Abstract: This paper proposes a concept of critical post-encroachment time (PET) for all-red clearance interval design at signalized intersections that aims at achieving the optimum performance in both safety and mobility. The critical PET is defined as the minimum accepted PET at the conflict point by the first entering drivers. Variability of the accepted PETs in the case of late exit was analyzed at a study intersection in order to estimate the critical PET. Results showed that they follow a two-parameter Weibull distribution and tend to increase as the entering distance rises. Its 15th percentile value, approximately 2.0s, was then used as the critical PET to discuss the implication in the design of all-red time through numerical calculations. Conclusions supported that the calculated all-red time based on the critical PET could achieve significant operational and safety benefits, as compared with the all-red times based on the existing methods.

Key Words: critical post-encroachment time, all-red clearance interval, late exit, signalized intersection

1. INTRODUCTION

In Japan, over 40.0% of the total accidents occurred at or near intersections, and approximately 16.2% of the total fatal accidents took place inside signalized intersections in the year of 2009 (Japan National Police Agency, 2010). A significant portion of the fatalities were due to the violation of traffic signals, e.g., red-light-running and hurry start (Hagita et al, 2005). Those indiscipline or law-violating behavior at the change of phases is closely associated with the design of intergreen times, i.e., the yellow change and all-red clearance intervals, in addition to traffic enforcement. Meanwhile, intergreen times are a direct cause of lost time and critical for determining the optimum cycle length in the planning stage (Tang et al, 2010). Therefore, rational design of intergreen times is of great importance for both safety and operational efficiency at signalized intersections.

However, the all-red clearance interval in Japan is often set longer than sufficient because of the current intersection planning and design philosophy (Tang and Nakamura, 2008). It is particularly true for the right-turn traffic with comparably low clearing speed. Unnecessarily long all-red time not only induces the aforementioned risky behavior of drivers being aware of its abundant length (Shikata et al, 2003; Tang and Nakamura, 2007), but also results in larger lost time, cycle length, and thus control delay. On the other hand, insufficient all-red time may produce clearance failure and lead to accidents.

In view of that, the purpose of this study is to propose a concept of critical post-encroachment
time (PET) for all-red time design in order to achieve the optimum performance in both safety and mobility. Furthermore, this research also intends to explore the variability of critical PET and its implication in the design of all-red time, based on empirical results at an intersection located in Tokyo. The remaining of the paper is organized as follows. Chapter 2 reviews past research on the all-red time design as well as the related driver behavior. Chapter 3 illustrates the concept of critical PET. Chapter 4 describes the study site and data collection. Chapter 5 analyzes the variation of critical PETs at the study site and its relationships with multiple factors. Chapter 6 discusses the implication in the design of all-red time through numerical calculations. Finally, conclusions and future works are summarized in Chapter 7.

2. PAST RESEARCH

The theoretical basis for the calculation of intergreen times was made by Gazis, Herman, and Maradudin in 1960. In accordance with their theory, a few researchers have proposed other calculation methods of intergreen times, e.g., Crawfoard and Taylor (1961), Parsonson and Santiago (1980), Chang et al (1985), and Lin (1986). At present, determination of the yellow change interval is pretty similar all over the world, which is based on the dilemma zone theory. However, determination of the all-red clearance interval considerably varies from country to country both in guidelines and practice, owing to distinguished driver behavior and signal operation polices. Table 1 highlights calculation methods of the all-red clearance interval in the major developed countries (Tang and Kuwahara, 2009). Note that clearance distance, $S_c$, is commonly defined as the distance between the stop-line and furthest point of potential conflict with vehicles or pedestrians of the next phase. Thus, it should be measured from the furthest clearing lane and the nearest entering lane if multiple lanes exist, and from the position of the pedestrian crossing if the following phase is a pedestrian phase.

Those methods can be generally categorized into three types: the methods considering entering distance, the methods considering entering time, and the methods without considering entering distance or time. Marginal status at the end of the all-red time interval under each method is also illustrated in the table, assuming that the last clearing vehicle crosses the stop-line at the end of the yellow change interval and the first entering vehicle makes a normal start to move. In the first and second types of methods, i.e., the methods in the United Kingdom and Germany, the first entering vehicle will almost collide with the last clearing vehicle. However, entering drivers rarely accept that degree of risk in reality, and thus may have to take evasive actions and wait for crossing after the clearing vehicle is far enough from the conflict point. Consequently, extra start-up lost time occurs (Suzuki et al, 2010) and meanwhile risk exposure to the following vehicle increases due to the evasive actions. In the third type of methods, the first entering vehicle is not permitted to enter the intersection until the last clearing vehicle departs from the conflict point or from the entire intersection. As a result, extra clearance lost time may be generated (Sasaki et al, 2010), particularly when the entering distance or entering time is large.

Therefore, the optimum all-red clearance interval, i.e., enabling to avoid the evasive actions and eliminate the extra clearance lost time, can be found out if we know the minimum acceptable post-encroachment time (PET) of the entering drivers at the conflict point, i.e., the critical PET. In fact, a few similar concepts have been proposed and widely used in the field of traffic engineering. For instance, critical gap is one of the major parameters for capacity
Table 1 Design of the all-red clearance interval in the major developed countries

<table>
<thead>
<tr>
<th>Country</th>
<th>Design of the all-red clearance interval (AR)</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>The United Kingdom</td>
<td>$AR=2s$, $(S_c-S_e)&lt;9m$; $AR=3s$, $10m&lt;(S_c-S_e)&lt;18m$; $AR=9s$, $65m&lt;(S_c-S_e)&lt;73m$. Where, $S_c$=clearance distance, m; $S_e$=entering distance, m.</td>
<td>Type 1: the methods considering entering time (※AR includes the all-red and the red-and-amber, 2s)</td>
</tr>
<tr>
<td>Germany</td>
<td>$AR = t_c - t_e$ Where, $t_c$=clearance time, s; $t_e$=entering time, s. (※it holds true only when crossing time, $t_u$, is equal to yellow time. AR includes the all-red and the red-and-amber, 1s)</td>
<td>Type 2: the methods considering entering time</td>
</tr>
<tr>
<td>The United States</td>
<td>$AR = \frac{W + L}{V_c}$ Where, $W$=intersection width, m; $L$=vehicle length, m; $V_c$=clearing speed, m/s.</td>
<td>Type 3: the methods without considering entering distance or entering time</td>
</tr>
<tr>
<td>Japan</td>
<td>$AR = \frac{S_c + L}{V_c}$ Where, $S_c$=clearance distance, m; $L$=vehicle length, m; $V_c$=clearing speed, m/s.</td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>$AR = \frac{S_c}{V_c}$ Where, $S_c$=clearance distance, m; $V_c$=clearing speed, m/s.</td>
<td></td>
</tr>
</tbody>
</table>

analysis at intersections. It is typically defined as the minimum time needed for a vehicle in
the minor road to enter safely the major road stream, normally ranges between 4.1s and 7.5s (TRB, 2001). The critical gap is the median of the minimum accepted gaps that divides the data set into two parts of equal size. Another definition given by Raff and Hart (Salter, 1974) defines the critical gap as that gap of which the number of accepted gaps shorter than it is equal to the number of rejected gaps longer than it, i.e., crossing point of the accepted and rejected gaps’ cumulative curves. With respect to safety, time-to-collision (TTC) is an effective measure for rating the severity of traffic conflicts, which is defined by Hayward (1972) as the time required for two vehicles to collide if they continue at their present speed and on the same path. Some threshold values of TTC have been utilized for discriminating critical from normal behavior in Collision Avoidance Systems, such as 1.5s for the minimum value and 4.0s for the criteria to produce a direct warning to the driver (Grayson et al, 1984). Those values were derived based on field experiments and/or driving simulator.

Hence, the concept of critical PET could be implemented in the design of all-red clearance interval to achieve the maximum benefits. This idea was initially suggested by Tang et al in 2011 that attempted to develop a reliability-based intergreen time design method and the value of critical PET was simply assumed to be 1.0s due to the lack of empirical results. As a follow-up study, this paper is going to further develop the concept of critical PET and investigate its variability through empirical analysis.

3. CRITICAL POST-ENCROACHMENT TIME

At the change of phases, incompatible traffic movements have to pass a number of common areas inside the intersections, known as conflict areas or points. Accordingly, traffic conflicts, e.g., merging, diverging, and crossing, may take place. Out of which, the conflict between the clearing right-turn traffic and the entering through traffic is perhaps most significant at signalized intersection in Japan, in terms of its high occurrence frequency and important effects on operational efficiency because of conventional signal phasing plans in Japan as well as its comparably long all red clearance intervals. A time-based surrogate measure of PET (Post-Encroachment Time), originally proposed by Allen in 1972, was applied to evaluate such type of traffic conflicts by Tang and Nakamura (2009). PET is uniquely defined for a conflict point and refers to the time between the departure of the encroaching vehicle from the conflict point and the arrival of the vehicle with the right-of-way at the conflict point.

In this study, the minimum PET accepted by the first entering drivers that will not cause evasive actions or unnecessary waiting time is regarded as the critical PET. Figure 1 shows hypothetical clearing and entering vehicle’s trajectories at the change of phases and the critical PET at the conflict point. Where, it is assumed that the last clearing vehicle crosses the stop-line exactly at the beginning of all-red clearance interval at a constant speed, and the first entering vehicle makes a usual start at a stable acceleration rate. Insufficient all-red was considered to be the value based on the German method presented in Table 1, i.e., $AR = t_c - t_e$.

Meanwhile, a concept of Exit Time is also defined to quantify how late the clearing vehicle exits from the conflict point in order to facilitate the following analysis. It represents the elapsed time from when the all-red time ends up to when the clearing vehicle completely leaves the conflict point. A positive value translates that the clearing vehicle fails to clear from the conflict point before the start of green for traffic movements on the crossing road, i.e., a late exit (Schattler et al, 2003). For a late exit, drivers may have either entered the intersection at the end of the yellow change interval or during the red signal.
According to the figure, the optimum all-red time can be derived by Eq. (1). It is basically the sum of the critical PET and the calculated all-red time based on the German method.

\[
AR_{\text{optimum}} = t_e - t_e' + PET_{\text{critical}}
\]  

(1)

Where, \(AR_{\text{optimum}}\) = the optimum all-red time, s; \(t_e\) = clearance time, s; \(t_e'\) = entering time without taking any evasive action, s; \(PET_{\text{critical}}\) = the critical PET, s.

Note that \(t_e\) happens to be identical to Exit Time, i.e., \(t_2-t_1\), because the last clearing vehicle enters the intersection at the beginning of all-red time. Exit Time will become smaller than \(t_e\) if the last clearing vehicle enters earlier. It is also important to know that \(PET_{\text{critical}}\) can be approximately measured in the real world based on the below equation when a late exit occurs.

\[
PET_{\text{critical}} = t_e' - t_e = t_3 - t_2
\]  

(2)

Where, \(t_e'\) = entering time with taking evasive action; \(t_e, t_2,\) and \(t_3\) are defined previously.

4. STUDY SITE AND DATA COLLECTION

Hibiya intersection exhibited in Figure 2, located in Tokyo metropolitan area, was chosen as the study site to empirically analyze the variability of accepted PETs in the case of late exit. A dual-lagging protected-only right-turn phasing plan was operated at Hibiya during the survey period, which is 7am~10am covering a morning peak on a normal weekday in 2007. The conflict between the clearing right-turn traffic at Approach 1 and the entering through traffic at Approach 4 was selected as the subject conflict. Both of the approaches have a speed limit of 60km/h. For the subject right-turn movement, the yellow change interval was 2.0s and the
all-red clearance interval was 3.0s. Totally, there are three conflict points correspondent to three entering lanes respectively, as indicated in the figure.

Figure 2 Study intersection and subject conflicting movements

Signal control parameters and driver behavior data were separately collected by utilizing two video cameras. For the sake of accuracy, one high resolution camera was placed at a high building near the intersection to observe vehicle trajectories. The other camera was put on roadside to capture the signal heads for the subject approaches, Approach 1 and 4. Video files taken by the two different cameras were first synchronized and then reduced by carefully reviewing the video with a time step of 1/30s in the Windows Media Player. Collected essential information related to driver behavior includes $t_0$, $t_2$, $t_3$, $t_c$, $t_e$, $t_e'$, and Exit Time at each conflict point. The accepted PETs at each conflict point in the case of late exit were approximately calculated based on Eq. (2).

5. ACCEPTED POST-ENCOUCHMENT TIME WITH LATE EXIT

5.1 Variability
During the entire observation period of 54 cycles, totally 66 late exit events occurred at the conflict points for the subject conflicting movements. Figure 3 presents histogram and probability density of the accepted PETs with late exit. Statistical analysis results showed that variability of the accepted PET can be well described by a 2-parameter Weibull distribution with a scale parameter of 3.146 and a shape parameter of 4.265, whose basic probability density function is given in Eq. (3). The Weibull distribution is one of the most commonly used distributions in reliability engineering because of the many shapes it attains for various values of $\beta$ (slope). It can therefore model a great variety of data and life characteristics (Dimitri, 1991). For example, it has been adopted in modeling breakdown probability at bottlenecks on expressways (Brilon et al, 2007).
\[
f(x, \alpha, \beta) = \begin{cases} 
\frac{\alpha}{\beta} \left( \frac{x}{\beta} \right)^{\alpha-1} e^{-x/\beta} & x \geq 0 \\
0 & x < 0 
\end{cases}
\]  \tag{3}

Where, \( \alpha \) = scale parameter, \( \beta \) = shape parameter.

A Chi-Square test, also known as Pearson's Chi-Square test, was performed to measure the goodness-of-fit. This Chi-Squared test is used to determine if a sample comes from a population with a specific distribution. It is usually applied to binned data, so the value of the test statistic depends on how the data is binned, as explained in the following equation. The calculated statistic, \( \chi^2 \), was 4.262 based on a sample size of 66. It implies that the test was passed at the significance level of 0.05.

\[
\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} \tag{4}
\]

Where, \( O_i \) = the observed frequency for bin \( i \); \( E_i \) = the expected frequency for bin \( i \) in Eq. (5).

\[
E_i = F(x_2) - F(x_1) \tag{5}
\]

Where, \( F \) = cumulative distribution function; \( x_1, x_2 \) = limits for bin \( i \).

Furthermore, Figure 4 presents survival function of the accepted PETs with late exit, based on the specified Weibull distribution. The survival function shown in Eq. (6) is displayed similarly to the cumulative density function, and is often used in reliability and related fields to denote the probability a unit survives beyond time \( x \).

\[
S(x) = P(X > x) = 1 - F(x) \tag{6}
\]
Where, \( S=\) survival probability function; \( F=\) cumulative distribution function.

As far as the accepted PETs here are concerned, survival probability of a PET represents the likelihood of the PET to be the minimum acceptable value. As exhibited in the figure, the estimated survival possibilities of 2.40s, 3.85s, and 5.30s are 85%, 50%, and 15% respectively, and the observed minimum and maximum accepted PETs are 1.13s and 7.13s. Hence, considering the practical range of accepted PETs (1.13, 7.13), 2.40s would be approximate for the critical PET of the subject traffic conflict if the “85%” principle in traffic engineering is applied. However, it will be 3.85s if the method of determining critical gap introduced at the beginning of the paper is applied.

5.2 Influencing Factors
As the accepted PETs reflect risk perception of drivers, a few factors are supposed to influence its variability. First of all, the first entering drivers normally wait with a full stop or proceed at a low speed before the last clearing vehicle departures from the conflict point in the case of late exit. Hence, entering vehicles may need more time to reach the conflict point and the PET is thus possible to increase if the entering distance is relatively long. Secondly, entering drivers may become impatient and more likely to accept higher risk, i.e., a smaller PET, if Exit Time of the last clearing vehicle is too long. Thirdly, the faster the clearing vehicle crosses the intersection, the greater the perceived conflict severity by the entering drivers is. Thus, a larger clearing speed may lead to a bigger PET.

5.2.1 Entering distance
In order to understand the impacts of entering distance, Table 2 summarizes statistical characteristics of the accepted PETs by conflict point. An apparently bigger mean PET, 4.46s, was observed at the furthest conflict point, conflict point 1. However, there is no significant difference between the PETs at conflict point 2 and 3, 3.46s and 3.78s respectively. In addition, the minimum, the median, and the 15\textsuperscript{th} and 85\textsuperscript{th} percentile values all appear to be remarkably higher at conflict point 1. It reveals that the accepted PET tends to be greater as the entering distance rises.

![Figure 4 Survival function of the accepted PETs with late exit](image-url)
Table 2 Statistic features of the accepted PETs by conflict point

<table>
<thead>
<tr>
<th></th>
<th>Conflict point 1</th>
<th>Conflict point 2</th>
<th>Conflict point 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>4.46 s</td>
<td>3.46 s</td>
<td>3.78 s</td>
</tr>
<tr>
<td>Std.ev.</td>
<td>0.87 s</td>
<td>1.11 s</td>
<td>1.62 s</td>
</tr>
<tr>
<td>Min.</td>
<td>3.03 s</td>
<td>1.39 s</td>
<td>1.13 s</td>
</tr>
<tr>
<td>15th percentile</td>
<td>3.74 s</td>
<td>2.35 s</td>
<td>2.03 s</td>
</tr>
<tr>
<td>Median</td>
<td>4.49 s</td>
<td>3.42 s</td>
<td>3.78 s</td>
</tr>
<tr>
<td>85th percentile</td>
<td>5.04 s</td>
<td>4.46 s</td>
<td>5.22 s</td>
</tr>
<tr>
<td>Max.</td>
<td>6.66 s</td>
<td>5.70 s</td>
<td>7.13 s</td>
</tr>
<tr>
<td>Sample size</td>
<td>18 veh</td>
<td>22 veh</td>
<td>26 veh</td>
</tr>
</tbody>
</table>

5.2.2 Exit Time

Figure 4 shows the relationship between the accepted PETs and Exit Time, where the accepted PETs are evenly distributed over Exit Time, ranging between 0.00s and 3.80s. Despite a decreasing trend can be easily identified from the figure, the relationship is rather weak and not statistically significant ($R^2=0.15$). A possible explanation is that the range of observed Exit Time is too narrow to distinguish driver’s difference in tolerance to long Exit Time.

$$y = -0.5842x + 4.7526$$

5.2.3 Clearing speed

Figure 5 shows the relationship between the accepted PETs and clearing speed. Although the range of observed clearing speeds is quite wide, no significant correlation with the accepted PETs was found. The result might be understandable if one agrees that entering drivers may not be able to properly perceive the difference in clearing speed, due to their waiting positions as well as physical constraints of human being.
6. IMPLICATION IN ALL-RED CLEARANCE INTERVAL DESIGN

The critical PET has important implication in the design of all-red time. Taking Hibiya intersection as a test bed, Figure 6 compares the optimum all-red time based on the critical PET and the calculated all-red times based on the existing methods highlighted in Table 1. Since conflict point 3 is the critical point when designing the all-red clearance interval, entering speed, distance and time as well as clearance speed, distance and time were measured from conflict point 3. For the calculations of the existing methods, only those data without late exit was used in order to exclude the impacts of late exit clearing vehicles on entering speed and entering time, etc. For the calculation of the proposed method, i.e., Eq. (1), the observed 15th percentile value of accepted PETs at conflict point 3, 2.03s, was adopted, which is approximately equal to an ordinary saturated headway of discharge flow at signalized intersections.

Both the actual calculated values and the adjusted values complying with each guideline’s principles are provided in the figure. More specifically, the recommended value is set to 0.0s for the German method, because the actual calculated value is -0.10s. The actual calculated value is rounded to the next 0.5s for the recommended value in the Australian method. Regarding the method in the United States, since the actual calculated value, 6.9s, exceeds the maximum value regulated the Institute of Traffic Engineers and it is thus reduced to 6.0s. With respect to the method in Japan, the calculated value, 4.9s, is converted to 4.0s for the recommended value, and in return the yellow time is usually extended by 1.0s in practice. Note that the real observed all-red time at the subject approach was 3.0s as introduced in Chapter 3, which is different from the recommended value by the Japanese method, 4.0s. The reason could be that the measured parameters such as clearing speed in this study are somehow varied from the used parameters by the police when setting the all-red time at the subject intersection. Therefore, for the purpose of comparability, the recommended value of 4.0s is referred in the following discussions.
Figure 6 Comparison of the optimum all-red time and the calculated all-red times based on the current methods in the major developed countries

It was found that the proposed all-red time, 2.0s, is longer than the value based on the methods considering entering time (Type 2), 0.0s. This extension of all-red is useful to provide the last clearing vehicles with more time to leave the conflict point, so that the first entering drivers don’t need to take brake and slow down their speeds after the onset of green. It can also be considered as the extra start-up lost time that may occur given a zero all-red time. Therefore, it improves safety by eliminating evasive actions of entering drivers, while maintaining the operational efficiency.

It was also found that the proposed all-red time is considerably shorter than the values based on Type 2 and Type 3 methods. The differences fall between 1.0s and 4.0s, which might cause a variation of the optimum cycle lengths from 20s to 80s according to the Webster formula (Webster, 1966) presented below, if simply assuming that the unnecessary part of all-red will be the extra clearance lost time and the total critical demand ratio is 0.85.

\[
C = \frac{5 + 1.5L}{1 - \lambda}
\]

(7)

Where, \(C\)=optimal cycle length (s); \(L\)=lost time of one cycle (s); \(\lambda\)=total critical demand ratio.

The rise of cycle length may further lead to a rapid increase of control delay. As illustrated in Figure 7, the total control delay within an identical time period will simply become 1/2 if the cycle length is reduced by half.
7. CONCLUSIONS AND FUTURE WORKS

This paper proposes a concept of critical post-encroachment time (PET) for all-red clearance interval design at signalized intersections that aims at achieving the optimum performance in both safety and mobility. The critical PET is defined as the minimum accepted PET at the conflict point by the first entering drivers during the change of phases. Following a theoretical analysis of the critical PET, variability of the accepted PETs with late exit was investigated in order to estimate the critical PET, based on empirical results at a study intersection. Conclusions supported that the critical PET can be well described by a two-parameter Weibull distribution, and tends to increase as the entering distance rises. However, its relationships with exit time and clearing speed were not found to be significant. Finally, its implication in the design of all-red time was discussed through numerical calculations. Results indicated that the calculated all-red time based on the critical PET is longer than the all-red time based on the German method, and however considerably shorter than the values based on the methods applied in Japan and other developed countries.

To facilitate a wide application of the critical PET in the design of all-red time, a general method of deriving the critical PET must be developed. To achieve that goal, the exact evasive actions taken by the entering drivers in the case of late exit needs to be explicitly interpreted. However, entering driver’s behavior under such a situation is rather complex and highly depends upon the local conditions, and is hard to be explained by a single formula.

In addition, in most of Asian developing countries, multiple transportation modes including vehicle, motor cycle, bike, and pedestrian exist at signalized intersections, and many of the road users often ignore lane discipline and traffic signals. Under such sort of traffic situations, traffic conflicts among different movements at the change of phases are more complicated and difficult to handle for traffic engineers. To seek the optimum all-red clearance intervals, the critical PETs must be separately estimated in terms of conflict types and transportation modes, because they have distinct maneuvering trajectories and behavioral characteristics such as clearing speed and acceleration rate. Furthermore, to achieve the optimum operational and safety performance, traffic conflicts analysis also needs to be done to determine the optimal signal phasing scheme that produces the minimum lost time and/or the shortest cycle length. In return, rational design of all-red clearance intervals and cycle length helps to eliminate traffic violations and thus improve the whole performance of signalized intersections.
As future works, the authors are now collecting data widespreadly in order for modeling the critical PETs under a variety of traffic situations. Afterwards, the authors plan to conduct experiments by the use of Driving Simulator at the institute of Industrial Science, the University of Tokyo (Yamaguchi, 2008), to look into the mechanism of interaction between the entering and clearing drivers.

ACKNOWLEDGEMENT

The authors would like to acknowledge the Grant-in-Aid for Japan Society for the Promotion of Science (JSPS) Fellows (No. P08396) for financial support of this research. The authors would also like to convey their sincere appreciations to Dr. Kazufumi Suzuki at National Institute for Land and Infrastructure Management of Ministry of Land, Infrastructure, Transport and Tourism, Japan, and Dr. Axel Wolfermann at Nagoya University for their helpful comments to shape this research.

REFERENCE


