Acquisition of New Aircraft with Probabilistic Dynamic Programming

Lay Eng TEOH
Lecturer
Department of Mathematical and Actuarial Sciences
Faculty of Engineering and Science
Universiti Tunku Abdul Rahman
Setapak Campus,
Off Jalan Genting Kelang,
Setapak, 53300 Kuala Lumpur, Malaysia
Fax: +6-03-41079803
E-mail: teohle@utar.edu.my

Hooi Ling KHOO
Assistant Professor
Department of Civil Engineering
Faculty of Engineering and Science
Universiti Tunku Abdul Rahman
Setapak Campus,
Off Jalan Genting Kelang,
Setapak, 53300 Kuala Lumpur, Malaysia
Fax: +6-03-41079803
E-mail: khoohl@utar.edu.my

Abstract: This study deals with an aircraft acquisition decision model to tackle the stochastic demand of the air transportation system. Probabilistic dynamic programming is applied to develop an optimization model with the aim to maximize the operational profit of the airline company. Correspondingly, a probable phenomenon is defined to comprehend the uncertain state variables so that the targeted level of service by the airline company could be met satisfactorily. The proposed probabilistic dynamic programming model is then converted into a non-linear programming model as the objective function and the constraints have non-linear expression with respect to the decision variables. Later, the proposed model and the solution method are examined with an illustrative case study to determine the number and the types of new aircraft that should be purchased at every time period. The results show that the proposed methodology is viable in providing the optimal solution.

Key Words: Stochastic demand, Aircraft acquisition, Probabilistic dynamic programming

1. INTRODUCTION

Level of passenger demand which varies now and then due to unpredicted events such as the outbreaks of flu diseases could affect the operation of airline companies. In order to maximize the profit and to sustain a certain level of service, the airline companies need to look into the uncertainty of travel demand when carrying out their operational planning. This includes one foremost operational decision upon the acquisition of new aircraft. In such a case, the airline companies have to decide on the fleet size of their aircraft based on the anticipated level of demand. In other words, how to determine the number and the types of the aircraft to be acquired in meeting the uncertain level of demand is utmost important.

Earlier literatures revealed that in order to obtain the optimal solution for the aircraft acquisition model, it is important to consider the quantity of travellers’ demand (New, 1975; Wei and Hansen, 2005). In particular, Listes et al. (2005) mentioned that by considering stochasticity, the solution obtained is more robust and closer to practical implementation. More recently, by using probabilistic dynamic programming, Khoo and Teoh (2011) developed a mathematical model to solve a linear programming model for an aircraft acquisition decision model. They developed constraints that could consider lead and lack time of the model subject to several operational constraints, including the lead time and selling time constraints. This model is capable of deciding the aircraft to be purchased and sold. Besides, they formed the order delivery constraint as the sum of $n$ types of aircraft. Different from their study, this paper developed a non-linear programming model to optimize the
acquisition process. In addition, this paper considered the order delivery constraint for individual aircraft.

To account for the inconsistency of the stochastic demand in airline operational planning, Pitfield et al. (2009) adopted simultaneous equations approach to evaluate the real airline data. They found that the level of demand elasticity could affect the aircraft size as well as the service frequency. Apart from this, uncertainty of future demand was inspected by List et al. (2003) by using a partial moment measure of risk to solve the robust optimization problem. Then, Listes and Dekker (2005) used scenario aggregation-based approach to inspect the best fleet composition, i.e. choice of aircraft for the most profitable operation subject to the airline’s planned schedule. They looked into the robustness of an airline fleet configuration that accounts explicitly for short-term stochastic demand fluctuations. In the study, they found that the stochastic approach is pertinent and viable in capturing the robustness of a larger set of realistic data. However, the limitation of the study is that only short term planning horizon is considered. Besides, there are some studies in fleet assignment and scheduling problem that consider stochastic demand (Feldman, 2002; Yan et al., 2008).

Apparently there are limited studies for aircraft acquisition decision model that consider stochastic demand. Undeniably, this is essential as, in reality, the airline company has to consider uncertain demand in planning aircraft acquisition as a long term planning problem. As such, the approach and models developed for short term planning might not be functional and effective in such circumstances. Motivated by this fact, an aircraft acquisition decision model by considering stochastic demand is proposed in this study. By using the probabilistic dynamic programming, an optimization model is developed to maximize the earned profit of the airline companies. This mathematical approach is selected as it is capable of decomposing the proposed model into a chain of simpler single-period sub-problems during the planning horizon. More importantly, this approach considers states (i.e. decision variables) and the corresponding profits which are probabilistic (not deterministic) at each stage in order to comprehend the level of demand that is uncertain. To capture the demand uncertainty, it is assumed that the travel demand could be described by some probabilistic distributions while the decision variables of the acquisition model are the number and types of aircraft that need to be purchased in order to maximize the operational profit. Besides, a probable phenomena is defined in response to the targeted confidence level as the state variables are probabilistic due to uncertain demand. This is indispensible to capture the uncertainty of state variables properly. Only with the probable phenomena, the targeted level of service by the airline company could be met satisfactorily for a profitable return. It is then shown that the probabilistic dynamic programming model could be converted into a non-linear programming model if the objective function and the constraints have non-linear relationship with respect to the decision variables. An illustrative case study is illustrated to test the proposed model and methodology. For simplicity, only 2 types of aircraft are considered. Gu et al. (1994) mentioned the problem might become a NP-hard problem which can only be solved with meta-heuristic methods if more than two aircraft types are considered.

2. METHODOLOGIES

In order to comprehend demand uncertainty, this section demonstrates the application of probabilistic dynamic programming in formulating an optimization model for the decision making to acquire new aircraft.
2.1 Nomenclature

For the operating period \( t \), the notations used in this study (for \( n \) types of aircraft at age \( y \)) are listed as follows:

**Parameters**

- \( T \) : Horizon length for the planning period
- \( \text{MAX}_{\text{budget}(t)} \) : Budget constraint allocated for the acquisition of new aircraft
- \( D_i^y \) : Stochastic demand corresponds to phenomenon \( S \)
- \( \text{ORDER}_t \) : Total number of aircraft that could be purchased in the market
- \( \text{PARK}_t \) : Area of hangar (as geometry limitation)
- \( r_i \) : Discount rate for which the discount factor is \( (1 + r_i)^{-t} \)
- \( \alpha \) : Significance level of demand constraint
- \( E(f_{\text{fare}}) \) : Expected value of flight fare per passenger (at phenomenon \( S \))
- \( E(f_{\text{cost}}) \) : Expected value of flight cost per passenger (at phenomenon \( S \))
- \( p_s \) : Probability to own \( I_i \), as the initial number of aircraft at phenomenon \( S \)
- \( A_i^n \) : Total of aircraft owned

**Functions**

- \( P(t) \) : Function of discounted profit earned
- \( f(D_i^y, A_i^n) \) : Function of number of flights in terms of \( D_i^y \) and \( A_i^n \)
- \( hgf(D_i^y, A_i^n) \) : Maintenance cost function in terms of the function of total mileage travelled, \( g \), and the function of number of flights, \( f \)

**Sets**

- \( X_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \) : Number of \( n \) types of aircraft to be purchased
- \( I_i = (I_{i1}, I_{i2}, \ldots, I_{in}) \) : Initial number of \( n \) types of aircraft (at age \( y \))
- \( U_i = (u_{i1}, u_{i2}, \ldots, u_{in}) \) : Setup cost for the acquisition of \( n \) types of aircraft
- \( S = (s_1, s_2, \ldots, s_k) \) : Phenomenon of owning \( I_i \)
- \( \text{PURC}_i = (\text{purc}_{i1}, \text{purc}_{i2}, \ldots, \text{purc}_{in}) \) : Purchase cost for \( n \) types of aircraft
- \( \text{SEAT}_n = (\text{seat}_1, \text{seat}_2, \ldots, \text{seat}_n) \) : Number of seats for \( n \) types of aircraft owned
- \( \text{SOLD}_i = (\text{sold}_{i1}, \text{sold}_{i2}, \ldots, \text{sold}_{in}) \) : Number of \( n \) types of aircraft sold (at age \( y \))
- \( \text{RESALE}_i = (\text{resale}_{i1}, \text{resale}_{i2}, \ldots, \text{resale}_{in}) \) : Resale price for \( n \) types of aircraft (at age \( y \))
- \( \text{DEP}_i = (\text{dep}_{i1}, \text{dep}_{i2}, \ldots, \text{dep}_{in}) \) : Depreciation values for \( n \) types of aircraft
- \( \text{SIZE} = (\text{size}_1, \text{size}_2, \ldots, \text{size}_n) \) : Size of \( n \) types of aircraft

2.2 Problem Formulation

The objective of this study is to determine the number and types of aircraft that should be acquired with the aim to maximize the profit of the airline companies by assuming that there is a choice of \( n \) types of aircraft that could be acquired and operated for a set of origin-
destination (OD) pairs for the operating route network. The passenger demand for the mentioned OD pairs is assumed to be stochastic and could be expressed by some random distributions. To deal with this stochastic element, the problem is formulated as probabilistic dynamic programming problem by considering numerous practical constraints encountered in the operation planning.

2.3 Probabilistic Dynamic Programming Model

2.3.1 Stage, State Variables and Optimal Decision

The stage of the model is the planning horizon of the aircraft acquisition period for which the planning period, \( t \), in terms of years is the stage variable of the model. The state variable at each stage \( t \) consisted of various interrelated variables, namely the number of aircraft to be purchased as the main decision variable for this study, initial number of aircraft owned, number of aircraft to be sold and stochastic demand. The optimal decision (i.e. alternatives at each stage) is the acquisition decision of new aircraft to meet stochastic demand while making decision to sell ageing aircraft in order to maximize the expected profit.

2.3.2 Constraints

Some constraints that need to be considered for the efficiency of the operational planning of airline companies are explained as follows:

(i) Budget constraint
This is the most practical constraint in order to ascertain that the solution obtained is financially feasible for the airline companies. Accordingly, the total purchase cost of the aircraft which should not be more than the allocated budget can be expressed as follows:

\[
\sum_{i=1}^{n} purc_i x_i \leq \text{MAX}_{budget(t)} \text{ for } t = 1, 2, ..., T
\]  

(ii) Demand constraint
The stochastic demand can be represented by some probability distributions. Let \( \alpha \) indicates the significance level to meet stochastic demand, the following expression can be formulated to achieve the targeted level of service (for \( k \)-th possible phenomenon).

\[
P \left( \sum_{i=1}^{n} (\text{SEAT}_i) \left( f \left( D^s, A' \right) \right) \right) \geq D^s \geq 1 - \alpha \text{ for } t = 1, 2, ..., T, S = s_1, s_2, ..., s_k
\]  

where \( P \) is the probability of the occurrence of the desired level of service while \( 1 - \alpha \) is the confidence level (i.e. targeted level). If the demand is assumed to follow the normal distribution with mean \( \mu \) and standard deviation \( \sigma \), the demand constraint could be expressed by,

\[
\sum_{i=1}^{n} (\text{SEAT}_i) \left( f \left( D^s, A' \right) \right) \geq F^{-1}(1 - \alpha) \sigma + \mu \text{ for } t = 1, 2, ..., T, S = s_1, s_2, ..., s_k
\]  

where \( F^{-1}(1 - \alpha) \) is the inverse cumulative probability of \( 1 - \alpha \).

(iii) Parking constraint
The choice of the aircraft would sometimes be constrained by the geometry design of the airports. For instance, the jumbo plane A380 could only be used at certain airports due to its large body size and heavy weight. Besides, when the aircraft is “off-duty”, it has to be
grounded at the hanger of the airport. As such, it is a feasible constraint to be considered. The constraint is expressed as follows:

\[
\sum_{i=1}^{n} \sum_{y=0}^{m} (\ln{u_{iy}} + x_{iy}) \times SIZE_{i} \leq PARK_{t} \text{ for } t = 1, 2, \ldots, T
\]  

(iv) Sales of aircraft constraint

For some airlines, ageing aircraft which is less cost-effective might be sold at the beginning of a certain operating period, \( t \) when the airlines make the decision to acquire new aircraft. However, to sustain a certain level of operational efficiency, the number of aircraft sold should not be more than the aircraft owned by the aircraft’ companies. It is expressed as follows:

\[
sold_{iy} \leq In_{(t-1)i(y-1)} \text{ for } t = 1, 2, \ldots, T, i = 1, 2, \ldots, n, y = 1, 2, \ldots, m
\]

(v) Order delivery constraint

Sometimes, there might be a delay in fleet supply due to the delivery of the new aircraft ordered, which is dependent on the operations and the efficiency of the manufacturing company. As such, the aircraft that one could purchase should not be more than the number of aircraft available in the market, which is expressed as follows:

\[
x_{iy} \leq ORDER_{i} \text{ for } t = 1, 2, \ldots, T, i = 1, 2, \ldots, n
\]

2.3.3 Objective Function

The objective of this study is to maximize the expected profit of the airline companies which could be derived by the subtraction of the total operating cost from the total revenue obtained. For an airline company, basically the total revenue is generated from the operational income (i.e. the sales of the flight tickets) and the sales of ageing aircraft while the total operating cost considers the total purchasing cost of new aircraft, the total operational cost of aircraft owned, the total maintenance cost of aircraft owned and the total depreciation expenses of aircraft owned.

The total revenue (corresponds to \( k \)-th possible phenomenon) for the operating period \( t \), \( TR(I_{t}) \), is expressed as follows:

\[
TR(I_{t}) = E\left(fare_{i}^{s}\right)D_{i}^{s} + \sum_{i=1}^{n} \sum_{y=1}^{m} sold_{iy} \times resale_{iy} \text{ for } t = 1, 2, \ldots, T, S = s_{1}, s_{2}, \ldots, s_{k}
\]

where the first term of the right hand side of eqn. (7) indicates the expected income obtained from the sale of the flight tickets by considering the stochastic demand \( D_{i}^{s} \) for which \( D_{i}^{s} \geq F^{-1}(1-\alpha)\sigma + \mu \). The second term indicates the revenue obtained from the sale of ageing aircraft.

The total operating cost for the operating period \( t \), \( TC(I_{t}) \) is expressed by,

\[
TC(I_{t}) = \sum_{i=1}^{n} u_{i} + \left( purc_{i} \right) (x_{i}) + E\left(\cosl_{i}^{s}\right)D_{i}^{s} + \sum_{i=1}^{n} hgf\left(D_{i}^{s}, A_{i}\right) + \sum_{i=1}^{n} \sum_{j=1}^{m} \left(ln_{ij}\right) \left(dep_{ij}\right) \text{ for } t = 1, 2, \ldots, T, S = s_{1}, s_{2}, \ldots, s_{k}
\]

where the first, second, third, fourth and the last term of the right hand side of eqn. (8) indicates the setup cost for the acquisition of \( n \) types of aircraft, the purchasing cost of the new aircraft, the expected operating cost, the maintenance cost and the total depreciation expenses respectively, corresponding to \( k \)-th possible phenomenon.
2.3.4 The Probable Phenomena, \( s_1, \ldots, s_k \)

Since the demand is stochastic, its actual value remains unknown until the day of operation. As a consequence, the state variables are probabilistic and this implies that the probable phenomenon for which the likely state variables to be occurred should be defined accordingly in order to capture the uncertainty of state variables properly. To account for the possible phenomenon appropriately, the airline company ought to consider all possible levels of service (i.e. actual level of demand) in order to plan their profitable operations strategically. Only by recognizing the probable phenomenon, the targeted level of service by airline companies could be met satisfactorily. This signifies that the possible phenomenon to be happened correlates intimately with the level of service achieved by airline companies. In general, let \( s_1, \ldots, s_k \) be \( k \) possible phenomenon to meet the level of service (i.e. actual level of demand) at a targeted confidence level. In spite of this, the phenomena, \( s_1, \ldots, s_k \) differ in terms of the attainment of the targeted level of service. In particular, \( s_1, \ldots, s_k \) is the concerned phenomena for which the actual level of demand is achieved differently subject to the targeted confidence level.

In fact, component of \( s_1, \ldots, s_k \) in the developed model is extremely significant as it turns out to be an essential indicator to imply the possession of aircraft in order to capture the actual occurrence in reality. Only with this indicator, the actual operation under uncertainties will then be monitored closely with the developed optimization model. Correspondingly, the possibility for the phenomena \( s_1, \ldots, s_k \) to be happened is necessary to be taken into account and hence \( p_{s_1}, \ldots, p_{s_k} \) (i.e. the possibility for \( s_1, \ldots, s_k \) to be occurred) is included necessarily in the developed optimization model. For the real practice, the phenomenon \( s_1, \ldots, s_k \) and the corresponding probability \( p_{s_1}, \ldots, p_{s_k} \) ought to be treated strategically by considering the company’s decision policy, qualitative judgement from experts or consultants, the past operational performance as well as the feedbacks from air transportation users (i.e. travellers).

2.3.5 The Optimization Model

With the aim to maximize the expected profit earned by acquiring new aircraft to meet the travellers demand under uncertainty, the formulation of the optimization model can be phrased as follows:

For \( t = 1, 2, \ldots, T \)

\[
P(I_t) = \max_{x_t} \frac{1}{1 + r} \left\{ \begin{array}{ll}
P_{s_k} & \left[ E(\text{fare}_t^{s_k})D_t^{s_k} + \sum_{i=1}^{n} \sum_{y=1}^{m} \text{sold}_{i,y} \text{resale}_{i,y} - \sum_{i=1}^{n} u_i + \left( \text{purc}_i \right)(x_i) \right] \\
- E(\cos t_t^{s_k})D_t^{s_k} - \sum_{i=1}^{n} \text{hgf}(D_t^{s_k}, A_t^{s_k}) - \sum_{i=1}^{n} \sum_{y=1}^{m} (\text{In}_{i,y}) (\text{dep}_{i,y}) \\
E(\text{fare}_t^{s_1})D_t^{s_1} + \sum_{i=1}^{n} \sum_{y=1}^{m} \text{sold}_{i,y} \text{resale}_{i,y} - \sum_{i=1}^{n} u_i + \left( \text{purc}_i \right)(x_i) \right) \right. \\
- E(\cos t_t^{s_1})D_t^{s_1} - \sum_{i=1}^{n} \text{hgf}(D_t^{s_1}, A_t^{s_1}) - \sum_{i=1}^{n} \sum_{y=1}^{m} (\text{In}_{i,y}) (\text{dep}_{i,y}) \\
+ P_{s_1}(I_t) \end{array} \right. 
\]

subject to (1) and (3)-(6) for which \( D_t^{s}, X_t, I_t, \text{SOLD}_{t} \in Z^{+} \cup \{0\} \). Note that,
$D_i^S \geq F^{-1}(1-\alpha)\sigma + \mu$. The term, \(\frac{1}{(1+r)^t}\) is needed in order to obtain the discounted value across the period of time while \(k\) indicates the \(k\)-th possible phenomenon for owning \(I_i\) as the initial number of aircraft. Only two phenomenon, namely \(s_1\) and \(s_2\) are considered in this study in order to reduce the complexity.

It is important to note that the model formulation is formed by assuming that the developed model drives operational decision of the airline companies, particularly from the perspective of flight’s frequency and its scheduling to meet stochastic demand. In other words, the acquisition decision of new aircraft will subsequently lead to the optimal operational decision of a fleet routing at a desired level of service.

3. **SOLUTION METHOD**

The proposed probabilistic dynamic programming can be solved by decomposing it into a series of simpler sub-problems. With the working backward (i.e. from the last period to the first period), the solution method commences by solving the sub-problem at the period of \(T\) for which \(T\) is the last period of the planning horizon. The current optimal solutions found for the decision variables (i.e. the states of dynamic programming) at current stage leads to the problem solving at the period of \(T - 1, T - 2, ..., 1\). This procedure continues until all the sub-problems have been solved optimally so that the decision policy to acquire new aircraft can be determined eventually. For the developed optimization model (9), the type of solution method (i.e. linear programming problem or non-linear programming problem) can be identified clearly particularly based on the key components as follows:

- function of the number of flights, \(f(D_i^S, A_i^e)\);
- function of the maintenance cost, \(hgf(D_i^S, A_i^e)\);
- constraints (1), and (3)-(6).

If the components shown in above are non-linear in nature, the proposed model (9) could be converted to as a non-linear programming model. In reality, the linearity of these components is based on the data collected for a particular airline company. It shall then be validated by using the regression test with the aid of some mathematical software. In the illustrative case study as shown in the following section, non-linear relationship is adopted for the above mentioned components. Nonetheless, due to the nature of probabilistic dynamic programming, the non-linear programming model obtained from the conversion could not be solved directly using any conventional methods available for solving non-linear programming model and hence an algorithm is developed as Excel 2007 spreadsheet to determine the optimal solution.

For a more complicated optimal system, i.e. when the problem size is getting bigger, the proposed solution method is still feasible in generating results. Unavoidably, the computational efficiency reduces when the problem size gets larger due to the additional modelling parameters and variables. As such, more computational effort is necessary for larger state and stage spaces. In particular, the extension of planning horizon, \(T\) would results in an increment ratio of \(\frac{1}{T}\) i.e. approximately an increment of 10-20% possible solutions that to be identified as the optimal solution. As such, this requires an additional 10-20% of
computational efforts. Numerically, there is \((ORDER_n + 1)\) times more possible solutions for each additional type of aircraft, \(n\) for which \(ORDER_n\) refers to the order delivery constraint. As a result, additional computational effort increases at this proportion. This implies that more computational time is needed (for more computational efforts) to obtain the optimal solution when the problem size as well as the number of possible solution increases proportionally with the increment of \(n\) and \(t\) for a larger scale of the problem.

4. AN ILLUSTRATIVE CASE STUDY

Table 1 Expected value of flight fare and flight cost per passenger for the operating period of \(t\)

<table>
<thead>
<tr>
<th>Period, (t)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E(fare_t^n)), $</td>
<td>238</td>
<td>286</td>
<td>295</td>
<td>310</td>
<td>402</td>
</tr>
<tr>
<td>(E(fare_t^{z_2})), $</td>
<td>206</td>
<td>210</td>
<td>226</td>
<td>317</td>
<td>340</td>
</tr>
<tr>
<td>(E(cost_t^n)), $</td>
<td>219</td>
<td>222</td>
<td>235</td>
<td>249</td>
<td>270</td>
</tr>
<tr>
<td>(E(cost_t^{z_2})), $</td>
<td>125</td>
<td>140</td>
<td>150</td>
<td>181</td>
<td>195</td>
</tr>
</tbody>
</table>

Table 2 Resale price, depreciation values and purchase prices of aircraft

<table>
<thead>
<tr>
<th>(y)</th>
<th>resale$_{51y}$ ($ millions)</th>
<th>resale$_{52y}$ ($ millions)</th>
<th>dep$_{51y}$ ($ millions)</th>
<th>dep$_{52y}$ ($ millions)</th>
<th>purc$_{51}$ ($ millions)</th>
<th>purc$_{52}$ ($ millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>76</td>
<td>202</td>
<td>5</td>
<td>4</td>
<td>82</td>
<td>213</td>
</tr>
<tr>
<td>2</td>
<td>71</td>
<td>198</td>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>66</td>
<td>194</td>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>62</td>
<td>189</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>58</td>
<td>184</td>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>4.6</td>
<td>4.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

An illustrative case study is shown on how the proposed model could provide an optimal solution for the decision making upon the acquisition of new aircraft, i.e. to decide when and which type of aircraft should be purchased over the planning horizon, i.e. 5 years. For a set of OD pairs, assume that there are two types of aircraft choice for which \(n=1\) and \(n=2\) for A320-200 and A330-300 respectively. This paper examines the applicability of the developed model with a case study that consist of 2 types of aircraft for 5 operating years due to the fact that many of the low cost carriers operate their business with few varieties of aircraft types (O’Connell and William, 2005). In addition, a planning horizon of 5 years is also justified as according to Malaysia Airlines (2010a) and AirAsia (2010), in average, new aircraft acquired requires a period of 5 years to be delivered completely. This reveals the new aircraft will be received and in operation in 5 years. As such, the two types of aircraft considered for the planning horizon of 5 years in this paper are reasonably practical for the real operation of the airline companies.

To account for unreality value for the parameters and functions, relevant information has been obtained from the published reports and accessible websites. Table 1 and 2 show the data input into the model. From the Airbus published statement (Airbus, 2010b; Airbus, 2010c), it is obtained that the capacity of aircraft A320-200 and A330-300 is 180 and 295 respectively.
while the size of A320-200 and A330-300 is $1282\ m^2$ and $3836\ m^2$ respectively. The expected flight fare and cost as shown in Table 1 is generated based on the available financial reports of Malaysia Airlines (MAS) (MAS, 2010b). In addition, the purchase prices of aircraft as shown in Table 2 were obtained from the published data of Airbus (Airbus, 2010a). However, the resale price and depreciation values are assumed data, which are generated respectively for A320-200 and A330-300 due to unavailable information from the accessible websites and reports.

There are many variables and parameters in the developed model. Since not all real data can be obtained, it is interesting to investigate how the results of the model vary if the values of the variables are changed. Therefore, six scenarios are created besides the benchmark scenarios in order to test the variation of the obtained results.

4.1 Benchmark Scenario

The values chosen for parameters and functions in benchmark scenario are described as follows:

- Two possible phenomenon are considered, where $k = 2$ for the model shown in (9).
- At the first operating period i.e. $t = 1$, the initial number of A320-200 and A330-300 are $In_{11} = 50$ and $In_{12} = 50$ respectively.
- The probability of posses these aircraft at initial period is $0.5$ respectively, i.e. $p_{s_1} = 0.5$ and $p_{s_2} = 0.5$.
- Budget, $MAX_{budget(t)} = $1,500,000,000.
- Hangar area, $PARK_t = 450,000m^2$.
- Order delivery, $ORDER_t = 5$ for A320-200 and A330-300.
- Discount rate is fixed, $r = 5\%$ per annum.
- Confidence level, $1 - \alpha = 95\%$.
- At the first operating period i.e. $t = 1$, the initial number of A320-200 and A330-300 to be 4 years old is 3 respectively i.e. $In_{114} = In_{124} = 3$.
- Setup cost to acquire $n$ types of new aircraft, $u_{i_0} = 0$ for $t = 1, 2, \ldots, T$, $i = 1, 2, \ldots, n$.
- $D_i^t = 0.95D_i^n$ for $t = 1, 2, \ldots, T$ (10)
- For $n$ types of aircraft,
  - Number of flights, $f\left(A_i^n\right) = 78300 - 977.6A_i^n + 22.57\left(A_i^n\right)^2$ for $t = 1, 2, \ldots, T$ (11)
  - For $n$ types of aircraft,
  - Maintenance cost, $hgf\left(A_i^n\right) = 5177 - 23313A_i^n + 535\left(A_i^n\right)^2 + 3.33\left(A_i^n\right)^3$ for $t = 1, 2, \ldots, T$ (12)
  - Number of aircraft, $NA = -73.6 + 0.00001NP$ (13)

where $NP$ is the number of passengers.

Based on the published reports of Malaysia Airlines (2010a) and AirAsia (2010), eqns. (11)-(13) are obtained via polynomial regression analysis. Theoretically or practically, eqns. (11) and (12) are expected to be correlated with stochastic demand, $D_i^t$ and the total of aircraft owned, $A_i^n$. Analytically, the analysis shows that eqns. (11) and (12) are fitted fairly well as non-linear functions in terms of the total of aircraft owned, $A_i^n$. Similarly, the regression
analysis shows that the function of number of aircraft is best fitted as a linear function in terms of the number of passengers. Specifically, eqn. (11) indicates that number of operated flights is associated closely with the total of aircraft owned in the form of quadratic equation while eqn. (12) denotes a cubic equation that relates maintenance cost and the total of aircraft owned. Eqn. (12) displays that $5,177 is the overall estimated maintenance cost without considering additional aircraft. These functions signify that the respective function is strongly affected by the total of aircraft owned by the airline companies. Eqn. (13) implies that each additional 100,000 passengers require one additional aircraft (or one passenger requires 0.00001 aircraft). The constants of 78300 (in eqn. (11)) and -73.6 (in eqn. (13)) have no practical interpretation.

With the backward working, model (9) is simplified to model (14)-(20) when $t = T = 5$.

$$P(I_s) = \max_{x_s} \frac{1}{(1.05)^5} \left\{ \begin{array}{c} 132D_5^s + (5.8\times10^7 sold_{515} + 1.84\times10^8 sold_{525}) - (8.1\times10^7 x_{s1} + 2.06\times10^8 x_{s2}) \right\} +$$

$$\left\{ \begin{array}{c} p_s \left\{ (5177 - 23313 A_5^s + 535 (A_5^s)^2 + 3.33 (A_5^s)^3) - (4.6\times10^6 In_{s1} + 4.4\times10^6 In_{s2}) \right\} \\ 130.5D_5^s + (5.8\times10^7 sold_{515} + 1.84\times10^8 sold_{525}) - (8.1\times10^7 x_{s1} + 2.06\times10^8 x_{s2}) \right\} \right\} \right\}$$

subject to

$$82x_{s1} + 213x_{s2} \leq 1,500 \quad (15)$$

$$22.57(In_{s1} + In_{s2} + x_{s1} + x_{s2})^2 - 977.6(In_{s1} + In_{s2} + x_{s1} + x_{s2}) - 44,821 \geq 0 \quad (16)$$

$$D_5^s \geq 10,645,000, \quad D_5^s \geq 10,645,000 \quad (17)$$

$$(In_{s1} + x_{s1})(1282) + (In_{s2} + x_{s2})(3836) \leq 450,000 \quad (18)$$

$$sold_{515} \leq In_{s1}, \quad sold_{525} \leq In_{s2} \quad (19)$$

$$x_{s1} \leq 5, \quad x_{s2} \leq 5 \quad (20)$$

Eqn. (15) takes the budget constraint of $1,500 millions. The total demand simulated for $t = 5$ is to follow normal distribution i.e. $D_5^s \sim N(9\times10^6, 1\times10^6)$ and at a 95% confidence level, eqn. (16) signifies demand constraint, which could be derived from eqn. (3) and (11) for which $A_5^s = In_{s1} + In_{s2} + x_{s1} + x_{s2}$. Eqn. (17) indicates that the actual level of demand for $t = 5$ is predicted to be at least 10,645,000 at a confidence level of 95%, which is derived by (2)-(3). Eqn. (18) is the parking constraint; eqn. (19) is the sales of aircraft constraint, which is derived with the assumption the aircraft at the age of 5 years old or more are considered to be sold. The sales of aircraft constraint as depicted in eqn. (5) gives $sold_{515} \leq In_{414}$ and $sold_{525} \leq In_{424}$. Since $In_{414} \leq In_{s1}$ and $In_{424} \leq In_{s2}$, these expressions subsequently result in $sold_{515} \leq In_{s1}$ and $sold_{525} \leq In_{s2}$ as could be seen in eqn. (19). Eqn. (20) indicates the order delivery constraint. Similarly, this procedure can be repeated to formulate the optimization model for the operating period $t = 1, 2, 3, 4$. As elaborated in the previous section, this model appears as a non-linear programming model as the objective function and constraints are expressed in the form of non-linear functions in terms of the decision variables.

4.2 Other Scenarios
Another six scenarios with variations to some of the parameters used in the benchmark scenarios are developed to investigate the impact of the changes on the results obtained. The following lists the scenario developed and the value of parameters used.

- Scenarios A and B have a confidence level of 90% and 99%, respectively.
- Scenarios C and D have the probability of owning the initial aircraft of 0.6:0.4 and 0.4:0.6, respectively.
- Scenarios E and F have the order delivery value i.e. $ORDER_i = 4$ and $ORDER_i = 6$, respectively.

4.3 Results and Discussion

Table 3 Benchmark scenario

<table>
<thead>
<tr>
<th>Period, ( t )</th>
<th>Future value</th>
<th>Discounted annual profit of period ( t )</th>
<th>Number of aircraft to be purchased</th>
<th>Initial number of aircraft</th>
<th>Number of aircraft sold</th>
<th>Total demand, ( D_t^a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t + (t + 1) )</td>
<td>( t )</td>
<td>Discounted annual profit of period ( t )</td>
<td>A320</td>
<td>A330</td>
<td>A320</td>
<td>A330</td>
</tr>
<tr>
<td>----------------</td>
<td>-------------</td>
<td>-----------------------------------------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>1</td>
<td>$7,107,908,970</td>
<td>$1,365,091,546</td>
<td>0</td>
<td>0</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>$6,216,982,737</td>
<td>$1,600,754,164</td>
<td>5</td>
<td>4</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>$5,093,360,859</td>
<td>$1,734,656,271</td>
<td>0</td>
<td>0</td>
<td>52</td>
<td>51</td>
</tr>
<tr>
<td>4</td>
<td>$3,891,926,900</td>
<td>$1,501,919,821</td>
<td>5</td>
<td>5</td>
<td>52</td>
<td>51</td>
</tr>
<tr>
<td>5</td>
<td>$2,908,720,716</td>
<td>$2,908,720,716</td>
<td>5</td>
<td>5</td>
<td>57</td>
<td>56</td>
</tr>
</tbody>
</table>

In addition to the benchmark scenario, the graphical results of Scenarios A-F are illustrated in Figure 1 while Tables 3 to 9 show the detailed outputs. The results obtained for benchmark scenario is shown in Table 3. It could be seen that the proposed model and solution method could produce the optimal solution for aircraft acquisition decision problem. Generally Table 3 shows a consistent increasing trend of discounted annual profit earned for growing demand. This finding is similar to what Khoo and Teoh (2011) found in their study of a linear programming model for aircraft acquisition problem. In addition, the results in Table 3 illustrate that a higher level of demand is not certainly guaranteeing a higher return for a particular operating period (i.e. operating period 4 in this study) due to the necessity to
acquire new aircraft to meet increasing demand. This implies that the level of service targeted by the airline companies to meet increasing demand will generate a higher return of profit practically, yet subject to the operational cost from all aspects, including the acquisition cost of new aircraft. This explains why there is a drop of profit in the operating year 4 (as displayed by Figure 1 and Table 3). Therefore, it could be seen that in the real practice, how to make a wise operational decision by considering the trade-offs among these practical constraints is really important to guarantee an optimal profit. It is anticipated that the illustrated benchmark scenario is able to capture the demand fluctuation in the real practice in a better manner as well as to demonstrate a better view for the airline companies in making decision for aircraft acquisition, particularly to capture the fluctuation of stochastic demand.

As shown in Figure 1 and Tables 3-5, it is observed that the change of confidence level influences the value of the total demand and hence in order to achieve a targeted level of service, the airline companies ought to control the targeted level within a quantified range namely confidence level in this study. Theoretically or practically, the confidence level indicates the level of a service targeted by an airline company. The level of profit earned by the airline company is affected if the targeted level of service varies. Subsequently, this will result in different decision making to comprehend these scenarios. In other words, a higher confidence level means that the airline companies intend to service a higher level of demand. As such, the level of profit obtained is higher. As shown by benchmark scenario as well as Scenarios A and B, the fact for which a higher profit is gained at a higher value of confidence level is established based on the tendency of the profit trend which is increasingly consistently when the value of confidence level is increasing. Additionally, the comparison of results shows that there is a propensity for the airline company to acquire more aircraft to meet a higher increase of demand but yet subject to the operational constraints as elaborated earlier. Additionally, it is important to note that an increase in demand does not indeed produce a higher return due to the acquisition cost of new aircraft, which is quite costly. This fact explains the displayed results by indicating how many and which type of aircraft that should have to be acquired at a particular operating period to obtain the optimal profit. Particularly,
for Scenario A, there is a drop of earned profit in operating year 5 due to the charge of aircraft acquisition cost. Similarly, the operational profit of Scenario B decreases in operating year 2 and 4 (because of the aircraft acquisition impose the costly purchase cost). Briefly, the sensitivity analysis shows that the airline company has to set their goal strategically (by setting the targeted confidence level appropriately) in order to maximize the expected profit at a desired level of service.

From Tables 6 and 7, it is observed that the setting of the probability of owning an initial number of aircraft could affect the profit level of the airline company. An interesting point to highlight is that the number of aircraft should be purchased at each year is the same for both scenarios (due to the same level of demand). In contrast to Scenario D, expected profit generated by Scenario C is higher as it is outlined at a higher probability of \( s_i \) i.e., \( p_{s_i} = 0.6 \) which is 20% higher than \( p_{s_i} \) for Scenario D. Correspondingly, the profit gained by Scenario C is higher than benchmark scenario (at a higher value of 10% for \( p_{s_i} \)) during the planning horizon. This shows that a higher value of \( p_{s_i} \) which corresponds to a higher probability of owning an initial number of aircraft to meet stochastic demand would consequently, result in a higher return. Apart from this, Figure 1 displays parallel lines of operational profit for benchmark scenario as well as Scenarios C and D during the planning horizon. This reveals that the earned profit behaves proportionally with the possession of aircraft. Therefore, the proposed model is relatively sensitive to the setting of the initial number of aircraft owned by the airline companies. Directly or indirectly, this implies that the setting of the probability of owning an initial number of aircraft has an influence upon the operations as well as the profit earned. The similar judgment could be seen in Khoo and Teoh (2011).

Tables 8 and 9 show the results for Scenarios E and F. It is observed that the order delivery constraint, i.e. the number of aircraft to be purchased for each operating period could affect the decision making and the profit of the airline company. This explains the discrepancy for
the results shown by benchmark scenario as well as Scenarios E and F. Comparatively, Scenario E (with order delivery constraint of 4) produces the highest profit due to the availability to purchase new aircraft to meet the stochastic demand. On the other hand, Scenario F (with order delivery constraint of 6) generated the lowest profit by purchasing more aircraft at an allowable availability in the market. Hence, it is important to note that it’s not indeed profitable to acquire more aircraft as higher purchasing cost and maintenance cost will occur. Perhaps, purchase lesser aircraft contributes higher expected profit due to the less charged costs. For some circumstances, the decision making to purchase new aircraft is also affected by the consideration of the airline companies in getting the least number of aircraft as long as the total number of aircraft operated is sufficient to provide the targeted level of service.

Concisely, it could be seen that the parameters setting in the developed model has an effect on the results, to some extent. It is relatively important to note that there is no ideal means to obtain an extreme profit as the optimal acquisition decision is decidedly dependent on several factors, i.e. management policy practiced by the airline companies, the desired scenarios to be optimized as well as some uncertainties, which could take place unpredictably. Therefore, in order to make a wise decision by managing stochastic demand under uncertainties, those aspects as mentioned and illustrated at the earlier sections should be taken into account considerably.

5. CONCLUSIONS

This study formulated an aircraft acquisition decision model for which the traveller demand appears to be stochastic in nature. It adopted probabilistic dynamic programming approach to incorporate the stochastic demand, which is assumed to follow the normal distribution. By considering some practical constraints, the model aims to maximize the airline companies’ profit i.e. the model is solved to find the number and types of new aircraft that should be

<table>
<thead>
<tr>
<th>Period, $t$</th>
<th>Discounted annual profit of period $t+1$</th>
<th>Future value</th>
<th>Discounted annual profit of period $t$</th>
<th>Future value</th>
<th>Number of aircraft to be purchased</th>
<th>Initial number of aircraft</th>
<th>Number of aircraft sold</th>
<th>Total demand, $D^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$7,692,869,341</td>
<td>$1,365,091,546</td>
<td>0</td>
<td>0</td>
<td>50</td>
<td>50</td>
<td>0</td>
<td>12,000,000</td>
</tr>
<tr>
<td>2</td>
<td>$6,644,166,685</td>
<td>$1,674,397,085</td>
<td>4</td>
<td>4</td>
<td>50</td>
<td>50</td>
<td>3</td>
<td>15,000,000</td>
</tr>
<tr>
<td>3</td>
<td>$5,479,170,984</td>
<td>$1,738,795,194</td>
<td>0</td>
<td>0</td>
<td>51</td>
<td>51</td>
<td>0</td>
<td>13,500,000</td>
</tr>
<tr>
<td>4</td>
<td>$4,329,952,524</td>
<td>$1,742,363,335</td>
<td>4</td>
<td>4</td>
<td>51</td>
<td>51</td>
<td>0</td>
<td>17,000,000</td>
</tr>
<tr>
<td>5</td>
<td>$3,145,230,832</td>
<td>$1,452,363,335</td>
<td>4</td>
<td>4</td>
<td>55</td>
<td>55</td>
<td>0</td>
<td>21,000,000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period, $t$</th>
<th>Discounted annual profit of period $t+1$</th>
<th>Future value</th>
<th>Discounted annual profit of period $t$</th>
<th>Future value</th>
<th>Number of aircraft to be purchased</th>
<th>Initial number of aircraft</th>
<th>Number of aircraft sold</th>
<th>Total demand, $D^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5,307,866,113</td>
<td>$1,365,091,546</td>
<td>0</td>
<td>0</td>
<td>50</td>
<td>50</td>
<td>0</td>
<td>12,000,000</td>
</tr>
<tr>
<td>2</td>
<td>$4,139,913,296</td>
<td>$1,109,128,727</td>
<td>6</td>
<td>5</td>
<td>50</td>
<td>50</td>
<td>3</td>
<td>15,000,000</td>
</tr>
<tr>
<td>3</td>
<td>$3,341,439,987</td>
<td>$1,421,878,347</td>
<td>0</td>
<td>0</td>
<td>53</td>
<td>53</td>
<td>0</td>
<td>13,500,000</td>
</tr>
<tr>
<td>4</td>
<td>$2,222,132,543</td>
<td>$1,134,252,850</td>
<td>5</td>
<td>2</td>
<td>53</td>
<td>53</td>
<td>0</td>
<td>17,000,000</td>
</tr>
<tr>
<td>5</td>
<td>$1,322,324,566</td>
<td>$1,322,324,566</td>
<td>6</td>
<td>3</td>
<td>58</td>
<td>54</td>
<td>0</td>
<td>21,000,000</td>
</tr>
</tbody>
</table>
purchased at every time period during the planning horizon. The proposed model and solution method is tested with an illustrative case study for which the input data and functions is either obtained or simulated by using the real data. The results obtained indicated that the proposed methodology is viable. With reasonable assumptions that pertain closely to the actual operation, the results reveal that aircraft acquisition decision is strongly influenced by stochastic demand and also the policies of the airline companies, for instance, pre-determined age of aircraft to be sold in this study. In general, the profit earned is increasing when the level of demand is on the rise with the exception of unexpected drop of demand, which could be taken place in the real practice. Additionally, six scenarios are created to test the sensitivity of the modelling parameters to the outcomes. Remarkably, order delivery constraint has a little impact for aircraft acquisition decision as compared to the benchmark problem. Apart from this, the acquisition decision is comparatively influenced by the confidence level and the probability of owning the initial number of aircraft. It is shown that the significant findings in this study are able to steer the relevant authorities at the management level as well as the decision makers in making a wise profitable operational decision to perform better in such a competitive airline industry. For the future work, the proposed model will be tested with a set of real data collected from the airline company. In addition, the extension of the planning horizon to be longer than 5 years will be taken into consideration in order to test the applicability of the developed model. The service frequency assignment will be considered as well.

REFERENCES

Available at: http://www.airasia.com/my/en/corporate/irannualreport.html


Airbus. (2010b) Specifications Airbus A320. [Online]


Malaysia Airlines. (2010a) Annual Reports. [Online]
Available at: http://malaysiaairlines.listedcompany.com/ar.html


