MODELING TRANSIT PASSENGER TRAVEL BEHAVIORS IN CONGESTED NETWORK WITH EN-ROUTE TRANSIT INFORMATION SYSTEMS

Hualing REN
Research Assistant
Department of Civil and Structural Engineering
The Hong Kong Polytechnic University
Hong Kong, China
School of Traffic and Transportation
Beijing Jiaotong University
Beijing, 100044, P. R. China
Fax: 86-10-5168-7127
E-mail: renhualing@jtys.bjtu.edu.cn

William H. K. LAM
Chair Professor
Department of Civil and Structural Engineering
The Hong Kong Polytechnic University
Hong Kong, China
Fax: 852-2334-6389
E-mail: cehklam@polyu.edu.hk

Abstract: This paper proposes a multi-class schedule-based dynamic transit assignment model to investigate the impacts of En-route Transit Information Systems (ETIS). The proposed model considers simultaneously the departure time and route choices of passengers in congested transit network with ETIS. There are two classes of passengers: those equipped and those unequipped with ETIS. These passengers would make their travel choices to follow the stochastic dynamic user optimal principles, with the equipped passengers having a lower perception variation of the travel cost due to the availability of better information. The numerical example indicates some important insights on passenger travel behaviors and the performance of the transit network with ETIS. The effects of the service cost and service quality of ETIS on the market penetration of ETIS and the total passenger travel cost are assessed under different conditions with various levels of transit passenger demand.

Key Words: En-route transit information systems, Schedule-based dynamic transit assignment, Market penetration

1. INTRODUCTION

In the past decades, many studies have been undertaken to model the impacts of Advanced Traveler Information Systems (ATIS) on road network (Hall, 1996; Yang, 1998; Lo et al., 1999; Lo and Szeto, 2002; Chan and Lam, 2002). These studies focused on assessment of the effects of ATIS on traveler route choice behaviors. The travelers are categorized into two classes: those equipped and those unequipped with ATIS. Their route choice behaviors follow User Equilibrium (UE), System Optimum (SO), or Stochastic User Equilibrium (SUE) principles. Among these studies, Yang (1998) viewed the demand for ATIS service as an elastic function of user benefits. The equipped travelers were modeled to make route choices following the UE principle whereas the unequipped travelers choose paths following the SUE criterion. In Lo and Szeto (2002), both the equipped and unequipped travelers were assumed to follow the SUE principles. However, the equipped travelers would have a lower perception variation of the network travel time. The extent of this travel time perception variation for the equipped travelers was considered as a measure of the ATIS information quality.

Most of the previous related studies have employed static models for assessing the effects of ATIS. As a result, the travelers’ departure time choice could not be modeled properly under non-recurrent dynamic congestion conditions with ATIS. More recently, some studies began
to analyze the problem with a dynamic paradigm (Al-Deek and Kanafani, 1993; Al-Deek et al., 1998; Arnott et al., 1991; Ben-Akiva et al., 1991; Chen and Mahmassani, 1991; Emmerink et al., 1995; Hall, 1993; Mahmassani and Liu, 1997). Lo and Szeto (2004) further investigated the impacts of ATIS in static and dynamic scenarios. It provided some insights on the differences of the ATIS effects between the two modeling approaches.

In order to enhance the transit network performance and assist passengers to plan for their travel, relevant transit information can be made available at some transit stations. For instance, transit route, time-schedule and fare information can be provided. Recently, the En-route Transit Information Systems (ETIS) have been proposed as an effective tool to alleviate congestion and enhance the transit network performance in Hong Kong. This ETIS would provide passengers with real-time information on multiple transit modes via the third generation (3-G) mobile telephones, personal digital assistants or website. Two samples of such technology application through internet are shown in Figure 1 (EasyGo – a research and development product by the Department of Land Surveying and Geo-Informatics, The Hong Kong Polytechnic University).

![Figure 1 The ETIS samples (Website: http://158.132.187.49/eng/index.asp)](image)

Similar to ATIS, one fundamental requirement for ETIS applications is the development of user travel choice behavior models for investigating the effects of the information systems (Reddy et al., 1995; Yang, 1998). However, little attention has been paid to the effects of ETIS on transit networks (Asakura and Hato, 2001; Luk and Yang, 2001). It may be partially attributed to the difficulties in modeling passenger route choice behaviors in dynamic transit networks with congestion. In recent decades, the schedule-based approach for modeling dynamic transit assignment problem has been examined (Nuzzolo et al., 2001; Poon, 2002). This approach can provide precise and accurate details by fully utilizing the detailed transit schedule information. So it can reflect the constantly changing and microscopic nature of the dynamic transit network characteristics. The schedule-based approach will be used in this paper to evaluate dynamically the impacts of ETIS on transit passenger travel behaviors in congested transit network.

In order to capture the congestion effects in transit network, the maximal capacity of each transit line is considered explicitly in this paper. If the transit line is congested, not all of the passengers waiting at transit station can get on the first arriving vehicle, and some of them may have to wait for the next or the third one. There are two classes of passengers: those equipped and those unequipped with ETIS. These passengers would make their travel choices to follow the Stochastic Dynamic User Optimal (SDUO) principles, with the equipped passengers having a lower perception variation of the travel cost due to the availability of better transit information from ETIS.
In this paper, a new multi-class SDUO transit assignment model is proposed to investigate the effects of ETIS under various conditions. The market penetration of ETIS is defined as the proportion of transit passengers using ETIS service. It is formulated elastically to reveal the ETIS effects on passenger route choice behavior, departure time choice behavior and transit network performance. The proposed model is formulated as an equivalent Fixed-Point (FP) problem for modeling transit passenger travel choice behaviors in congested network with ETIS. A numerical example is designed to analyze the impacts of ETIS on the equipped and unequipped passengers’ departure time and route choice behaviors as well as the network performance. The latter is expressed as the total transit passenger travel cost. The service cost and service quality of ETIS are considered as two important parameters to affect the performance of the transit network.

This paper is organized as follows. In Section 2, a multi-class dynamic transit assignment model is proposed for congested transit network with ETIS. A heuristic solution algorithm for solving the proposed dynamic transit assignment model is presented in Section 3. Section 4 illustrates the application of the proposed model and solution algorithm on an example network. The concluding remarks are given in Section 5 together with suggestions for further studies.

The case study is on the transit network connecting the urban area Kowloon to the Hong Kong International Airport (HKIA). There are two OD demands from two origins the HKIA. The study period is the morning rush hour of the transit network from 8:00 am to 12:00 noon. We will discuss the convergence of the results, the average travel cost saving, the market penetration of ETIS, and the network performance in terms of total passenger travel cost, etc.

2. MODEL FORMULATION

2.1 Transit Network Description

A transit network is composed of a set of stations (nodes) and transit lines joining the stations. For modeling the complicated transit network, we adopt in this paper the concept of transit “link” introduced in Le Clercq (1972). The transit link is a segment of a transit line between two stations, which may unnecessarily consecutive. When there are several common lines directly joining two nodes, each of them forms a distinct transit link (Poon, 2002; Poon et al., 2004). With this concept, the common line problem does not exist, and we can compute the number of on-boarding and waiting passengers for the transit line at each station.

Given a transit network $G(N, A, L)$, where $N$ is the set of nodes, $A$ is the set of transit links, and $L$ is the set of transit lines. Let $N_o$ and $N_d$ be the sets of origin and destination nodes respectively. Suppose that the study period is $[0, T_S]$, which is divided into unit time intervals. Some variable notations adopted in this paper are listed as follows:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^m_{j,l}$</td>
<td>departure time of the $j$th vehicle of transit line $l$ from node $m$ (which is determined by transit line schedule)</td>
</tr>
<tr>
<td>$\bar{h}<em>{rs,p}(t,t^*)$, $\hat{h}</em>{rs,p}(t,t^*)$</td>
<td>passenger flow equipped (unequipped) with ETIS on route $p$ between OD pair $rs$ departing at time $t$ with preferred arrival time $t^*$</td>
</tr>
<tr>
<td>$\bar{w}<em>{rs,p}(t,t^*)$, $\hat{w}</em>{rs,p}(t,t^*)$</td>
<td>proportion of equipped (unequipped) passengers on route $p$ between OD pair $rs$ departing at time $t$ with preferred arrival time $t^*$</td>
</tr>
<tr>
<td>$\eta_{rs,p}(t,t^*)$</td>
<td>actual travel cost on route $p$ between OD pair $rs$ for passengers</td>
</tr>
</tbody>
</table>
\[ K_{l}(l \in L) \] departing at time \( t \) with preferred arrival time \( t^{*} \)
\[ d^{rs} \] maximal capacity of transit line \( l \)
\[ d^{rs}, (\hat{d}^{rs}) \] total transit passenger demand between OD pair \( rs \)
\[ d^{rs}(t^{*}), (\hat{d}^{rs}(t^{*})) \] equipped (unequipped) transit passenger demand between OD pair \( rs \) with preferred arrival time \( t^{*} \)
\[ \pi^{rs} \] market penetration of ETIS between OD pair \( rs \)

Obviously, the following formula holds:
\[
\sum_{t} \sum_{rs} \sum_{p} \hat{h}_{rep}(t, t^{*}) = d^{rs}(t^{*}), \quad \sum_{t} \sum_{rs} \sum_{p} \hat{h}_{rep}(t, t^{*}) = \hat{d}^{rs}(t^{*}), \quad \forall r, s, t^{*}.
\] (1)
\[
\sum_{t} d^{rs}(t^{*}) = d^{rs}, \quad \sum_{t} \hat{d}^{rs}(t^{*}) = \hat{d}^{rs}, \quad \forall r, s.
\] (2)

Note that, for the example network in this paper from the urban area Kowloon to the HKIA, there are many preferred arrival times at the destination. For the general commuter transit network, there maybe only one preferred arrival time for one destination.

2.2 Departure Time and Route Choice Behaviors
The departure time and route choice behaviors of the two classes of transit passengers are characterized by two sets of transit SDUO conditions. The SDUO conditions of the equipped and unequipped passengers can be expressed as:
\[
\bar{h}_{rep}(t, t^{*}) = \bar{w}_{rep}(t, t^{*}) \cdot d^{rs}(t^{*}), \quad \forall r, s, p, t, t^{*},
\] (3)
\[
\hat{h}_{rep}(t, t^{*}) = \hat{w}_{rep}(t, t^{*}) \cdot \hat{d}^{rs}(t^{*}), \quad \forall r, s, p, t, t^{*},
\] (4)
where \( \bar{h}_{rep}(t, t^{*}) \) and \( \hat{h}_{rep}(t, t^{*}) \) are respectively the equipped and unequipped transit passenger flows who choose to depart at time \( t \) on route \( p \) between OD pair \( rs \) with preferred arrival time \( t^{*} \). \( d^{rs}(t^{*}) \) and \( \hat{d}^{rs}(t^{*}) \) are the OD demands of equipped and unequipped transit passengers, respectively, who have the same preferred arrival time \( t^{*} \). \( \bar{w}_{rep}(t, t^{*}) \) and \( \hat{w}_{rep}(t, t^{*}) \) are respectively the proportions of equipped and unequipped passengers choosing route \( p \) between OD pair \( rs \) departing at time \( t \) with preferred arrival time \( t^{*} \).

2.3 Modeling Market Penetration of ETIS
The market penetration of ETIS \( \pi^{rs} \), modeled in an elastic manner, is defined as the proportion of the equipped passengers to the total demand for each OD pair. An exponential function has been proposed by previous research (Yang, 1998) to formulate the relationship between the market penetration of ATIS and the users’ travel time saving. Similarly, the market penetration of ETIS between OD pair \( rs \) can be defined as follows:
\[
\pi^{rs} = \frac{d^{rs}}{\bar{d}^{rs}} = \frac{1}{1 + \exp(\lambda - \theta S^{rs})}, \quad \forall r, s.
\] (5)

Where \( d^{rs} \) is the total transit passenger demand between OD pair \( rs \), while \( \bar{d}^{rs} \) is the demand of equipped transit passengers between OD pair \( rs \). The parameter \( \lambda \) is the service
cost of ETIS, and \( \theta \) is the value of time (VOT). 

\( S^{rs} \) is the average travel cost saving generated by the ETIS service for travel between OD pair \( rs \), which can be converted into equivalent unit of time. As the equipped passengers always receive more accurate travel information from ETIS to minimize their travel cost, \( S^{rs} \) is usually positive and can be obtained from:

\[
S^{rs} = \Phi^{rs} - \hat{\Phi}^{rs}, \quad \forall r,s,
\]

Where \( \Phi^{rs} \) and \( \hat{\Phi}^{rs} \) are the average travel costs of equipped and unequipped transit passengers between OD pair \( rs \), respectively. They are calculated as below:

\[
\Phi^{rs} = \sum_{r'} \sum_{s'} \sum_{p} \sum_{t} \bar{w}_{rs} (t, t^*) \cdot \eta_{rs} (t, t^*), \quad \forall r,s
\]

\[
\hat{\Phi}^{rs} = \sum_{r'} \sum_{s'} \sum_{p} \sum_{t} \hat{w}_{rs} (t, t^*) \cdot \eta_{rs} (t, t^*), \quad \forall r,s
\]

Where \( \eta_{rs} (t, t^*) \) is the travel cost on route \( p \) between OD pair \( rs \) for passengers departing at time \( t \) with preferred arrival time \( t^* \).

Once the service cost \( \lambda \) is given, the market penetration of ETIS between OD pair \( rs \) can be determined by eqn. (4). Then the equipped demand between OD pair \( rs \) with preferred arrival time \( t^* \) is:

\[
\pi^{rs} (t^*) = \pi^{rs} (t^*) \cdot d^{rs} (t^*), \quad \forall r,s,t^*.
\]

And the unequipped demand between OD pair \( rs \) with preferred arrival time \( t^* \) is:

\[
\hat{\pi}^{rs} (t^*) = d^{rs} (t^*) - \pi^{rs} (t^*), \quad \forall r,s,t^*.
\]

From eqn. (4), it should be noted that even there is no travel cost saving, the ETIS market penetration would not be equal to zero if the service cost of ETIS is not very high. On the other hand, if both the value of time and travel cost saving are kept on increasing, the ETIS market penetration would approach to 100%. However, higher travel cost saving would attract more subscribers to use the ETIS service. Subsequently, the benefit of using ETIS service will be diminished and the travel cost saving will then be reduced. This elastic market penetration is analogous to a supply-demand equilibrium (Yang, 1998; Lo and Szeto, 2002).

### 2.4 Transit Travel Cost Determination

In this paper, the passengers’ generalized cost of travel consists of five components: (i) the in-vehicle travel cost in unit of time, including the in-vehicle discomfort cost; (ii) the waiting time at the station (node); (iii) the out-pocket fare on the transit route; (iv) the penalty of transfer times; (v) the cost of schedule delay. Mathematically, the generalized cost of travel on route \( p \) between OD pair \( rs \) for passengers departing at time \( t \) with preferred arrival time \( t^* \) is defined as below:

\[
\eta_{rs} (t, t^*) = \eta_{rs}^z (t) + \beta^1 \eta_{rs}^w (t) + \beta^2 \eta_{rs}^f (t) + \beta^3 \eta_{rs}^p (t) + \beta^4 \eta_{rs}^d (t, t^*), \quad \forall r,s,p,t,t^*.
\]

Where \( \eta_{rs}^z (t) \), \( \eta_{rs}^w (t) \), \( \eta_{rs}^f (t) \), \( \eta_{rs}^p \), and \( \eta_{rs}^d \) represent the five components, respectively. The parameters \( \beta^1 \), \( \beta^2 \), \( \beta^3 \) and \( \beta^4 \) are the weight coefficients of waiting time, transit fare, transferring penalty and schedule delay cost so as to convert these components to equivalent unit of the in-vehicle travel cost.

Denote \( \eta_{rs}^z (t) \) as the in-vehicle travel cost for passengers boarding transit vehicle on transit link \( a \) at time \( t \), including the in-vehicle discomfort cost. The latter is a function of the number of passengers on-board the vehicle traveling on the transit link. It can be expressed as follows...
by adapting the BPR-type function for the in-vehicle travel cost with taking account of the in-vehicle discomfort effects (Tian et al., 1997; Uchida et al., 2005):

$$\eta^z_a(t) = \eta^{z0}_a \left[1 + \alpha \left( \frac{v^m_l(t)}{K_l} \right)^\gamma \right],$$

(12)

Where $\eta^{z0}_a$ is the free flow in-vehicle travel time on transit link $a$. $v^m_l(t)$ is the number of passengers on the vehicle of transit line $l$ at node $m$ at time $t$. $K_l$ is the maximal capacity of transit line $l$. Then the in-vehicle travel cost $\eta_{rs}^z(t)$ on route $p$ between OD pair $rs$ for passengers departing at time $t$ can be calculated recursively by the in-vehicle travel costs on transit links along the transit route $p$.

Denote $\eta^w_a(t)$ as the waiting time for passengers arriving at the tail-node station of transit link $a$ at time $t$. It consists of two components (De Cea and Fernández, 1993; Tong and Wong, 1999; Poon, 2002): one is the waiting time between the arrival time of the passengers and the schedule arrival time of the vehicle by transit line; the other is the congestion delay caused by the transit line capacity constraints. Several models have considered the passenger waiting time in congested transit network as a function of the frequency of the crowded transit lines and the occupancy of the vehicle (De Cea and Fernández, 1993; Wu et al., 1994). However, when these models are applied to the congested transit networks, they may lead to some unrealistic results. For example, some transit lines may be loaded beyond their physical capacities while others serving the same OD pair may remain under-utilized (Poon, 2002). In order to capture the congestion effects on transit network, it is assumed in this paper that each transit line $l$ has an explicit maximal capacity, say $K_l$. That is, the total number of passengers boarding the transit vehicle cannot exceed its maximal capacity.

$$v^m_l(t) \leq K_l, \quad \forall m, l, t,$$

(13)

Where $v^m_l(t)$ is the number of passengers on the vehicle of transit line $l$ at node $m$ at time $t$. These passengers are categorized into two groups: the passengers who have boarded this transit vehicle before node $m$ and will not alight at this node; and the passengers boarding this transit vehicle at node $m$ at time $t$. If the transit line is congested and there are too many passengers arriving at the station, then not all of them can board the first arriving vehicle. That is to say, some of them have to wait for the next or the third vehicle. In this case, the passenger waiting time at the station on transit link can be calculated as the weight average of all the passengers arriving at the station at the same time. Then the waiting time $\eta_{rs}^w(t)$ for passengers departing at time $t$ on route $p$ between OD pair $rs$ can be calculated recursively using the passenger waiting time for vehicle on transit links along the transit route $p$.

$\eta_{rs}^f$ is the out-pocket fare on transit route $p$. $\eta_{rs}^w$ is the penalty cost relative to the transfer times on route $p$ and is assumed to be different by transit mode. $\eta_{rs}^d(t,t^{**})$ is the schedule delay cost of early or late arrival at the destination for passengers departing at time $t$ on transit route $p$ between OD pair $rs$. The travel time on route $p$ between OD pair $rs$ for passengers departing at time $t$ is defined as follows:

$$T_{rs}(t) = \eta^{z0}_{rs} + \eta^w_{rs}(t) + \eta^d_{rs}(t,t^{**}), \quad \forall r, s, p, t,$$

(14)

Where $\eta^{z0}_{rs}$ is the free flow in-vehicle travel time on route $p$ between OD pair $rs$. Then the schedule delay cost for passengers departing at time $t$ on route $p$ between OD pair $rs$ with preferred arrival time $t^{**}$ is defined as below (Huang and Lam, 2002; Lam et al., 2006):
\[ n^d_{rsp}(t, t^*) = \begin{cases} 
\beta^s [t^* - \Delta^1_s - t - T_{rsp}(t)] & \text{if } t + T_{rsp}(t) < t^* - \Delta^1_s \\
\beta^o [t + T_{rsp}(t) - t^* + \Delta^2_s] & \text{if } t + T_{rsp}(t) > t^* + \Delta^2_s, \forall r, s, p, t, t^* \\
0 & \text{otherwise} 
\end{cases} \]

Where \( t^* \) is the preferred arrival time at destination \( s \). \( \Delta^1_s \) and \( \Delta^2_s \) are two given time slices which satisfy that \([t^* - \Delta^1_s, t^* + \Delta^2_s]\) is the desired arrival time window around \( t^* \) for passengers arriving at destination \( s \) without any schedule delay penalty. \( \beta^s (\beta^o) \) is the unit cost of arriving early (late) (i.e. schedule delay) at destination.

**Note:** The parameters \( \beta^s, \beta^o, \beta^t \) in eqn. (11), and \( \alpha, \gamma \) in eqn. (12) are all different by transit mode.

The perceived travel costs on route \( p \) between OD pair \( rs \) for equipped and unequipped passengers departing at time \( t \) with preferred arrival time \( t^* \) are:

\[
\Psi_{rsp}(t, t^*) = \eta_{rsp}(t, t^*) + \bar{\epsilon}_{rsp}(t, t^*), \quad \forall r, s, p, t, t^*, \quad (16)
\]

\[
\hat{\Psi}_{rsp}(t, t^*) = \hat{\eta}_{rsp}(t, t^*) + \hat{\epsilon}_{rsp}(t, t^*), \quad \forall r, s, p, t, t^*, \quad (17)
\]

Where \( \eta_{rsp}(t, t^*) \) is defined in (11). \( \bar{\epsilon}_{rsp}(t, t^*) \) and \( \hat{\epsilon}_{rsp}(t, t^*) \) are the perceived travel cost errors on route \( p \) between OD pair \( rs \) for equipped and unequipped passengers departing at time \( t \) with preferred arrival time \( t^* \). It is assumed that \( \bar{\epsilon}_{rsp}(t, t^*) \sim N(0, \mu_{\bar{\epsilon}_{rsp}(t, t^*)}) \) and \( \hat{\epsilon}_{rsp}(t, t^*) \sim N(0, \mu_{\hat{\epsilon}_{rsp}(t, t^*)}) \) for simplicity. It is also assumed that \( \mu < \hat{\mu} \). It implies that equipped passengers would have more precise travel information than unequipped passengers. The probabilities of equipped and unequipped passengers choosing route \( p \) for travel between OD pair \( rs \) departing at time \( t \) can be expressed as follows:

\[
\bar{\pi}_{rsp}(t, t^*) = \Pr\{\bar{\Psi}_{rsp}(t, t^*) \leq \bar{\Psi}_{rsp}(t', t^*), \forall p' \in P_{rs}, p' \neq p, \forall t' \neq t\}, \quad \forall r, s, p, t, t^*, \quad (18)
\]

\[
\hat{\pi}_{rsp}(t, t^*) = \Pr\{\hat{\Psi}_{rsp}(t, t^*) \leq \hat{\Psi}_{rsp}(t', t^*), \forall p' \in P_{rs}, p' \neq p, \forall t' \neq t\}, \quad \forall r, s, p, t, t^*. \quad (19)
\]

### 2.5 Fixed-Point Problem Presentation

Denote \( \bar{h} \) and \( \hat{h} \) as the vectors of equipped and unequipped passenger flows on route \( p \) between OD pair \( rs \) departing at time \( t \), respectively. The set of feasible passenger flows are \( \Omega = \{ (\bar{h}, \hat{h}) \} \), where \( \bar{h} \geq 0, \hat{h} \geq 0 \) and satisfy eqn. (1). Let \( f \) denote an element in this set, i.e. \( f = (\bar{h}, \hat{h}) \in \Omega \), then the following fixed-point problem can be derived for the proposed multi-class dynamic transit assignment model. In the proposed model, both the route and departure time choices are considered simultaneously.

\[
f^* - dP(f^*) = 0, \quad f^* \in \Omega, \quad (20)
\]

Where \( d \) is the vector of passenger O-D demand; and \( P \) is the vector of passenger departure time and route choice probabilities (see eqns. (2) and (3)).

Note that, the route travel time is essentially non-linear and non-convex. It means that the fixed-point problem (20) is non-convex and multiple local solutions may exist according to Brouwer’s fixed point theorem.

### 2.6 Performance Measure

As shown in Yang (1998), the public authority can choose an equilibrium in network with ATIS so as to generate the largest benefit with the use of an exogenous parameter such as the
service cost of ATIS. In other words, the level of market penetration can be brought to a 
desirable one through adjusting the service cost of ATIS. Lo and Szeto (2004) discussed the 
benefit of ATIS service from three perspectives: users, service providers, and the traffic 
management agency. Similarly, we can extend the idea to investigate the benefit of ETIS from 
different perspectives. With the use of the proposed model for transit network with ETIS, we 
will analyze the benefit of ETIS service in terms of total passenger travel cost by varying the 
service cost and service quality of ETIS.

The total passenger travel cost in this paper is the sum of the travel costs of all the equipped 
and unequipped passengers on all feasible transit routes between all OD pairs during the study 
horizon. It is defined as follows:

\[ TPTC = \sum_{t} \sum_{s,t,p} \sum_{p'} (\hat{h}_{rp}(t,t^*) + \hat{\eta}_{rp}(t,t^*)) \eta_{rs}(t,t^*) . \]  

(21)

The relative reduction in total passenger travel cost is defined as below to capture the changes 
in total passenger travel cost before and after the introduction of ETIS (Lo and Szeto, 2004):

\[ RC = \frac{TPTC^b - TPTC^a}{TPTC^b} \times 100\% . \]  

(22)

The superscripts \( b \), \( a \), respectively, refer to the cases of “before” and “after” the 
implementation of ETIS. A positive value refers to the case of total travel cost reduction due 
to the provision of ETIS service in the transit network.

3. SOLUTION ALGORITHM

A heuristic solution algorithm is adapted to solve the proposed multi-class dynamic transit 
assignment model for congested network with elastic market penetration of ETIS. The OD 
demands of equipped and unequipped transit passengers are determined by eqns. (5), (9) and 
(10), in which the service cost of ETIS is incorporated as an exogenous variable (Yang, 1998). 
For each given service cost of ETIS, the proposed solution algorithm is outlined as follows:

**Step 0** Given an initial value of ETIS market penetration, calculate the equipped and 
unequipped OD demands \( \bar{d}^{''}(t^*) \) and \( \hat{d}^{''}(t^*) \). Set iteration number \( n = 0 \).

**Step 1** Perform the multi-class dynamic transit assignment with given feasible transit route set 
to each OD pair:

**Step 1.1** Initialize the transit passenger flows;

**Step 1.2** Perform the network loading and obtain the transit travel costs;

**Step 1.3** Obtain the passenger flows of the two classes based on the current transit travel 
costs, using the Monte Carlo simulation;

**Step 1.4** Update the transit passenger flows of the two classes using the method of 
successive averages (MSA);

**Step 1.5** Check the convergence of the inner iteration.

**Step 2** Update the ETIS market penetration.

**Step 3** Check the convergence of the outer iteration.

It should be pointed out that since the proposed solution algorithm is a route-based method, 
and therefore explicit route enumeration is required. Since the common line problem does not 
exist in the schedule-based dynamic transit assignment model, we can employ the column 
generation technique in the road network (Damberg et al., 1996) to automatically generate the 
feasible (or effective) transit route set for each OD pair.
The dynamic network loading process in step 1.2 is to load the passenger flows \( \mathbf{h}_{rs}(t,t^*) \) and \( \mathbf{h}_{rs}(t,t^*) \) \((\forall r,s,p,t,t^*)\) onto the transit network in a time-incremental manner. Some important issues in the loading process are elaborated as follows.

Define \( A^m_l(t) \) as the cumulative number of passengers who arrive and wait for transit line \( l \) at node \( m \) at time \( t \). \( D^m_l(t) \) is the cumulative number of passengers who board transit line \( l \) and leave node \( m \) at time \( t \). \( A^m_l(t) \) can be calculated as below:

\[
A^m_l(t) \left\{ \begin{array}{ll} 
\sum_{t'=1}^t \sum_{a=1}^{\text{B}(r)} \sum_{r,s,p,t^*} h_{rs}(t',t^*) \delta_{ap} \bar{\delta}^{al} & \text{if } m = r \\
\sum_{t'=1}^t \sum_{s=1}^{\text{B}(m)} \sum_{r,t^*} x_{rs,lp}(t'-\eta_{b^0}^r) \delta_{b^0} \bar{\delta}^{bl} & \text{if } m \neq r
\end{array} \right., \quad \forall m,l,t,
\]  

where \( x_{rs,lp}(t'-\eta_{b^0}^r) \) \((b \in B(m))\) is the number of passengers who board the upstream transit link \( b \) using route \( p \) to travel between OD pair \( rs \) at time \( t'-\eta_{b^0}^r \) and arrive at node \( m \) at time \( t' \). \( B(m) \) is the set of transit links leading to node \( m \). \( \eta_{b^0}^r \) is the free flow in-vehicle travel time on transit link \( b \). \( \delta_{ap} \) is the transit link/route incidence indicator whose value is 1 if transit link \( a \) is on transit route \( p \), and 0 otherwise. \( \bar{\delta}^{al} \) is the transit link/line incidence indicator whose value is 1 if transit link \( a \) is on transit line \( l \), and 0 otherwise.

\( u^m_l(t) \) is denoted as the number of passengers who can get on the vehicle of transit line \( l \) at node \( m \) at time \( t \), which can be updated as below:

\[
u^m_l(t) \left\{ \begin{array}{ll} 
\min \{ A^m_l(t) - D^m_l(t-1), K - x^m_l(t) \} & \forall m,l,t,
\end{array} \right.
\]

Where \( x^m_l(t) \) is the number of passengers who have boarded this vehicle at an upstream node and will not alight at this node at time \( t \). It implies in eqn. (24) that, if there is no spare capacity on the transit vehicle, some of the waiting passengers would have to wait for the next one. As such, the physical capacity constraint of transit vehicle is incorporated in the proposed model explicitly.

The number of all passengers on vehicle of transit line \( l \) at node \( m \) at time \( t \) is calculated as:

\[
u^m_l(t) = x^m_l(t) + u^m_l(t), \quad \forall m,l,t.
\]

And \( D^m_l(t) \) is determined as:

\[
D^m_l(t) = \sum_{t'=1}^t u^m_l(t'), \quad \forall m,l,t.
\]

### 4. NUMERICAL RESULTS

A simple transit network is used to facilitate the presentation of the essential ideas in this paper. The transit network connects urban area Kowloon to the Hong Kong International Airport (HKIA) as shown in Figure 2(a). Four transit lines are considered: Airport Express Line (AEL), Mass Transit Railway (MTR), Bus line 1 (Bus-1) and Bus line 2 (Bus-2). There are two OD demands from two origins (Kowloon and Tsing Yi) to one destination (HKIA).

This numerical example is designed to: (1) demonstrate the performance of the proposed
model and solution algorithm in terms of convergence of the results; (2) analyze the effects of ETIS on the multi-class transit passengers’ departure time and route choice behaviors; (3) show how the service cost and service quality affect the average travel cost saving, the market penetration of ETIS, and the network performance in terms of total passenger travel cost.

(a) The simplified transit network between Kowloon and HKIA

(b) The alternative representation of the transit network by transit links

Figure 2 The transit network of the numerical example

Table 1 Basic transit line data for the example transit network

<table>
<thead>
<tr>
<th>Transit line</th>
<th>AEL</th>
<th>MTR</th>
<th>Bus-1</th>
<th>Bus-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_i) (pass/veh)</td>
<td>500</td>
<td>1500</td>
<td>650</td>
<td>200</td>
</tr>
<tr>
<td>(\alpha, \gamma)</td>
<td>2, 3</td>
<td>0.1, 3</td>
<td>3, 3</td>
<td>3, 3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Transit link</th>
<th>S_1</th>
<th>S_2</th>
<th>S_3</th>
<th>S_4</th>
<th>S_5</th>
<th>S_6</th>
<th>S_7</th>
<th>S_8</th>
</tr>
</thead>
<tbody>
<tr>
<td>In-vehicle travel time (min)</td>
<td>20</td>
<td>8</td>
<td>12</td>
<td>20</td>
<td>11</td>
<td>9</td>
<td>15</td>
<td>40</td>
</tr>
<tr>
<td>Transit fare (HK$)</td>
<td>90</td>
<td>9</td>
<td>60</td>
<td>17</td>
<td>14</td>
<td>9</td>
<td>3.5</td>
<td>33</td>
</tr>
</tbody>
</table>

Table 2 Transit route list by transit links

<table>
<thead>
<tr>
<th>Route</th>
<th>Order of transit links</th>
<th>OD pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>R_1</td>
<td>S_1</td>
<td>N_1---N_4</td>
</tr>
<tr>
<td>R_2</td>
<td>S_3</td>
<td>N_1---N_4</td>
</tr>
<tr>
<td>R_3</td>
<td>S_4 – S_7</td>
<td>N_1---N_4</td>
</tr>
<tr>
<td>R_4</td>
<td>S_5 – S_1</td>
<td>N_1---N_4</td>
</tr>
<tr>
<td>R_5</td>
<td>S_7 – S_4 – S_7</td>
<td>N_1---N_4</td>
</tr>
<tr>
<td>R_6</td>
<td>S_3</td>
<td>N_2---N_4</td>
</tr>
<tr>
<td>R_7</td>
<td>S_5 – S_7</td>
<td>N_2---N_4</td>
</tr>
</tbody>
</table>

The peak period for air flight departure in Hong Kong is from 11:00 am to 1:00 pm, while the rush hour regarding the transit network to HKIA should be around 2 hours before the air flight departure. So the study period of the example transit network is chosen to be the morning rush hour from 8:00 am to 12:00 noon. The total passenger demands during this rush hour from two origins to one destination are: the demand from Kowloon (node N_1) to HKIA (node N_4) \(q_{1-4} = 20000\) (pass), and the demand from Tsing Yi (node N_2) to HKIA (node N_4)
$q_{2-4}=10000\text{(pass)}$. Figure 2 (b) shows the alternative representation of the example transit network in terms of transit links.

Table 1 gives the basic transit line data for the example transit network. All transit routes available in the network are listed in Table 2 with respect to transit link and OD pair. An input parameter, OD multiplier, is denoted as $\vartheta$ to represent various passenger demand levels. Some other input data are: $\beta^1=3$, $\beta^2=0.3$, $\beta^4=1$ for all the four transit modes; $\beta^3=5$, 7, 4, 4, for the four transit modes respectively; $\beta^5=0.5$, $\beta^6=3$, $\hat{\mu}=1$.

The convergence results of the example at two different levels of OD demand are presented in Figure 3. The OD multipliers are $\vartheta=0.5$ and $\vartheta=2$, respectively. The ETIS service cost and service quality are $\lambda=\text{HK}$ 1 and $\mu=0.2$, respectively. It shows that the market penetration of ETIS at each demand level approaches a steady level within 6 iterations. It also shows that at higher level of OD demand $\vartheta=2$, the market penetration of ETIS converges to a higher steady level of 65.3%. On the contrary, at lower level of OD demand $\vartheta=0.5$, the ETIS market penetration converges to a lower steady level of 46.8%. It means that when the transit network becomes more congested due to the higher demand level, the market penetration of ETIS will be increased. In other words, passengers are more willing to use the ETIS service to help them make choices on their departure time and route under congested travel condition.

![Figure 3 Convergence results under different levels of OD demand](image)

Figure 4 compares the transit passenger flow results before and after the provision of ETIS. The y-axis is the passenger flow proportion, which is defined as the proportion of passengers (out of the total passengers during the study period) departing at each time interval for travel between a given OD pair (N2--N4). It can be seen in Figure 4 that after the provision of ETIS the passenger flow proportion at the best departure time of 10:10 am is 24.5%, which is about double of the 12.9% before the provision of ETIS. It means that after the provision of ETIS, equipped passengers will choose better departure times because of the more reliable travel information available from ETIS. On the other hand, with the provision of ETIS, fewer passengers will depart early at 9:30 am: only 3.76%, which is smaller than 6.06% before the provision of ETIS.

Table 3 shows the proportions of passengers waiting at the Kowloon station (i.e. N1) before and after the provision of ETIS. The values in the table are the proportions of passengers who cannot board the first arriving transit vehicle of transit line AEL at node N1 along the departure time. It shows that about 67% of the waiting passengers at 10:10 am cannot board the first arriving vehicle before the provision of ETIS due to the insufficient capacity of transit.
line AEL. After the provision of ETIS, about half of the passengers on transit route 1 during the period of 10:00-10:20 am would switch to transit route 3, and only 40% of the waiting passengers will wait for the next transit vehicle. It is because many of the equipped passengers choose the less congested transit route 3 although it is slower and requires a transfer to another transit line.

![Figure 4 Passenger flow proportion before and after the provision of ETIS](image)

Table 3 Proportion of passengers waiting at N1 before and after the provision of ETIS

<table>
<thead>
<tr>
<th>Departure time</th>
<th>8:00</th>
<th>8:30</th>
<th>9:00</th>
<th>9:30</th>
<th>10:10</th>
<th>10:30</th>
<th>11:00</th>
<th>11:30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before service</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>21%</td>
<td>67%</td>
<td>27%</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>After service</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>7%</td>
<td>40%</td>
<td>9%</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 5 displays the results of the average travel cost savings at different demand levels. It is noted in this figure that one of the two parameters is \( \mu \), which represents the service quality of ETIS. The value of \( \mu \) is in the range between 0 and 0.8. The lower the value, the better is the service quality of ETIS. In other words, the value of 0 implies the best service quality of ETIS. The other parameter is the service cost of ETIS \( \lambda \), ranging from 0 to HK$ 5. The upper layer shown in Figure 5 is the result at demand level \( \vartheta = 2 \), while the lower layer is that at \( \vartheta = 0.5 \). It shows that the travel cost savings at higher demand level are always greater than those at lower demand level. It also indicates that, higher service quality of ETIS always leads to higher travel cost saving for the two demand levels, and vice versa.

There is also an important result about the differences in average travel cost savings under the two levels of OD demand. It shows that, if the ETIS service quality is improved or the service cost is increased, the difference of the average travel cost saving between lower and higher levels of OD demand will increases. At the point of \( \mu = 0 \) and \( \lambda = 5 \), the difference is the largest, with the value of \( 8 - 4.72 = 3.28 \).

Figure 6 illustrates the results of the market penetration of ETIS at different levels of OD demand. The parameters are still the service quality and service cost of ETIS. It can be observed in Figure 6 that, the values of ETIS market penetration at higher demand level is always greater than that at lower demand level. It implies that, with the same service cost and service quality of ETIS, passengers in congested transit network (with higher level of OD demand) are more willing to use the ETIS service for their travel. It is also found that higher service cost always leads to lower market penetration at both demand levels. On the other hand, better service quality will attract more passengers to purchase the ETIS service.
Figure 5 Average travel cost saving of ETIS service

Figure 6 also shows the differences in market penetration of ETIS at the two levels of OD demand. It can be seen that, when the service quality is improved or the service cost is reduced, the market penetration increases more rapidly under congested condition (with higher demand) than under uncongested condition. It means that the difference of market penetration at the point of $\bar{\mu} = 0$ and $\lambda = 5$ is larger than that of any other points, with the value of 15.7% ($= 85.1% - 69.4%$).

Figure 6 Market penetration of ETIS service

Capturing the insights aforementioned, it can be expected to put the market penetration in control by adjusting the service cost or improving the service quality of ETIS so as to minimize the total passenger travel cost of the transit network.

Finally, we examine the effects of ETIS on reduction in total passenger travel cost. Figure 7(a) and (b) show the measures of the RC obtained at different levels of OD demand: $\vartheta = 0.5$ and $\vartheta = 2$. From the numerical results, we can see that at the lower level of OD demand, if the service quality is improved and the service cost of ETIS is reduced, the RC is always increased within a limited range from less than 0.5% to more than 3%. It implies that although higher service quality and lower service cost would attract more passengers to purchase the ETIS service, the RC is still increased. It is because that there is no congestion in the network at lower demand level.
However, the findings are quite different for higher level of OD demand $\vartheta = 2$. Given ETIS service quality parameter $\mu = 0.2$, when the service cost $\lambda$ is reduced from HK$ 5, the RC is increasing at first, from 1% to 3.4%. If $\lambda$ is further reduced, the RC decreases instead, from 3.4% to less than 3%. It implies that there exists an optimal service cost of ETIS, say HK$ 1, which would maximize the RC at the value of 3.4%. That is, the total passenger travel cost is minimized at the optimal service cost in this example. Similar results can be found for other fixed $\mu$. On the other hand, given ETIS service cost $\lambda = 1$, if the service quality parameter $\mu$ is improved from 0.8 to 0.2, the RC is increasing from 1% to 3.4%. But beyond $\mu = 0.2$, the RC will decrease when $\mu$ continues to reduce. That is to say, there exists an optimal service quality of ETIS ($i.e., \mu = 0.2$) to maximize the RC in this example. It means that, at higher demand level, low service cost or good service quality of ETIS will reduce the TPTC at the beginning. However, if the service cost is reduced to $\lambda = 1$, and the service quality is improved to $\mu = 0.2$, there will be too many passengers equipped with ETIS in the transit network. As a result, the transit network will get congested and the TPTC cannot be reduced anymore. Either ETIS service cost lower than HK$ 1 (i.e. $\lambda = 1$), or better service quality with $\mu$ less than 0.2 will lead to more severe congestion in the transit network and therefore the TPTC will be increased instead. The TPTC at the optimal service cost and service quality of ETIS is 65,167(pass·hr), and the reduction in total passenger travel cost is 2,294(pass·hr) in comparison to the one without ETIS. Similar to Lo and Szeto (2002), the proposed model can also be used to investigate the effects of ETIS on some other benefit perspectives, such as users, and service providers.

![Figure 7 Reduction in total passenger travel cost resulted from ETIS service](image)

**5. CONCLUDING REMARKS**

This paper has formulated an equivalent fixed-point problem for modeling passengers’ departure time and route choice behaviors in congested transit network with ETIS. Two classes of passengers, equipped and unequipped with ETIS, were assumed to follow the SDUO conditions. The equipped passengers would have a lower perception variation of travel cost due to the availability of better information from ETIS. A heuristic solution algorithm was adapted to solve the FP problem. The service cost and service quality of ETIS service were shown to be two important parameters to affect the performance of the transit network. The numerical results have illustrated the convergence of the solution. Some important insights have also been shown in the numerical example regarding the impacts of ETIS on the transit passenger travel behaviors and the performance of the transit network.
It was found in the numerical example that lower service cost and higher service quality of ETIS can attract more subscribers to the service, especially at higher level of passenger demand with congestion in transit network. The total passenger travel cost was adopted as a performance measure of the transit network with and without ETIS. At lower level of passenger demand, the lower the service cost (or the higher the service quality), the more the total passenger travel cost is reduced. However, the findings are different for congested transit network. At the higher level of passenger demand, high service cost or poor service quality of ETIS will increase the total passenger travel cost. On the other hand, too low service cost and too good service quality of ETIS will attract too many passengers using the ETIS service. As a result, it would bring congestion to the transit network and also increase the total passenger travel cost. The results indicated that an optimal service cost or service quality of ETIS can be found to minimize the total passenger travel cost of the transit network.

It is well known that transit service reliability has a great impact on transit passenger travel behaviors, particularly in congested transit network with uncertainty in travel time. So, further study may extend the proposed model to investigate the impacts of ETIS and the transit service reliability simultaneously on transit passenger travel behaviors.

ACKNOWLEDGEMENTS

The work described in this paper was jointly supported by grants from the Research Grants Council of the Hong Kong Special Administrative Region (Project No. PolyU 5184/05E and PolyU 5202/06E), and the National Basic Research Program of China (2006CB705500).

REFERENCES


