d (\delta_{jd}^{-1} Z_{id})$$

(11)

It should be noted that this part of transformations is applicable to any variable based on the OD matrix, notably the trip cost matrix, the proportionality factor and the destination balancing factor, in addition to the OD matrix.

Normalization. To achieve the logical consistency such that the sum (over destinations) of the estimated trips for each origin $i$ is equal to the observed trips generated at $i$ and similarly, for the sum (over origins) of the estimated trips for each destination $j$ is equal to the observed trips generated at $j$, then the two following constraints are required.

$$O_i = \sum_j Z_{ij}$$

(12a)

$$D_j = \sum_d (\delta_{jd} D_d)$$ and

$$D_d = \sum_i \left[ \sum_j (\delta_{jd}^{-1} Z_{ij}) \right]$$

(12b)

Transformations. In order to provide a monotonic scaling of variables in such a manner as to generate families of specific functional forms, the Box-Cox transformations is used. The direct Box-Cox transformation of a variable $y$ can be defined as:

$$y^{(\varepsilon)} = \begin{cases} \frac{(y^\varepsilon - 1)}{\varepsilon} & \varepsilon \neq 0 \\ \log y & \varepsilon = 0 \end{cases}$$

(13)

and the inverse Box-Cox transformation as:

$$y^{1/\varepsilon} = \begin{cases} (y^{1/\varepsilon} + 1)^{1/\varepsilon} & \varepsilon \neq 0 \\ \exp y & \varepsilon = 0 \end{cases}$$

(14)

These transformations may be combined into a new function which we introduce as a convex combination in $\mu$.

$$y^{(\varepsilon, \mu)} = \mu y^{(\varepsilon)} + (1 - \mu) y^{(1/\varepsilon)} \quad \text{with } 0 \leq \mu \leq 1$$

(15)

2.2.3 Specification of the Opportunity Function

A key step in the integration of both models is the specification of an opportunity function which has as arguments destination-attribute variables such as population, income or some other measures of opportunities and generalized cost or trip impedance variables relating origin and destination. The opportunity function $U_{ip}$ relates $i$ and the $p^{th}$ destination away from $i$ and is defined generally as:

$$U_{ip} = \exp \left((1 - \varepsilon) \alpha D_p \cdot \Omega - \beta C_{ip} \cdot \Phi \right)$$

(16)

$U_{ip}$ is defined here as a combined vector of intervening-opportunity factors and impedances. The term $(1-\varepsilon)$ ensures that, when $\varepsilon=1$ then the gravity model is obtained and the destination intervening-opportunity effect is removed. These impedances weight the intervening-opportunity by their location to origin and destination, generally the closer the intervening-
opportunity to an origin the greater the impact on travel between i and j. Table 1 shows the specification of the opportunity function depending on the value of parameters Ω and Φ.

### Table 1 Specification of the Opportunity Function

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Intervening-Opportunity</th>
<th>Impedance</th>
<th>$U_{ip}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega$</td>
<td>$\Phi$</td>
<td>$\exp[(1-\varepsilon) \cdot \alpha \cdot D_{ip}^{(\Omega)}]$</td>
<td>$\exp(-\beta \cdot C_{ip}^{(\Phi)})$</td>
<td>$\exp[(1-\varepsilon) \cdot \alpha \cdot D_{ip}^{(\Omega)} - \beta \cdot C_{ip}^{(\Phi)}]$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$\exp[(1-\varepsilon) \cdot \alpha \cdot D_{ip}^{(\Omega)}]$</td>
<td>$\exp(-\beta \cdot C_{ip}^{(\Phi)})$</td>
<td>$\exp[(1-\varepsilon) \cdot \alpha \cdot D_{ip}^{(\Omega)} - \beta \cdot C_{ip}^{(\Phi)}]$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$D_{pi}^{\alpha \cdot (1-\varepsilon)}$</td>
<td>$C_{ip}^{-\beta}$</td>
<td>$D_{pi}^{\alpha \cdot (1-\varepsilon)} \cdot C_{ip}^{-\beta}$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$\exp[(1-\varepsilon) \cdot \alpha \cdot D_{ip}^{(\Omega)}]$</td>
<td>$C_{ip}^{-\beta}$</td>
<td>$\exp[(1-\varepsilon) \cdot \alpha \cdot D_{ip}^{(\Omega)} - \beta \cdot \log \varepsilon C_{ip}^{(\Phi)}]$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$D_{pi}^{\alpha \cdot (1-\varepsilon)}$</td>
<td>$\exp(-\beta \cdot C_{ip}^{(\Phi)})$</td>
<td>$\exp[(1-\varepsilon) \cdot \alpha \cdot \log \varepsilon D_{ip}^{(\Omega)} - \beta \cdot C_{ip}^{(\Phi)}]$</td>
</tr>
</tbody>
</table>


#### 2.2.4 Structure of the Proportionality Factor

The opportunity function is incorporated into a general proportionality factor $F_{ij}$, which is defined by the difference in functions of the cumulative opportunities from i to the jth destination away from i, and from i to the (j-1)th destination away from i, and can be defined as:

$$F_{ij} = X_{ij} - X_{ij-1}$$ (17)

The most general form of the cumulative opportunities to be considered here defines $X_{ij}$ and $X_{ij-1}$ as:

$$X_{ij} = \left(\sum_p U_{ip}\right)^{(\varepsilon, \mu)} \quad \text{and} \quad X_{ij-1} = \left(\sum_p U_{ip}\right)^{(\varepsilon, \mu)}$$ (18)

Where $(\varepsilon, \mu)$ transformation is defined by equations (13)-(15). Substitution of equation (18) into equation (15) leads to the general proportionality factor form as:

$$F_{ij} = \left(\sum_p U_{ip}\right)^{(\varepsilon, \mu)} - \left(\sum_p U_{ip}\right)^{(\varepsilon, \mu)}$$ (19)

The general proportionality factor is subjected to a convex combination of direct and inverse Box-Cox transformations. The form given by equation (18) generates two branches of special cases: the direct-opportunity (DO) model, with $\mu=1$, and the inverse-opportunity (IO) model, with $\mu=0$. The DO model is significant because it contains the important special case of the logarithmic-opportunity (LO) model, with $\varepsilon=0$. That is:

$$F_{ij} = \log \varepsilon \left(\sum_p U_{ip}\right) - \log \varepsilon \left(\sum_p U_{ip}\right)$$ (20)

The IO model is particularly important because it contains the exponential-opportunity (EO) model, again with $\varepsilon=0$. That is:

$$F_{ij} = \exp \left(\sum_p U_{ip}\right) - \exp \left(\sum_p U_{ip}\right)$$ (21)

We can also consider blends of the LO and EO models, without going to the full GO model, by taking a convex combination of equations (19) and (20) with the mixture depending on values of $\mu$. This blended form, the blended-opportunity (BO) model, is given in equation (22) as:

$$F_{ij} = \mu \left[\log \varepsilon \left(\sum_p U_{ip}\right) - \log \varepsilon \left(\sum_p U_{ip}\right)\right] + (1-\mu) \left[\exp \left(\sum_p U_{ip}\right) - \exp \left(\sum_p U_{ip}\right)\right]$$ (22)
Finally, we observe that if $\varepsilon=1$, for $0 \leq \mu \leq 1$, the gravity (GR) model is revealed as:

$$F_{ij} = \left( \sum_{p} U_{ip} \right) - \left( \sum_{p} U_{jp} \right) = U_{ij}$$

(23)

showing that the standard GR model can be obtained as a special case of the GO model. As mentioned, different values of the parameters controlling these transformations generate contrasting families of models, notably the exponential-opportunity (EO) model, the logarithmic-opportunity (LO) model and the gravity (GR) model, see Table 2. All models are shown to be embedded in a transformed triangular region over which likelihood function, response surface or simultaneous confidence interval contours may be plotted as shown in Figure 2.

Figure 2 Diagrammatic Structure of the Proportionality Factor and Its Special Cases Source: Wills (1986)

Having all the assumptions, the proposed GO model is therefore:

$$T_{id} = \sum_{k} \left( b_{k} \cdot O_{id}^{k} \cdot D_{di}^{k} \cdot B_{d}^{k} \cdot f_{id}^{k} \right)$$

(24)

where:

- $A_{i}$ and $B_{d}$ are defined as equations (5)
- $f_{id}^{k} = \sum \left( \delta_{i,j}^{k-1} \cdot F_{ij}^{k} \right)$
- $F_{ij}^{k} = \left( \sum_{p} U_{ip}^{k} \right)^{\left( \varepsilon, \mu \right)} - \left( \sum_{p} U_{jp}^{k} \right)^{\left( \varepsilon, \mu \right)}$
- $U_{ip}^{k} = \exp \left[ \left( 1 - \varepsilon \right) \cdot \alpha_{k} \cdot D_{pk}^{(\Omega)} - \beta_{k} \cdot C_{ip}^{(\Phi)} \right]$
- $D_{jk}^{i} = \sum_{d} \left( \delta_{i,j}^{k} - D_{d}^{k} \right)$
- the $(\Omega, \Phi)$ parameters were chosen, in advance, externally to the main calibration process, see Table 1.
- the $(\varepsilon, \mu)$ transformation is defined by equations (13)–(15), see also Table 2.

3. ESTIMATION METHODS
3.1 Non-Linear-Least-Squares estimation method (NLLS)

The main idea in the calibration procedure is to estimate the unknown parameters by minimizing the sum of the squared differences between the estimated and observed OD matrix. That is to minimize the following equation:
Table 2 The Specification of Proportional Factor

<table>
<thead>
<tr>
<th>Form</th>
<th>$\mu$</th>
<th>$\varepsilon$</th>
<th>Cumulative Opportunities ($X_{ij}$)</th>
<th>Proportional Factor ($F_{ij}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GO</td>
<td>$0 \leq \mu \leq 1$</td>
<td>$0 \leq \varepsilon \leq 1$</td>
<td>$(\sum_p U_{ip})^{(\varepsilon, \mu)}$</td>
<td>$(\sum_p U_{ip})^{(\varepsilon, \mu)} - (\sum_p j^{-p} U_{ip})^{(\varepsilon, \mu)}$</td>
</tr>
<tr>
<td>LO</td>
<td>1</td>
<td>0</td>
<td>$\log_e (\sum_p U_{ip})$</td>
<td>$\log_e (\sum_p U_{ip}) - \log_e (\sum_p j^{-p} U_{ip})$</td>
</tr>
<tr>
<td>DO</td>
<td>1</td>
<td>$0 \leq \varepsilon \leq 1$</td>
<td>$(\sum_p U_{ip})^{(\varepsilon)}$</td>
<td>$(\sum_p U_{ip})^{(\varepsilon)} - (\sum_p j^{-p} U_{ip})^{(\varepsilon)}$</td>
</tr>
<tr>
<td>GR</td>
<td>$0 \leq \mu \leq 1$</td>
<td>1</td>
<td>$(\sum_p U_{ip})$</td>
<td>$U_{ip}$</td>
</tr>
<tr>
<td>IO</td>
<td>0</td>
<td>$0 \leq \varepsilon \leq 1$</td>
<td>$(\sum_p U_{ip})^{(1/\varepsilon)}$</td>
<td>$(\sum_p U_{ip})^{(1/\varepsilon)} - (\sum_p j^{-p} U_{ip})^{(1/\varepsilon)}$</td>
</tr>
<tr>
<td>EO</td>
<td>0</td>
<td>0</td>
<td>$\exp(\sum_p U_{ip})$</td>
<td>$\exp(\sum_p U_{ip}) - \exp(\sum_p j^{-p} U_{ip})$</td>
</tr>
<tr>
<td>BO</td>
<td>$0 \leq \mu \leq 1$</td>
<td>0</td>
<td>$\mu \log_e (\sum_p U_{ip}) + (1 - \mu) \exp(\sum_p U_{ip})$</td>
<td>$\mu \left[ \log_e (\sum_p U_{ip}) - \log_e (\sum_p j^{-p} U_{ip}) \right] + (1 - \mu) \left[ \exp(\sum_p U_{ip}) - \exp(\sum_p j^{-p} U_{ip}) \right]$</td>
</tr>
</tbody>
</table>

to minimize \( S = \sum_{i} \sum_{d} \left[ \frac{1}{T} (T_{id} - \hat{T}_{id})^2 \right] \) \( (29) \)

Where: \( \hat{T} = 1 \) for NLLS and \( \hat{T} = \hat{T}_{id} \) for WNLLS

The following set of equation (30) is required in order to find a set of unknown parameter \((\alpha_k, \beta_k)\) of the GO model which minimizes equation (29).

\[
\frac{\partial S}{\partial \alpha_k} = f\alpha_k = \sum_{i} \sum_{d} \left[ \frac{1}{T} \left( 2(T_{id} - \hat{T}_{id}) \frac{\partial T_{id}}{\partial \alpha_k} \right) \right] = 0
\]

\[
\frac{\partial S}{\partial \beta_k} = f\beta_k = \sum_{i} \sum_{d} \left[ \frac{1}{T} \left( 2(T_{id} - \hat{T}_{id}) \frac{\partial T_{id}}{\partial \beta_k} \right) \right] = 0
\]

The equations (30) are a system of \(2K\) simultaneous equations which has \(2K\) unknown parameters \(\alpha_k\) and \(\beta_k\) for estimation. Newton’s method combined with the Gauss-Jordan Matrix Elimination technique can then be used to solve equation (30).

3.2 Maximum-Likelihood estimation method (ML)

Let assume that \(p_{id}\) be the probability of having an estimated independent trip interchange for each particular origin \(i\) and destination \(d\) expressed as:

\[
p_{id} = \frac{T_{id}}{\hat{T}_{i}}
\]

where:

\[
\hat{T}_{i} = \sum_{d} \hat{T}_{id}
\]

The framework of the Maximum-Likelihood (ML) estimation method is that the choice of the hypothesis \(H\) maximizing equation (31), will yield a distribution of \(T_{id}\) giving the best possible fit to the survey data \((\hat{T}_{id})\). The objective function for this framework is expressed as:

\[
L = e^{\sum_{i} \sum_{d} p_{id} \hat{T}_{id}}
\]

(33)

By subtituting equation (31–32) into equation (33) and taking the natural logarithm term of equation (33) and using the following ‘Lagrangian Multiplier’ method, equation (33) can then be written into a single equation. That is:

\[
to maximise \quad L_1 = \sum_{i} \sum_{d} (\hat{T}_{id} \log_e T_{id}) - \hat{T}_{i} \log_e \hat{T}_{i} + \log_e e
\]

(34)

By omitting the constant value of equation (34), the objective function can then be simplified and expressed as:

\[
to maximise \quad L_2 = \sum_{i} \sum_{d} \hat{T}_{id} \log_e T_{id} \quad \text{with respect to parameters} \ (\alpha, \beta) \quad (35)
\]

In order to determine uniquely parameters \((\alpha_k, \beta_k)\) of the GO model which maximizes equation (35), the following sets of equations are then required. They are as follows:

\[
\frac{\partial L_2}{\partial \alpha_k} = f\alpha_k = \sum_{i} \sum_{d} \left[ \frac{\hat{T}_{id}}{T_{id}} \frac{\partial T_{id}}{\partial \alpha_k} \right] = 0
\]

\[
\frac{\partial L_2}{\partial \beta_k} = f\beta_k = \sum_{i} \sum_{d} \left[ \frac{\hat{T}_{id}}{T_{id}} \frac{\partial T_{id}}{\partial \beta_k} \right] = 0
\]

(36)
Equation (36) is in effect a system of 2K simultaneous equations which has 2K unknown parameters ($\alpha_k, \beta_k$) of the GO model need to be estimated. Again, the Newton’s method combined with the Gauss-Jordan Matrix Elimination technique can then be used to solve for equation (36).

### 3.3 Statistical Test

The development of good transport model needs an indication of how accurate are the resulting models and this requires a comparison between estimated and an independently observed OD matrix using appropriate statistical indicators of this fit. In this paper, the following Root mean square error (RMSE) and $R^2$ statistical test has been used, as suggested by Tamin (1988):

$$\text{RMSE} = \sqrt{\frac{\sum \sum (\hat{T}_{id} - T_{id})^2}{N(N-1)}} \quad \text{for } i \neq d \quad (37)$$

$$R^2 = 1 - \frac{\sum \sum (\hat{T}_{id} - T_{id})^2}{\sum \sum (T_{id} - T_1)^2} \quad \text{for } i \neq d \quad (38)$$

Where: $\hat{T}_{id}, T_{id} =$ observed and estimated OD matrix, respectively.

$$T_1 = \frac{1}{N(N-1)} \sum \sum T_{id}$$

### 4. TESTS USING ARTIFICIAL DATA SET

#### 4.1 Artificial Data Set

To demonstrate the applicability of the proposed estimation methods, an artificial data test case was devised and tested. The main objective of this test is to see how good the proposed estimation methods are in estimating the parameters of the given transport models, hence predicting the estimated OD matrices, and also to see how good the estimated OD matrices are compared with the original or real matrix.

Two different original OD matrices were generated in this data base. The first original OD matrix was generated by assuming that we have one trip purpose and the second one by assuming that we have three trip purposes or commodity types. The values of the OD matrices for each zone and for one- and three-Commodity Types cases are depicted below in Tables 3–4.

<table>
<thead>
<tr>
<th>Zone</th>
<th>901</th>
<th>902</th>
<th>903</th>
<th>904</th>
<th>1001</th>
<th>1002</th>
<th>Dd</th>
</tr>
</thead>
<tbody>
<tr>
<td>901</td>
<td>0</td>
<td>200</td>
<td>150</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>575</td>
</tr>
<tr>
<td>902</td>
<td>150</td>
<td>0</td>
<td>60</td>
<td>60</td>
<td>55</td>
<td>70</td>
<td>395</td>
</tr>
<tr>
<td>903</td>
<td>210</td>
<td>140</td>
<td>0</td>
<td>210</td>
<td>125</td>
<td>100</td>
<td>785</td>
</tr>
<tr>
<td>904</td>
<td>60</td>
<td>50</td>
<td>235</td>
<td>0</td>
<td>65</td>
<td>200</td>
<td>610</td>
</tr>
<tr>
<td>1001</td>
<td>100</td>
<td>75</td>
<td>150</td>
<td>110</td>
<td>0</td>
<td>50</td>
<td>485</td>
</tr>
<tr>
<td>1002</td>
<td>200</td>
<td>140</td>
<td>150</td>
<td>170</td>
<td>175</td>
<td>0</td>
<td>835</td>
</tr>
<tr>
<td>Dd</td>
<td>720</td>
<td>605</td>
<td>745</td>
<td>600</td>
<td>495</td>
<td>520</td>
<td>3685</td>
</tr>
</tbody>
</table>

Table 3 Artificial OD Matrix for Total Commodity (Single Commodity)
Using these sets of observed OD matrices, the proposed estimation methods were then used to estimate the parameter(s) of the model, hence the estimated OD matrices. Therefore, the purpose of this test is to see how good the proposed estimation methods are in estimating the parameters of the given transport models, and also to see how good the estimated OD matrices are compared with the original matrix. The GOF statistics mentioned were then used to compare the estimated OD matrices with the original one.

### 4.2 One-trip-purpose case

Table 5 shows the calibrated parameters of the GR model using estimation methods (NLLS, WNLLS and ML) for three different deterrence functions as well as the number of iterations required for convergence.

<table>
<thead>
<tr>
<th>Estimation Methods</th>
<th>Deterrence Functions</th>
<th>Exponential</th>
<th>Power</th>
<th>Tanner</th>
<th>Exponential</th>
<th>Power</th>
<th>Tanner</th>
<th>Exponential</th>
<th>Power</th>
<th>Tanner</th>
<th>Exponential</th>
<th>Power</th>
<th>Tanner</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β</td>
<td>No of iterations</td>
<td>β</td>
<td>No of iterations</td>
<td>α</td>
<td>β</td>
<td>No of iterations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NLLS</td>
<td>-0.0515</td>
<td>2</td>
<td>-0.4282</td>
<td>2</td>
<td>0.2169</td>
<td>-0.0767</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WNLLS</td>
<td>-0.04</td>
<td>2</td>
<td>-0.3987</td>
<td>2</td>
<td>-2.2601</td>
<td>0.2193</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ML</td>
<td>-0.0423</td>
<td>3</td>
<td>-0.3609</td>
<td>3</td>
<td>-0.0975</td>
<td>-0.0311</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It can be seen from Table 5, using the negative exponential and power function, all NLLS, WNLLS and ML methods give negative values of β meaning that any increment in travel cost between each corresponding zones will increase the number of flows traveling between them. This does not follow what the realism is; the higher travel cost, the lower will be the number of flows between those zones. However, using Tanner and WNLLS estimation method, the value of β is positive reflecting what is we expect.

The number of iteration required for convergence is very much depending on the starting value of the calibrated parameter. In this case, the starting values of all cases was set to zero. The closer the initial value of the calibrated parameters to the expected value will reduce the number of iterations for convergence. It is shown in Table 5 that Tanner function required more iteration to converge than Exponential and power functions. It is due to that Tanner function has more parameters to estimate.

Table 6 shows the RMSE value of estimated matrix compared with the observed matrix and Table 7 shows the R² value of estimated matrix compared with the observed matrix. It can be seen from Tables 6–7, for the negative exponential and power functions, the NLLS estimation method produces the best estimated OD matrix compared to the observed one (negative exponential function is better than power function).
### Table 6 The GoF Statistics of Estimated OD Matrices (RMSE)

<table>
<thead>
<tr>
<th>Estimation Methods</th>
<th>RMSE</th>
<th>Power</th>
<th>Tanner</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLLS</td>
<td>41.9866</td>
<td>42.0810</td>
<td>41.9750</td>
</tr>
<tr>
<td>WNLLS</td>
<td>42.0882</td>
<td>42.0902</td>
<td>43.4018</td>
</tr>
<tr>
<td>ML</td>
<td>42.0522</td>
<td>42.1290</td>
<td>42.0638</td>
</tr>
</tbody>
</table>

### Table 7 The GoF Statistics of Estimated OD Matrices (R²)

<table>
<thead>
<tr>
<th>Estimation Methods</th>
<th>R²</th>
<th>Power</th>
<th>Tanner</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLLS</td>
<td>0.7209</td>
<td>0.7196</td>
<td>0.7210</td>
</tr>
<tr>
<td>WNLLS</td>
<td>0.7195</td>
<td>0.7195</td>
<td>0.7017</td>
</tr>
<tr>
<td>ML</td>
<td>0.7200</td>
<td>0.7190</td>
<td>0.7198</td>
</tr>
</tbody>
</table>

Compared to other estimation methods, the best estimated OD matrix (GR model) is obtained by using Non-Linear-Least-Square (NLLS) estimation method and Tanner’s deterrence function. The use of Tanner’s function gives better fit compared with the use of negative exponential and power functions. This was expected since Tanner’s function has more parameters than those other functions (negative exponential and power). In fact, the negative exponential and power functions can be obtained as a special case of the Tanner’s function. However, this does not guarantee it will always give better fit in other model.

### 4.3 Three-trip-purpose case

The GOF statistics of the OD matrices estimated by the NLLS, WNLLS and ML estimation methods is shown in Table 8. Table 8 presents that all the estimation methods, using three commodities, produced more accurate estimated OD matrices compared with using one aggregated commodity. The best estimated OD matrix was obtained using the NLLS estimation method.

### Table 8 The GoF Statistics of Estimated OD Matrices

<table>
<thead>
<tr>
<th>COMMODITY</th>
<th>Estimation Methods</th>
<th>NLLS</th>
<th>WNLLS</th>
<th>ML</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>R²</td>
<td>RMSE</td>
<td>R²</td>
</tr>
<tr>
<td>All COM’S</td>
<td>41.9866</td>
<td>0.7209</td>
<td>42.0882</td>
<td>0.7195</td>
</tr>
<tr>
<td>3 COM’S</td>
<td>40.9480</td>
<td>0.7345</td>
<td>41.1822</td>
<td>0.7315</td>
</tr>
</tbody>
</table>

The GOF statistics (RMSE and R²) comparing the observed OD matrix for each commodity with the corresponding OD matrices using NLLS, WNLLS and ML methods are given in Tables 9–11.

### Table 9 The GoF Statistics of Estimated OD Matrices using NLLS Method

<table>
<thead>
<tr>
<th>Commodity</th>
<th>RMSE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>40.9480</td>
<td>0.7345</td>
</tr>
<tr>
<td>Type 1</td>
<td>10.9807</td>
<td>0.8189</td>
</tr>
<tr>
<td>Type 2</td>
<td>16.1827</td>
<td>0.6578</td>
</tr>
<tr>
<td>Type 3</td>
<td>19.7115</td>
<td>0.6069</td>
</tr>
</tbody>
</table>

### Table 10 The GoF Statistics of Estimated OD Matrices using WNLLS Method

<table>
<thead>
<tr>
<th>Commodity</th>
<th>RMSE</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>41.1822</td>
<td>0.7315</td>
</tr>
<tr>
<td>Type 1</td>
<td>14.0291</td>
<td>0.7044</td>
</tr>
<tr>
<td>Type 2</td>
<td>15.5377</td>
<td>0.6845</td>
</tr>
<tr>
<td>Type 3</td>
<td>20.1827</td>
<td>0.5879</td>
</tr>
</tbody>
</table>
Table 11 The GoF Statistics of Estimated OD Matrices using ML Method

<table>
<thead>
<tr>
<th>Commodity</th>
<th>RMSE</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>40.9872</td>
<td>0.7340</td>
</tr>
<tr>
<td>Type 1</td>
<td>11.0926</td>
<td>0.8152</td>
</tr>
<tr>
<td>Type 2</td>
<td>16.4665</td>
<td>0.6457</td>
</tr>
<tr>
<td>Type 3</td>
<td>19.7936</td>
<td>0.6036</td>
</tr>
</tbody>
</table>

It can be seen from those tables that the resulting OD matrices using NLLS method were found better compared with WNLLS and ML method except for commodity 2. For commodity 2, estimated OD matrices using WNLLS estimation method were found better compared with NLLS and ML method. The values of parameter \( \beta \) of the GR model, for each commodity are given as well as number of iterations required for convergence in Table 12. It can be seen from Table 12 that all estimation methods obtained stable values of parameter \( \beta \) for each commodity.

Table 12 The Estimated Parameters of GR Model

<table>
<thead>
<tr>
<th>Estimation Methods</th>
<th>Commodity 1 ( \beta )</th>
<th>Commodity 2 ( \beta )</th>
<th>Commodity 3 ( \beta )</th>
<th>No of iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLLS</td>
<td>-0.05866</td>
<td>0.01248</td>
<td>-0.12378</td>
<td>3</td>
</tr>
<tr>
<td>WNLLS</td>
<td>0.02998</td>
<td>-0.04007</td>
<td>-0.14196</td>
<td>3</td>
</tr>
<tr>
<td>ML</td>
<td>-0.04642</td>
<td>0.02305</td>
<td>-0.12742</td>
<td>4</td>
</tr>
</tbody>
</table>

5. APPLICATIONS

5.1 Domestic Sea Freight Movement Data Set

In order to validate the new transport model, a real data set of domestic sea freight movement was used. The data is obtained from the Study on the Development of Domestic Sea Transportation and Maritime Industry in the Republic of Indonesia (Stramindo, 2003). The data describes the movement pattern of domestic sea freight (in tones) for all commodities between 25 major ports (25 zones) in Indonesia during the year of 2003. Based on the domestic sea freight OD matrices, the existing number of loading and unloading for each major ports \( (O_i\) and \( D_j) \) can then be determined. The “desire lines” of the existing domestic sea freight movement is illustrated in Figure 2.

Figure 2 Domestic Sea Freight OD Movement (Desire Lines)
In addition to existing domestic sea freight movement, another data required for the purpose of estimating future OD matrices is “trip impedance”. In this study, port-to-port distance data of 25 major ports (25 zones) in Indonesia (in sea miles) is used as trip impedance. The calibration process are carried out by implementing gravity (GR) and gravity-opportunity (GO) model and using WNLLS, NLLS and ML estimation methods. The input used during the calibration process are as follows:
- Existing domestic sea freight OD matrix
- Loading and unloading data for each major port (O_1 and D_1)
- Initial value of parameters (α, β)

The result of the calibration process can be used to estimate the future domestic sea freight traffic demand using the current OD pattern and the forecasted loading and unloading volume for each port.

5.2 Results
In assessing the value of a new transport model one is of course interested in the accuracy of the estimated travel behavior. In this paper, the new approach offers a choice of demand models to represent trip making behavior. This flexibility enhances the value of the model but it is also important to have some feelings on how the choice of model form may affect the accuracy of the resulting OD matrix. Therefore, we use the domestic sea freight OD matrix to obtain an indication of the most suitable model in this case. For the GR model, three types of deterrence function (Exponential, Power and Tanner) have been used. Table 13 shows the calibrated parameters of the GR model using estimation methods (NLLS, WNLLS and ML), number of iterations required for convergence as well as three different deterrence functions.

<table>
<thead>
<tr>
<th>Estimation Methods</th>
<th>Deterrence Functions</th>
<th>Exponential</th>
<th>Power</th>
<th>Tanner</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β</td>
<td>No of iterations</td>
<td>β</td>
<td>No of iterations</td>
</tr>
<tr>
<td>NLLS</td>
<td>0.0599</td>
<td>4</td>
<td>0.2294</td>
<td>4</td>
</tr>
<tr>
<td>WNLLS</td>
<td>0.2477</td>
<td>4</td>
<td>1.4453</td>
<td>4</td>
</tr>
<tr>
<td>ML</td>
<td>0.1055</td>
<td>5</td>
<td>0.4434</td>
<td>5</td>
</tr>
</tbody>
</table>

It can be seen from Table 13, using the negative exponential function and power function, all NLLS, WNLLS and ML methods give positive values of β meaning that any increment in travel cost between each corresponding zones will decrease the number of flows traveling between them. This condition follow what the realism is; the higher travel cost, the lower will be the number of flows between those zones. However, using tanner function, the all NLLS, WNLLS and ML estimation method give positive values of β, and the negative values of α. It is shown in Table 13 that Tanner function required more iteration to converge that Exponential and power functions. It is due to that Tanner function has more parameters to estimate. Table 14 shows the RMSE value of estimated matrix compared with the observed matrix and Table 15 shows the R^2 value of estimated matrix compared with the observed matrix.

<table>
<thead>
<tr>
<th>Estimation Methods</th>
<th>RMSE</th>
<th>Exponential</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLLS</td>
<td>90.2211</td>
<td>91.0869</td>
<td>86.5243</td>
</tr>
<tr>
<td>WNLLS</td>
<td>102.7759</td>
<td>106.3208</td>
<td>103.5658</td>
</tr>
<tr>
<td>ML</td>
<td>91.0585</td>
<td>91.5957</td>
<td>89.3802</td>
</tr>
</tbody>
</table>
It can be seen from Tables 14–15, for the negative exponential and power functions, the NLLS estimation method produces the best estimated OD matrix compared to the observed one (negative exponential function is better than power function). Compared to other estimation methods, the best estimated OD matrix (GR model) is obtained by using Non-Linear-Least-Square (NLLS) estimation method and Tanner’s deterrence function.

The use of Tanner’s function gives better fit compared with the use of negative exponential and power functions. This was expected since Tanner’s function has more parameters than those other functions (negative exponential and power). In fact, the negative exponential and power functions can be obtained as a special case of the Tanner’s function. However, this does not guarantee it will always give better fit in other region or province.

From Table 15, it can be seen that the value of $R^2$ resulted from the calibration process is not very good (maximum 0.69). That could happen because there are a lot of zero values in the existing (observed) domestic sea freight OD matrix. In the GO model, the choice between gravity or opportunity is decided empirically and statistically by restriction on parameters, $\varepsilon$ and $\mu$, which control the global functional form of the trip distribution mechanism.

By setting $\varepsilon=1$, the GO model will behave as the GR model since the opportunity part is omitted from the opportunity function (16) and similarly, by omitting the cost part of equation (16), the OP model is then created. By using various values of $\varepsilon$ and $\mu$, we can then calculate the values of $S$. It is found that the minimum value of $S$ is obtained at points $\varepsilon=0.9$ and $\mu=1$. These parameter values will change depending upon the characteristic of movement in certain study area.

Table 16 shows the calibrated parameters of the GO model using estimation methods (NLLS, WNLLS and ML) and Table 17 shows the RMSE value of estimated matrices compared with the observed matrix. Table 18 shows the $R^2$ value of estimated matrices compared with the observed matrix.
Table 18 The GoF Statistics of Estimated OD Matrices (GO Model)

<table>
<thead>
<tr>
<th>Estimation Methods</th>
<th>$\Omega = 1, \Phi = 1$</th>
<th>$\Omega = 1, \Phi = 0$</th>
<th>$\Omega = 0, \Phi = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLLS</td>
<td>0.6579</td>
<td>0.6538</td>
<td>0.6564</td>
</tr>
<tr>
<td>WNLLS</td>
<td>0.5606</td>
<td>0.5326</td>
<td>0.5603</td>
</tr>
<tr>
<td>ML</td>
<td>0.6516</td>
<td>0.6500</td>
<td>0.6513</td>
</tr>
</tbody>
</table>

It can be seen from Tables 17–18 that the best estimated OD matrix (GO model) is obtained by using NLLS estimation method. It can be seen that GR model performs better that the GO model since the value of RMSE of GR is less than those for GO. It was unexpected that the performance of GR was better than that of GO since GO have more parameters than GR. In fact, the GR model can be obtained as a special case of the GO model. However, this does not guarantee it will always give better fit in other model. It is also found that the use of GO model requires longer computer time to run compared with the use of the GR model. Deviation between observed OD matrix and estimated OD matrix can be seen in Figure 3.

![Figure 3 Relationship Between Observed OD Matrix and Estimated OD Matrix](image)

The figure shows the relationship between observed OD matrix and estimated OD matrix for the model with the highest $R^2$ value, i.e. GR model by using NLLS estimation method and Tanner’s deterrence function. It can be seen from Figure 3 that in general, the estimation method produced “under-estimate” of estimated OD matrix where the value of estimated OD matrix relatively lower than the value of observed OD matrix.

6. CONCLUSIONS

Two types of model have been used in this study: gravity (GR) and gravity-opportunity (GO) model and tested using Domestic Sea Freight Movement, of which the study area was divided into 25 zones. Some conclusions can be drawn from the result obtained:

- Whereas the gravity model is deficient in intervening-opportunities effects, the opportunity model constructed is equally deficient in omitting the trip impedance. It seems logical that an ideal model (GO) should contain both these distinct effects since the behavior of people's movement in certain varies greatly.
- It is shown that the standard gravity (GR) model and the opportunity (OP) model can be obtained as a special case of the GO model. Different values of the parameters controlling these transformations generate contrasting families of models, notably the EO model, the LO model and the GR model.
• It is found that for the Domestic Sea Freight Movement, for the GR model, the combination of Tanner’s deterrence function with NLLS estimation method will give the best estimated OD matrix. For the GO model, the NLLS method is found to be the best estimation method.
• It is found that the GR model produces the best model for sea freight movement.
• It is found that the GO model is more time consuming that the GR model since they use more complicated algebra and procedures which require longer time to solve.
• The estimation method produced “under-estimate” of estimated OD matrix where the value of estimated OD matrix relatively lower than the value of observed OD matrix.

REFERENCES