A MULTI-USER CLASSES EMERGENCY EVACUATION NETWORK RECONSTRUCTION MODEL FOR LARGE-SCALE NATURAL DISASTERS

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Abstract: The present study is a follow-up study of Wang and Hu (2005). We extend the study of the previous study by decomposing the emergency network reconstruction problem into an emergency evacuation network reconstruction and an emergency rescue network reconstruction problem. In this study, the emergency evacuation network reconstruction problems are the main focus. We formulate the emergency evacuation network reconstruction for natural disasters as a bi-level programming model. In the model, the victims’ evacuation route choice behaviors and evacuation destinations choice behaviors are both considered at the same time. This concern is the most different point from Wang and Hu (2005). Moreover, the multi-class users’ evacuation route choice behaviors are also considered in the model. Finally, the optimal network reconstruction planning, multi-class users’ emergency evacuation routes and destinations planning can be obtained.

Key Words: Emergency evacuation, Network reconstruction, Multi-class users

1. INTRODUCTION

Emergency network reconstruction problems differ from general network design problems in several ways. First, emergency network reconstruction problems are usually investigated in the large-scale network destruction destroyed by numerous big uncertain emergency events, such as natural disasters. Second, the missions for emergency network reconstruction are generally required to rapidly restore the network’s basic capability in disaster area. That is, an efficient emergency network restore planning is necessary. Third, the maximum network emergency reconstruction is usually recovering to the original levels of network capacity before destruction. The emergency network reconstructive problems can be generally categorized into emergency rescue and emergency evacuation problems. The present study is a follow-up study of Wang and Hu (2005). Wang and Hu (2005) pointed out the important of emergency network reconstruction problems and developed an emergency network reconstruction model that was considered the multi-class users’ emergency route choice behaviors. But under a large-scale natural disaster situation, the behaviors between victims’ evacuation and the rescue personnel are very different. We extend the previous study and focus on emergency evacuation network reconstruction planning and designing after suffering from natural disasters. For practical application concerns, we also consider multi-class users’ route choice behaviors in the emergency evacuation network reconstruction problem.
The emergency evacuation network reconstruction for natural disasters problem is a network design problem. As developed in the study of Wang and Hu (2005), we can formulate the problem by a bi-level programming model in which the upper level problem is the decision of minimizing system cost under limited resources, and the lower level problem is one of constraints for the upper level problem that considers the victim evacuation influencing factors of evacuation travel behaviors. In this study, the lower level problem is different from that of Wang and Hu (2005). The lower level problem of Wang and Hu (2005) assumes that only the victims’ evacuation route choice behaviors are affected by emergency evacuation network reconstruction planning. In this study, we assume that the evacuation route choice and the destination choice are both major affecting factors for victim evacuation decision. Hence, the evacuation destination choice and route choice combined model should be adopted to reasonably describe the victim evacuation behaviors at the lower level problem. Further, for practical application concerns, the multi-user classes’ condition is also considered in the lower-level problem. Under multi-user classes’ evacuation trip distribution and route choice constraints, the bi-level problem regarding emergency evacuation network reconstruction for natural disaster is difficult to solve because the solution set of this problem is non-convex. In dealing with the non-convex problem, variational inequality sensitivity analysis method and the generalized inverse approach are considered for solving the non-convex, bi-level programming formulation. Moreover, the gradient projection method and diagonalization method are also incorporated in this research. Based on the results of the numerical example, the Stackelberg equilibrium solution of the bi-level programming model with multi-class evacuation trip distribution and route choice constraints application can be obtained. The results of this research are expected to be used in support of emergency evacuation rescue network reconstruction decisions when large-scale network is destroyed by natural disasters. In the following, Section 2 briefs the victim emergency evacuation network reconstruction problem and reviews the literatures related to emergency evacuation problem. The emergency evacuation network reconstruction bi-level programming model formulation is discussed in Section 3. A solution algorithm is developed and presented in Section 4. In Section 5, a numerical example illustrates the feasibility of the proposed framework. Finally, some concluding remarks are presented in Section 6.

2. PROBLEM DESCRIPTION AND LITERATURE REVIEW

Transportation network maintenance is an important issue for many large-scale events and/or special events, such as Olympic game, festivities, or natural disasters. Especially, the transportation network is often destroyed and deconstructed after suffering from natural disasters, such as typhoon or earthquake. Hence, the issue concerning network reconstruction due to natural disasters is a main research topic. No matter what emergency evacuation and rescue strategies are planned and implemented, the role of transportation network reconstruction is the kernel of natural disaster deconstruction issues. How to execute emergency evacuation and rescue tasks and maintain network’s basic capability, or reconstruct the destroyed transportation network systematically after suffering from natural disasters is significantly affecting the efficiency of emergency evacuation and rescue tasks. Owing to the network destruction is uncertain and unpredictable in nature, and the victim emergency evacuation and rescue route planning should be considered depending on the network reconstruction plan. The problem of emergency evacuation and rescue network reconstruction is complicated and difficult to formulate. In general, the victim emergency evacuation and disaster area rescue are two different issues. The issue of victim emergency evacuation concerns about how to escape from the disaster area as soon as possible, whereas the problem
of disaster rescue is dealing with how to effectively arrange the rescue personnel and/or team to enter the disaster area in a timely manner. That is, the trips directions are different between the emergency evacuation problem and the disaster area rescue problem. The disaster rescue problem should consider rescue route choice behaviors given fixed and known total flow generated at each rescue origin node and the total flow destined to each rescue destination node. The aim of the problem is to find the O-D flows, subject to the known total origin and destination flow constraints. The problem of victims’ emergency evacuation, however, considers only the evacuation destination choice and the route choice between each evacuation O-D pair. The problem assumes that the number of emergency evacuative trips leaving each origin is known and the aim is to find how these emergency evacuation trips are distributed among various evacuation destinations. The emergency network reconstruction operations are also different between the emergency evacuation problem and the disaster area rescue problem. In this research, we focus on the study of victims’ emergency evacuation network reconstruction model formulation, and the multi-user classes condition is also added in the present study. Note that the issue of disaster area rescue network reconstruction is not included in this study.

Wang and Hu (2005) developed a network reconstruction bi-level programming model for natural disaster emergency evacuation and rescue problem. In this research, they designed a network reconstruction process and added the multi-user classes condition. But they adopted a fixed evacuation and rescue O-D demands and ignored the difference in trip distribution between the evacuation and rescue problems. In this study, we employ the emergency network reconstruction model proposed by Wang and Hu (2005). For the characteristics of the evacuation problem, the emergency evacuation destination choice consideration should be added in model formulation. For practical application reasons, the multi-user classes route choice constraints should be also included in the emergency network reconstruction for victim evacuation from disaster area.

The victims’ emergency evacuation behaviors might be affected by two major factors. The two major factors are evacuation areas attraction and travel impedance. Therefore, evacuation destination choice and evacuation route choice combined model should be developed and to describe the victims’ emergency evacuation travel behaviors. Feng and Wen (2005) also adopted the trip distribution and traffic assignment combined model to formulate the victims’ evacuation problem. The multi-class users route choice behaviors are also considered in our emergency evacuation network reconstruction model. It is an asymmetric traffic assignment problem. Dafermos (1980) developed a more general traffic assignment model. The model has been designed in order to handle situations where there is interaction between traffic on different links or between different user classes of transportation in the same link. She established a variational inequality (VI) model to formulate the problems. Dafermos (1982) developed an elastic demand traffic assignment model and used a variational inequality (VI) model to formulate the problems. In the solution method phase, Sheffi (1985) transferred the trip distribution and traffic assignment combined model as a standard traffic assignment model and adopted the Frank-Wolfe method to solve the problem. Chen and Wang (1994) developed new solution algorithms for an asymmetric traffic assignment model with variable demand and compared the performance of those solution algorithms, including diagonalization method, streamlined diagonalization method, hybrid method, and streamlined hybrid method. However, these solution algorithms are again all link-based solution methods.

In the solution method investigation, Tobin and Friesz (1988) and Tobin (1986) applied the variational inequality sensitivity analysis method to solve a network equilibrium problem.
This method is also an efficient solution method for a bi-level programming model. But this method assumes that the strict complementary slackness conditions are satisfied and that the extreme point is non-degenerate. It is shown that if the perturbed problem is restricted to only those routes that are positive in the extreme point, it can still solve the original perturbed problem for small perturbations. In practice, many of the network equilibrium problems could not satisfy the strict condition of the non-degenerate condition. In conquering the technical problem, Cho (1991) provided the generalized inverse approach to advance the variational inequality sensitivity analysis method. The generalized inverse method has no non-degenerate assumption and has been adopted in solving network design problem by Wang (1999). The generalized inverse method used the network equilibrium solution characters in which path flow solutions are not unique, link flow solution is unique, and path flow solutions can be obtained through the path/link incidence matrix and transferred to link flow solution to avoid the solution non-unique problem. For more efficient algorithms in finding the relationship between path flow solution and link flow solution, the path-based solution algorithms such as the gradient projection method (Jayakrishnan, 1994) are more useful than link-based solution algorithms.

In view of the characteristics of the problem described above, we are conducting a follow-up research developed by Wang and Hu (2005) by adding the conditions of evacuation trips distribution choice. The problem can be also formulated as an emergency evacuation network reconstruction formulation by a bi-level programming model. The upper level problem is a network reconstruction system optimal problem. Under the upper level system objective, we can assure that the network reconstruction planning would minimize the emergency evacuation travel time. The lower level problem is a multi-user classes’ evacuation destination choice and evacuation route choice problem. It is also one of the constraints in the upper level problem. In the solution method development, in order to transfer the path flow solution into link flow solution, the gradient projection method is adopted. In the next section, we will formulate the emergency evacuation network reconstruction bi-level programming model and demonstrate the corresponding solution algorithms.

3. MODEL FORMULATION

The aim of the emergency evacuation network reconstruction problem is to find the optimal transportation network recovery strategies after suffering from natural disaster. The so-called “network reconstruction” is to recover highway capacities destroyed by natural disaster. The victim emergency evacuation travel choice behaviors would be impacted by different network reconstruction plans. On the other hand, the system performance of emergency evacuation would be impacted by disaster victims’ emergency evacuation travel choice behaviors. The disaster victims’ emergency evacuation travel choice behaviors include victims’ evacuation route choice and destination choice behaviors. Therefore, the optimal transportation network’s recovery strategies for the disaster victims’ emergency evacuation problem should consider travelers’ realistic behaviors. For practical applications, the multi-user classes travel choice affections are included in the evacuation network reconstruction problem. In this paper, we still assume that each disaster victim all wish to leave disaster area as soon as possible. The disaster commander understands victims’ wish, too. The multi-class users’ emergency evacuation network reconstruction bi-level model is formulated as follows:

\[
\min \sum_{a} \sum_{i} f_{ai} c_{ai} (f_{ai}, f_{a_{i+1}}, \ldots, f_{a_{i+k}}, f_{a_{i+k+1}}, y_{ai}) \forall a, i
\]  

(1)
subject to
\[ 0 \leq y_a \leq 1, \quad \forall a \in m \] (2)
\[ \sum_a \Delta y_a \text{CAP}_a \leq \Gamma, \quad \forall a \in m \] (3)
\[ \prod_{p=r,s} (I_a \delta_{ap}) \geq 1, \quad \forall p \in (r,s); \forall a \in p \] (4)
\[ I_a = \begin{cases} 
0, & \text{if } y_a = 0 \\
1, & \text{if } y_a > 0
\end{cases} \] (5)

Moreover, victim multi-user classes evacuation destination choice and evacuation route choice constraint is shown as follows:
\[ c(f^*, y)^T (f - f^*) - H^{-1}(q^*, y)^T (q - q^*) \geq 0, \forall (f, q) \in \Omega_y \] (6)

The above model adopts the basic model structure of the network reconstruction bi-level model proposed by Wang and Hu (2005). However, in that specific study, the evacuation destination choice behavior is added in lower-level problem. About the emergency evacuation network reconstruction model demonstrated in equation (1), the objective of the upper level model represents the minimization of the total system cost in reconstructing the destroyed network. The decision variable \( y_a \) is the ratio of reserve capacity after destroyed divided the original capacity of link \( a \). The decision variable \( f_{i_a} \) represents the flows of the \( i \)-th mode on link \( a \). \( f_{i_a} \) is also representing the decision variable of the lower-level multi-class users victim evacuation destination choice and evacuation route choice combined model. As we know, different network structure or link capacities will induce different user’s evacuation destination choice and route choice behaviors in a network. When the vector of the upper-level decision variables \( y \) is changed, the vector of the lower-level decision variables \( f \) will follow to change. We cannot exactly express this relationship as a complete functional form between \( y \) and \( f \). The variables \( y_a \) and \( f_{i_a} \) are implicit function and can be expressed as \( f(y) \). Moreover, equation (2) defines that the domain of the decision variable \( y_a \) should lie between 0 and 1. That is, the maximum highway recovery capacities are at most repaired to the original capacity. The variable of \( m \) is the set of destroyed links. Equation (3) represents the total highway repairing quantities must less than or equal to the given maximum repairing quantities. \( \Gamma \) is a vector of the maximum repairing quantities that system can provide and it is a well-known constant. \( \Delta y_a \) is the capacity repairing quantity on link \( a \). Equation (4) represents that there is at least one path between each O-D pair \( (r,s) \). \( I_a \) is an indicator variable in equation (5), when \( y_a = 0 \) then \( I_a = 0 \), representing link \( a \) has been destroyed and it can’t serve any flow. When \( 0 < y_a \leq 1 \) then \( I_a = 1 \), representing link \( a \) might be destroyed but it still can pass emergency evacuation and rescue traffic flow. Equation (6) is a VI model that represents multi-class users network equilibrium route choice and destination choice combined model, and it is the lower level model. The first term in equation (6) is the same term as in standard traffic assignment model, and the second term of \( H^{-1}(q^*_i) \) denotes a disutility function. The symbol \( \Omega_y \) denotes the feasible region of the lower level model which is associated with corresponding constraints, such as flow conservation, non-negativity and variable definition, as follows:
\[ \sum_{p} h_{p_i}^r = q_i^r, \quad \forall p, r, s, i \] (7)
\[
\begin{align*}
\mathbf{h}_{rs}^{\pi} & \geq 0, \quad \forall \, p, r, s, i \\
\sum_{s} q_{rs}^{\pi} &= \mathcal{O}_{r}, \quad \forall r, i \\
q_{rs}^{\pi} & \geq 0, \quad \forall \, r, s, i \\
f_{ai} &= \sum_{rs} \sum_{p} h_{ps}^{\pi} \delta_{a,pi}^{rs} \quad \forall \, a, p, r, s, i \\
\delta_{a,pi}^{rs} &= \{0,1\} \\
\end{align*}
\]

Equation (7) represents the summation of all the path flows between O-D pair \((r,s)\) must be equal to the O-D demands of the \(i^{th}\) mode. Equation (8) denotes all of the path flows about mode \(i\) must be non-negative. Equation (9) defines the evacuation trips relationship between evacuation origin and each evacuation destination for the \(i^{th}\) mode. Equation (10) denotes all of the \(i^{th}\) mode trip demands must greater than or equal to 0. Equation (11) expresses the relationship between link flows and path flows for the \(i^{th}\) mode. Equation (10) defines the link-path incidence index \(\delta_{a,pi}^{rs}\), if link \(a\) on path \(p\), then \(\delta_{a,pi}^{rs} = 1\); otherwise \(\delta_{a,pi}^{rs} = 0\).

Analyzing the lower-level problem under the monotonicity assumption of smooth functions \(c(f)\) and \(H^{-1}(q)\), the sub-problem of Equation (6) can be expressed as an optimal nonlinear problem as follows:

\[
\min z(f,q) = \sum_{a} \sum_{i} \int \int_{0} c_{a}(f_{a1},f_{a2},\ldots,f_{ai},\ldots,f_{au})d\omega \\
- \sum_{r} \sum_{i} \int \int_{0} H^{-1}(q_{1}^{r},q_{2}^{r},\ldots,q_{i}^{r},\ldots,q_{n}^{r})d\omega \\
\text{subject to (7)-(12)} (14)
\]

If the disutility function, \(H^{-1}(q_{i}^{r})\), the second of equation (13), can then be simplified into an entropy form, \(-\frac{\ln(q_{i}^{r})}{\beta} + 1\), then equation (13) becomes:

\[
\min z(f,q) = \sum_{a} \sum_{i} \int \int_{0} c_{a}(f_{a1},f_{a2},\ldots,f_{ai},\ldots,f_{au})d\omega + \frac{1}{\beta} \sum_{r} \sum_{i} q_{i}^{r} \ln(q_{i}^{r}) \\
\]

In solving the above optimization problem (13)-(14), one can incorporate the Lagrangean multipliers \(\pi_{r}^{i}\) and \(\mu_{r}^{f}\) into the objective function, and solve for the following Lagrangean function:

\[
\min L(f,q,\pi,\mu) = \sum_{a} \sum_{i} \int \int_{0} c_{a}(f_{a1},f_{a2},\ldots,f_{ai},\ldots,f_{au})d\omega \\
+ \frac{1}{\beta} \sum_{r} \sum_{i} q_{i}^{r} \ln(q_{i}^{r}) + \sum_{r} \sum_{i} \pi_{r}^{i} \left( q_{i}^{r} - \sum_{p} h_{ps}^{\pi} \right) + \sum_{r} \sum_{i} \mu_{r}^{f} \left( \delta_{r}^{f} - \sum_{s} q_{rs}^{\pi} \right) \\
\text{subject to}
\begin{align*}
\mathbf{h}_{rs}^{\pi} & \geq 0, \quad \forall \, p, r, s, i \\
q_{rs}^{\pi} & \geq 0, \quad \forall \, r, s, i \\
\end{align*}
\]

Taking the partial derivative of (16) with respect to path flow \(h\) and trip demand \(q\), one can obtain the first order condition of the formulation as follows:

\[
\mathbf{h}_{rs}^{\pi}(c_{p}^{\pi} - \pi_{r}^{i}) = 0 
\]
Equations (19)–(21) represent the compensated relaxation relationships in route choice behaviors for the $i^{th}$ mode, which means that the $i^{th}$ mode’s travel times of the paths being assigned traffic flows for a given O-D pair are equal to the minimum path travel times for the $i^{th}$ mode, otherwise the flows on the other paths would be forced to be zero. Equations (22)–(24) express the compensated relaxation relationships between destination attraction cost and specific O-D demands for the $i^{th}$ mode. The $i^{th}$ mode’s destination attraction cost is combined by minimal path cost and disutility between O-D pair ($r$, $s$). That is, when the $i^{th}$ mode’s trip demands on a specific O-D pair ($r$, $s$) which destination attraction cost are equal to the $i^{th}$ mode’s minimum destination attraction cost among all destination connecting the origin $r$, then the $i^{th}$ mode’s trip demands are greater than or equal to zero; otherwise, the $i^{th}$ mode’s trip demand $q_{ir}^s$ is equal to zero.

By employing the above optimality conditions of (19)–(24) in the lower-level problem, we can find that they are suitable to explain the criterion of multi-user classes victims’ evacuation destination choice and route choice behaviors. That is, the emergency evacuation network reconstruction model is under the multi-user classes’ evacuation behaviors to compute appropriate network reconstruction plan. Finally, by developing appropriate computing algorithms, one can solve the problem and obtain realistic network reconstruction strategies, and other crucial victim evacuation information for disastrous management purposes.

4. SOLUTION ALGORITHMS

In this study, we also adopt the variational inequality sensitivity analysis method (Tobin, 1986; Tobin and Friesz, 1988) and generalized inverse approach (Cho, 1991) to solve the victims’ emergency evacuation network reconstruction bi-level model. Analysis of the above emergency evacuation network reconstruction bi-level programming model with multi-user classes evacuation destination choice and evacuation route choice constraints, the feasible region delineated by expressions (2)–(12) is essentially non-convex because expression (6) and the implicit function $f(y)$ are nonlinear. Using the variational inequality sensitivity analysis, the corresponding implicit differentiation can be obtained. The descent search direction of the upper level objective function also can be calculated. Owing to the multi-user classes evacuation destination choice and evacuation route choice constraints are considered in the bi-level model, we can adopt the super network concept (see Figure 1) to transfer the lower-level problem about evacuation destination choice and evacuation route choice combined model into a standard fixed demand victims’ evacuation route choice model (Sheffi, 1985). This approach can simplify the evacuation network reconstruction model and apply the modifications for calculating the derivatives by the variational inequality sensitivity analysis.
In solving the standard fixed demand traffic assignment model, we can adopt gradient projection method. A simple network shown in Figure 2 is used for testing. The test network consists of 12 links and 6 nodes, in which nodes 1, 2, 5, 6 represent both origins and destinations, and nodes 3, 4 are intermediates. The origin trip distribution demands are assumed in Table 1. In order to express the asymmetric effect between different classes of users, we adopt the asymmetric link travel time functions (equations (25) and (26)) proposed by Wang and Hu (2005) and to emphasize the different effect of multi-class users emergency evacuation destination and route choice behavior. The disutility function used in the work by Chen and Wang (1994) is adopted, and it is defined as equation (27). The link free travel time assumed here is 1, and the parameter $\beta$ is with respect to location character and can notably be defined exogenously. Since the correct $\beta$ estimate is not of particular interest, $\beta$ is assumed here to be equal to 1 for simplicity. We further assume that each link in this network the initial ratio of capacities to saturation flows is 0.5 and the saturation flow is 50 (Table 2). In here, we extend the network as figure 3, where node 7, 8 and the links connecting nodes 5, 6 and nodes 7, 8 are dummy nodes and dummy links. The link cost function of those dummy links can be defined as equation (27), and the evacuation trips of vehicle type A and vehicle type B from node 1 to dummy node 7 are 50 and 40, respectively, and from node 2 to dummy node 8 are 40 and 30 (Table 3). When the results converge to the network equilibrium status, the link flows are shown in Table 4 and the link flows pattern can be obtained by the gradient projection method. In these results we can find that when network reaches equilibrium, given a specific O-D pair and mode, the travel costs of the paths being used are less than or equal to those of the paths are not used.

![Figure 2 Test network 1](image)

**Figure 1** Network representation for solving evacuation destination choice and route choice combined problem

In here, we extend the network as figure 3, where node 7, 8 and the links connecting nodes 5, 6 and nodes 7, 8 are dummy nodes and dummy links. The link cost function of those dummy links can be defined as equation (27), and the evacuation trips of vehicle type A and vehicle type B from node 1 to dummy node 7 are 50 and 40, respectively, and from node 2 to dummy node 8 are 40 and 30 (Table 3). When the results converge to the network equilibrium status, the link flows are shown in Table 4 and the link flows pattern can be obtained by the gradient projection method. In these results we can find that when network reaches equilibrium, given a specific O-D pair and mode, the travel costs of the paths being used are less than or equal to those of the paths are not used.

![Figure 3 Extension of test network 1](image)

**Table 1** Origin trip demands

<table>
<thead>
<tr>
<th>Origin</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>Demands</td>
<td>50</td>
<td>40</td>
</tr>
</tbody>
</table>

**Table 2** Parameters setting for each link

<table>
<thead>
<tr>
<th>Initial $y_a$</th>
<th>Saturation Flow Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>50</td>
</tr>
</tbody>
</table>

$$c_{ad}(f) = c_{ae} \left[1 + 0.15 \left(\frac{f_{ad} + 0.2 f_{ah}}{CAP_a}\right)^4\right], \forall d$$

(25)
Next, based on the network equilibrium solutions, we impose a small perturbation on the ratio of capacities to saturation flows, and compare the actual solution with the estimated solution calculated by variational inequality sensitivity analysis and generalized inverse approach. The results are summarized in Tables 5 and 6. In the results of Table 5 we can find that, when we impose a small perturbation to the capacity rate, the estimated solution of link flows is almost the same as actual solution. Actually, if the perturbation was too big, the difference between the estimated solution and actual solution would be increased. After the numerical test, we have demonstrated that the implicit differentiation can be calculated by variational inequality sensitivity analysis and generalized inverse approach. And the exactly descent search direction could be found. Therefore, by further design on the optimal step sizes, the Stackelberg solution of the bi-level programming model could be solved.

### Table 3 Extension of the O-D table for test network 1

<table>
<thead>
<tr>
<th>Origin-Dummy Destination</th>
<th>Vehicle A</th>
<th>Vehicle B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-7</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>2-8</td>
<td>40</td>
<td>30</td>
</tr>
</tbody>
</table>

### Table 4 Link flows

<table>
<thead>
<tr>
<th>Link</th>
<th>Flows</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vehicle A</td>
</tr>
<tr>
<td>1→2</td>
<td>13.18</td>
</tr>
<tr>
<td>1→3</td>
<td>44.98</td>
</tr>
<tr>
<td>2→1</td>
<td>8.16</td>
</tr>
<tr>
<td>2→4</td>
<td>45.02</td>
</tr>
<tr>
<td>3→5</td>
<td>45.04</td>
</tr>
<tr>
<td>4→3</td>
<td>0.06</td>
</tr>
<tr>
<td>4→6</td>
<td>44.96</td>
</tr>
<tr>
<td>5→6</td>
<td>11.50</td>
</tr>
<tr>
<td>6→5</td>
<td>11.45</td>
</tr>
</tbody>
</table>

### Table 5 Comparison of link flows between actual solutions and estimated solutions

<table>
<thead>
<tr>
<th>Link</th>
<th>$\varepsilon = 0$</th>
<th>$\varepsilon = 0.05$</th>
<th>$\varepsilon = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1→2</td>
<td>13.18</td>
<td>10.66</td>
<td>13.18</td>
</tr>
<tr>
<td>1→3</td>
<td>44.98</td>
<td>34.99</td>
<td>44.97</td>
</tr>
<tr>
<td>2→1</td>
<td>8.16</td>
<td>5.65</td>
<td>8.16</td>
</tr>
<tr>
<td>2→4</td>
<td>45.02</td>
<td>35.01</td>
<td>45.03</td>
</tr>
<tr>
<td>3→5</td>
<td>45.04</td>
<td>35.03</td>
<td>45.04</td>
</tr>
<tr>
<td>4→3</td>
<td>0.06</td>
<td>0.03</td>
<td>0.07</td>
</tr>
<tr>
<td>4→6</td>
<td>44.96</td>
<td>34.98</td>
<td>44.96</td>
</tr>
<tr>
<td>5→6</td>
<td>11.50</td>
<td>9.15</td>
<td>11.50</td>
</tr>
</tbody>
</table>


Table 6 Comparison of O-D demands between actual solutions and estimated solutions

<table>
<thead>
<tr>
<th>O-D Pair</th>
<th>$\varepsilon = 0$</th>
<th>$\varepsilon = 0.05$</th>
<th>$\varepsilon = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Vehicle A</td>
<td>Vehicle B</td>
<td>Vehicle A</td>
</tr>
<tr>
<td></td>
<td>25.31</td>
<td>20.19</td>
<td>25.31</td>
</tr>
<tr>
<td></td>
<td>24.69</td>
<td>19.81</td>
<td>24.69</td>
</tr>
<tr>
<td></td>
<td>19.67</td>
<td>14.80</td>
<td>19.67</td>
</tr>
<tr>
<td></td>
<td>20.33</td>
<td>15.20</td>
<td>20.33</td>
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<tr>
<td></td>
<td>25.31</td>
<td>20.19</td>
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<td>24.69</td>
<td>19.81</td>
<td>24.69</td>
</tr>
<tr>
<td></td>
<td>19.67</td>
<td>14.80</td>
<td>19.67</td>
</tr>
<tr>
<td></td>
<td>20.33</td>
<td>15.20</td>
<td>20.33</td>
</tr>
</tbody>
</table>

After the test about variational inequality sensitivity analysis and generalized inverse approach, we further design the solution algorithm for emergency evacuation network reconstruction bi-level programming model as follows:

Step 0: adding the dummy links and dummy nodes and extend the basic network as a super network.

Step 1: set the initial values of $\gamma^n$. Let $n=1$. $\gamma^n$ is the ratio of link reserve capacities after destroyed divided by the link original capacities.

Step 2: compute the link capacities based on $\gamma^n$.

Step 3: using the diagonalization method and gradient projection method to solve the network equilibrium variational inequality model as follows:

$$
\text{c}(f^+, y^+) (f - f^+) - H^{-1}\left(\text{q}^+, y^+\right) (\text{q} - \text{q}^+) \geq 0, \forall (f, q) \in \Omega
$$

where is the feasible solution areas under the conditions of equations (7) – (12).

Step 4: apply the variational inequality sensitivity analysis and generalized inverse approach to calculate the implicit differentiation and obtain $\nabla \text{f}(0)$.

Step 5: compute the link capacity modifies descent search direction as follows:

$$
d^n = -\nabla \varepsilon \left(\text{fc}(f, 0)\right)$$

For example, the asymmetric link cost function (25) in this study, the descent search direction can be derived as follows:

$$
d^n = -\nabla \varepsilon \left(\text{fc}(f, 0)\right)$$

$$
= -\left(\varepsilon_0 \nabla f_a (0) + 0.15 \varepsilon_0 \nabla f_d (0) (f_d + 0.2 f_b) \right)(\gamma^n)^4 + 0.6 \varepsilon_0 f_d (f_d + 0.2 f_b)^3 [\gamma^n (\nabla f_d (0) + 0.2 \nabla f_b (0)) - (f_d + 0.2 f_b)] S^{-4} (\gamma^n)^5 + \varepsilon_0 \nabla f_b (0) + 0.15 \varepsilon_0 \nabla f_b (0) (0.5 f_d + f_b) \right)^4 (\gamma^n)^4 + 0.6 \varepsilon_0 f_b (0.5 f_d + f_b)^3 [\gamma^n (0.5 \nabla f_d (0) + \nabla f_b (0)) - (0.5 f_d + f_b)] S^{-4} (\gamma^n)^5
$$

Step 6: update $\gamma^n$, where $\gamma^{n+1} = \gamma^n + \frac{1}{n+1} \text{d}^n$, $\Delta \gamma = \gamma^{n+1} - \gamma^n$, $0 \leq \Delta \gamma \leq 1$.

Step 7: revise the link repairing capability to satisfy the maximum repairing capability constraint as follows:

$$
\sum \Delta y_a \times \text{CAP}_a \leq \Gamma, \forall a \in m
$$

Step 8: convergence test. If $\{y^a\} \approx \{y^{a+1}\}$, stop. Otherwise, set $n=n+1$ and go to Step 2.

Applying the solution steps as above, we can obtain the link repairing rate $y$. And it can be
used in support of emergency evacuation network reconstruction decisions when large-scale network destruction under natural disasters.

In step 3 of the above solution algorithm, using the diagonalization method and gradient projection method to solve the network equilibrium variational inequality model can be transferred into optimal sub-problem as follows:

\[
\min z(f, q) = \sum_a \sum_i \int_0^{h^*_a} c_a(f_a, f_{a_1}, ..., f_{a_n}, \omega, f_{a_1}, ..., f_{a_n})d\omega + \frac{1}{\rho} \sum_{rs} \sum_i q^*_i \ln(q^*_i) \quad (32)
\]

Combine the diagonalization method and gradient projection method to solve the above optimal sub-problem. Details of the algorithm are summarized as follows:

Step 1: set \( n = 0 \), calculate the \( i^{th} \) mode shortest paths based on the free-flow link travel times \( \{c_{a_0}\} \) of each OD pair \( rs \), and generate a set of paths with path flow between \( rs \) of \( \forall = s, r, q, h, \), define the path flows as \( \{h^*_p\} \), set \( i = 0 \).

Step 2: set \( i = i + 1 \), fixed the cross effects at the current level and solving the main effect.

Step 3: solve the master problem:

Step 3.1: set \( n = n + 1 \), calculate the link travel times \( \{c_{a_n}\} \) based on the \( i^{th} \) mode route flows \( \{h^*_p\} \) between O-D pair \( (r, s) \).

Step 3.2: calculate the shortest paths based on the prevailing link travel times \( \{c_{a_i}(x)\} \), and label it as the first solution \( \hat{p}_i \) in the solution set \( \{p_{a_n}\} \).

Step 4: update the path flows \( \{h^*_p\} \) and link flows \( f^*_a \) in equation (33) ~ (36).

\[
\begin{align*}
\hat{p}_i^{(n+1)} &= \max \{0, h^*_p - \alpha^{(n)} d^{(n)}_p, \forall r \in R, s \in S, p_i \neq \hat{p}_i \} \\
\hat{p}_i^{(n+1)} &= \hat{q}_i^{(n)} - \sum_{p \neq \hat{p}_i} h^*_p, \forall r \in R, s \in S \\
d^{(n)}_p &= \left(c^{(n)}_p - c^{(n)}_{\hat{p}_i}\right), p_i \neq \hat{p}_i \\
\alpha^{(n)}_{p_i} &= \frac{\lambda}{\sum_a c'_a \delta^{(n)}_{a_p} + \sum_a c'_a \delta^{(n)}_{a_{\hat{p}_i}} - \sum_{a \neq p, \hat{p}_i} 2c'_a}, \forall r \in R, s \in S, p_i \neq \hat{p}_i, 0 < \lambda \leq 1
\end{align*}
\]

Step 5: mode convergence check, compare the difference of route cost between the same O-D pair and using the same mode. If the percentage difference is greater than a pre-specified value \( \theta \), then return to Step 3, otherwise check whether all of the modes have been tested in Step5, then go to Step 6, else set \( n = 0 \), go to Step 2.

Step 6: network equilibrium convergence check, compare the difference of link flows between two successive traffic assignment results, if the percentage difference is less than a pre-specified value \( \theta \), then STOP, otherwise set \( i = 0 \), and return to Step 2.

5. NUMERICAL TESTS

Based the model framework and solution algorithms described in the previous sections, the present section conduct various numerical analysis to demonstrate the feasibility of the
proposed framework and approach in formulating the optimal network reconstruction planning and design in case of natural disasters.

5.1 Data Input
The test network consists of a simplified freeway/expressway system as shown in Figure 4 designed by Wang and Hu (2005). The link lengths are referring to a digital map released by the government office. The free flow travel times are obtained indirectly from the transformation of corresponding speed limits on the observed links. In terms of the link capacities, it was set by various link capacities depending on the geometric characteristics of each link. We also adopt the asymmetric link cost functions proposed by Wang and Hu (2005) and they are shown in equations (25) and (26). The vehicle loadings are classified as two types, namely types A and B. The trip distribution disutility function is shown in equation (27). The computing environment is at the PC under Pentium III-500 platform, and the solution algorithms were implementing by applying Borland C++ Version 5.02 language.

![Figure 4 Test network 2](image)

5.2 Scenario Designs
In the design of test scenarios, it is supposed that a large-scale natural disaster causing different degrees of destruction of various links in the observed network. The road repairing and rescue unit has 1,000 man powers with 10 units of repairing capacity of each man. The emergency evacuation teams have two types of vehicles. The evacuation origin and destination locations have been investigated previously. The evacuation origin locations are nodes 11, 37 and 39. The evacuation destination locations are nodes 4, 24 and 32. The evocation origin trip demands are shown in Table 7, and the network destruction was surveyed and demonstrated in Table 8. The purposes of the numerical tests are to analyze the specific problem and obtain prioritized links needed to be repaired at the very first stage, and plan the optimal evacuation routes.

<table>
<thead>
<tr>
<th>Table 7 The evacuation demands</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origin</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>37</td>
</tr>
<tr>
<td>39</td>
</tr>
</tbody>
</table>
5.3 Results Analysis

According the results shown in Tables 8~10, we have found the following results:

1. The information obtained in Table 9 including various vehicle types’ trip distribution demands, used paths evacuation travel time and O-D choice cost. In the Table 10, we have obtained the used paths, traffic loadings, and path travel times for different vehicle types. In Tables 10, we also can find that they are under equilibrium state. That is, the path travel times of the used paths are essentially the same, and the O-D choice costs of the generated O-D distribution demands are also the same. That is the costs of the paths being used for a specific O-D pair are the same and the costs of the O-D pairs being distributed for a specific trip origin location are the same, too. The outcome is complying with Wardrop’s first principle of equilibrium. Under the situation, each victim can choose his/her best evacuative path and escape disasters area as soon as possible.

2. In the results of Table 10, it is found that the systems cost is 915667 when the repairing capacity is not optimally allocated. After the reallocation of the resources obtained by the present research, the system cost is reduced to 861723. Therefore, the test result has demonstrated the proposed framework is capable of predicting better resources allocation and routing strategies, which in turn one can improve the effectiveness and efficiency of total emergency evacuation mechanism.

3. The contents shown in Table 8 demonstrate the ratios of capacities and different vehicle types’ traffic flows on each link after repairing capacity optimization. One of the
important aspect of the present research is that for practical applications one can have a clear picture of degrees of repairing on each link by referring the ratios of capacities before and after disaster and arrange corresponding repairing capacities in reconstructing specific links.

Table 9 Trip distribution and travel cost between each O-D pair

<table>
<thead>
<tr>
<th>Vehicle Type</th>
<th>O-D Pairs</th>
<th>The O-D Demands Distribution</th>
<th>Used Path Travel Time ((c_{rs}^{\text{rs}})) with Mode (i)</th>
<th>Disutility Function ((-H^{-1}(q_{rs}^{\text{rs}})})</th>
<th>O-D Choice Cost ((\tilde{c}_{pi}^{\text{rs}})}) with Mode (i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>11-4</td>
<td>1033.45</td>
<td>28.53</td>
<td>8509.17</td>
<td>8537.70</td>
</tr>
<tr>
<td></td>
<td>11-24</td>
<td>1032.73</td>
<td>35.07</td>
<td>8502.63</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11-32</td>
<td>1033.82</td>
<td>25.23</td>
<td>8512.47</td>
<td></td>
</tr>
<tr>
<td></td>
<td>37-4</td>
<td>1167.67</td>
<td>31.66</td>
<td>9330.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>37-24</td>
<td>1166.96</td>
<td>38.21</td>
<td>9323.78</td>
<td>9361.99</td>
</tr>
<tr>
<td></td>
<td>37-32</td>
<td>1165.37</td>
<td>52.63</td>
<td>9309.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>39-4</td>
<td>899.82</td>
<td>39.73</td>
<td>7701.77</td>
<td></td>
</tr>
<tr>
<td></td>
<td>39-24</td>
<td>899.54</td>
<td>42.24</td>
<td>7699.26</td>
<td>7741.50</td>
</tr>
<tr>
<td></td>
<td>39-32</td>
<td>900.64</td>
<td>32.41</td>
<td>7709.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td>11-4</td>
<td>833.40</td>
<td>30.06</td>
<td>9731.66</td>
<td>9761.72</td>
</tr>
<tr>
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<td>11-24</td>
<td>832.99</td>
<td>36.40</td>
<td>9725.32</td>
<td></td>
</tr>
<tr>
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<td>11-32</td>
<td>833.61</td>
<td>26.87</td>
<td>9734.85</td>
<td></td>
</tr>
<tr>
<td></td>
<td>37-4</td>
<td>667.24</td>
<td>32.78</td>
<td>8922.42</td>
<td></td>
</tr>
<tr>
<td></td>
<td>37-24</td>
<td>666.80</td>
<td>39.31</td>
<td>8915.89</td>
<td>8955.20</td>
</tr>
<tr>
<td></td>
<td>37-32</td>
<td>665.96</td>
<td>52.64</td>
<td>8902.56</td>
<td></td>
</tr>
<tr>
<td></td>
<td>39-4</td>
<td>999.89</td>
<td>42.19</td>
<td>10553.38</td>
<td></td>
</tr>
<tr>
<td></td>
<td>39-24</td>
<td>999.76</td>
<td>44.50</td>
<td>10551.07</td>
<td>10595.57</td>
</tr>
<tr>
<td></td>
<td>39-32</td>
<td>1000.35</td>
<td>34.98</td>
<td>10560.59</td>
<td></td>
</tr>
</tbody>
</table>

Table 10 Path flows of the lower level problem

<table>
<thead>
<tr>
<th>OD Pair</th>
<th>Vehicle Type</th>
<th>Used Path</th>
<th>Traffic Loads</th>
<th>Path Travel Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>11→4</td>
<td>A</td>
<td>11→10→9→8→7→6→5→4</td>
<td>1033.45</td>
<td>28.53</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>11→10→9→8→7→6→5→4</td>
<td>833.40</td>
<td>30.06</td>
</tr>
<tr>
<td>11→24</td>
<td>A</td>
<td>11→10→9→8→7→51→50→49→24</td>
<td>336.84</td>
<td>35.07</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>11→12→13→14→15→31→32</td>
<td>1033.82</td>
<td>25.23</td>
</tr>
<tr>
<td>11→32</td>
<td>A</td>
<td>37→38→6→5→4</td>
<td>28.88</td>
<td>31.66</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>37→5→4</td>
<td>1138.80</td>
<td>32.28</td>
</tr>
<tr>
<td>37→4</td>
<td>A</td>
<td>37→38→6→5→4</td>
<td>33.82</td>
<td>32.78</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>37→5→4</td>
<td>633.43</td>
<td>32.78</td>
</tr>
<tr>
<td>37→24</td>
<td>A</td>
<td>37→38→6→5→4→3→4→23→24</td>
<td>421.44</td>
<td>38.21</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>37→5→4→9→24</td>
<td>2.49</td>
<td>39.31</td>
</tr>
<tr>
<td>37→32</td>
<td>A</td>
<td>37→38→9→10→52→29→30→31→32</td>
<td>1165.36</td>
<td>52.63</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>37→38→9→10→52→29→30→31→32</td>
<td>665.96</td>
<td>52.64</td>
</tr>
<tr>
<td>39→4</td>
<td>A</td>
<td>39→38→6→5→4</td>
<td>717.64</td>
<td>39.73</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>39→38→6→5→4</td>
<td>824.88</td>
<td>42.19</td>
</tr>
<tr>
<td>39→24</td>
<td>A</td>
<td>39→12→10→9→8→7→6→5→4</td>
<td>899.53</td>
<td>42.24</td>
</tr>
</tbody>
</table>
6. CONCLUSIONS AND REMARKS

The purpose of this research aims at how to reconstruct the emergency evacuation network reconstruction after the transportation network has been destroyed by natural disasters. Based on the network design and evacuation route choice and destination choice behavior constraints for multi-user classes’ considerations, a bi-level formulation has been proposed. In the meantime, we have developed a solution algorithm that applies the variational inequality sensitivity analysis method, generalized inverse matrix approach, diagonalization method, and gradient projection method to solve the problem. Based on the numerical tests, we can derive the conclusions and remarks as follows:

1. The bi-level programming model with asymmetric route choice and trip distribution constraints is a non-convex problem. By adopting the variational inequality sensitivity analysis method, generalized inverse approach, diagonalization method, and gradient projection method, one can obtain the Stackelberg solution.

2. We use the network representation skill and transfer the multi-user classes trips distribution and traffic assignment combined model into a standard fixed demand asymmetric traffic assignment model in the lower-level problem. By doing so, the problem can reduce its complexity of the emergency evacuation network reconstruction bi-level programming model. One can further use the diagonalization method and gradient projection method to solve the lower-level problem.

3. The outputs of the emergency evacuation network reconstruction bi-level programming model include: the repairing ratio suggestions for destroyed links, emergency evacuation routes planning, the demands of trip distribution between each O-D pair, route travel cost and demands, the travel demands on each link, and link travel time. These outputs are crucial information in executing emergency evacuation missions.

REFERENCES


Wang, C. Y., (1999), Dynamic Travel Choice Models with Link Capacity Side Constraints, Ph.D. Dissertation, Department of Civil Engineering, National Central University, Taiwan.

**NOTATIONS**

\( a \) link number

\( p \) path \( p \)

\( c_a \) vector of link travel cost

\( q \) vector of traffic demands

\( c_{rs}^p \) the \( p^{th} \) path’s travel cost between O-D pair

\( q_{ri}^{*} \) traffic demands between O-D pair \( rs \) with mode \( i \)

\( \hat{d}_{rs}^p \) O-D choice cost of the \( p^{th} \) path between O-D pair \( rs \) with mode \( i \)

\( \text{CAP}_a \) link capacity of link \( a \)

\( d \) vector of improved directions

\( S \) saturation flow rate or link original capacity

\( d_{rs}^p \) improved direction of the \( p^{th} \) path given O-D pair \( rs \)

\( f_a \) traffic flow of link \( a \) using the mode \( i \)

\( H^{-1}(q_i^{rs}) \) disutility function between OD pair \( rs \) using mode \( i \)

\( H(q) \) vector of disutility function

\( h \) vector of path flow

\( \beta \) location character parameter

\( h_{rs}^{p_i} \) path flow of \( p \) given OD pair \( rs \) using mode \( i \)

\( \gamma \) the maximum total links capacity reconstruction capability

\( I_a \) indicator variable; if link \( a \) is on path \( p \) between OD pair \( rs \), otherwise, \( \delta_{0a}^{rs} = 0 \).

\( i \) number of mode type

\( J \) vector of Jacobian matrix

\( J \) Jacobian matrix

\( m \) set of destroyed links

\( n \) number

\( \delta_{rs}^{pr} \) traffic demands from origin \( r \) with mode \( i \)

\( P \) set of path

\( \hat{\delta}_{rs}^{pr} \) OD pair \( rs \) with mode \( i \), \( \delta_{rs}^{pr} = 1 \); otherwise, \( \delta_{rs}^{pr} = 0 \).

\( \alpha \) step size

\( \beta \) improved pace of the step size

\( \delta_{0a}^{rs} \) indicator variable; if link \( a \) is on path \( p \) between OD pair \( rs \), otherwise, \( \delta_{0a}^{rs} = 0 \).

\( \epsilon \) perturbation parameters

\( \theta \) the threshold value of convergence

\( \lambda \) the threshold value of convergence

\( \Omega \) feasible solution area

\( \Omega \) feasible solution area

\( \Omega \) feasible solution area

\( \Omega \) feasible solution area

\( \Omega \) feasible solution area

\( \Omega \) feasible solution area