Applying the Catastrophe Model to Analyze Freeway Drivers' Route Switching Behavior Given a Congestion Charging Strategy

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Abstract: Along with the forthcoming implementation of an electronic distance-based charging strategy for the freeways in Taiwan, a congestion charging strategy may also be designed to help alleviate heavy traffic. Unlike previous studies where the discrete choice models are used, this study applies the cusp catastrophe model to discuss the drivers’ route switching behavior given the various rates and describe their non-linear characteristics. A questionnaire survey was conducted to obtain empirical results with 461 valid questionnaires were collected from freeway drivers traveling in the Taipei metropolitan area. The proposed behavioral model considered “switching intention” as the state variable, with “switching barrier” and “congestion charging rate” as the control variables. The behavioral state analyzed samples located in six areas of the control space. The catastrophe characteristics of switching behavior were also discussed. Higher switching barrier could easily cause discontinuous switching behavior. A bimodality phenomenon occurred especially when the rate charged was NT$1.2/km.

Key Words: route switching behavior, congestion charging strategy, catastrophe model

1. INTRODUCTION

1.1 Research background

To alleviate traffic delays in metropolitan areas and solve the unfairness issue for user payment, the Taiwan Area National Freeway Bureau will execute a full electronic toll collection (ETC) system on the freeway based on a distance-based charging strategy on December 2012. The implementation of a free-flow ETC system on freeways involves the construction of special gantries with sensors between interchanges for which vehicles need to be equipped with an on-board unit (OBU) and an IC card that will enable them to pay tolls without stopping. Provision of the free-flow ETC system should promote better traffic management, and it can be integrated into an Electronic Toll and Traffic Management (ETTM) system for further traffic applications. The ETC system would also facilitate the implementation of a congestion toll (DeCorla-Souza, 2002).

Many countries have always considered road pricing strategies as a promising method to help solve traffic congestion problems in urban areas (Schade and Schlag, 2003; Wen et al., 2005; Holguín-Veras et al., 2006). These strategies necessitate drivers to pay extra travel costs, especially during peak hours. It has been found that drivers respond to this pricing system by
altering their route, departure time, modes of transport, or even giving up their trips (Adler et al., 1999; Lam et al., 2001; Burris et al., 2002; Goh, 2002; Schade and Schlag, 2003; Wen et al., 2005). After a congestion charging strategy is put into practice in Taiwan, heavy traffic congestion and considerable delays at tollbooths during daily peak hours should be reduced, especially in metropolitan areas.

The congestion toll rates would depend on the time period, sections of roadway and traffic volume, and most unnecessary trips would be shifted accordingly. Before the congestion charging strategy is implemented on Taiwan freeways, attention should be paid to drivers’ potential diversion reactions and whether this would have the benefit of reducing traffic volume in problem areas. The amount of the congestion charging rate would also consequently influence the degree of diversion. Whether drivers are willing to pay the extra tolls has an impact on their route switching behavior (Brownstone, 2003). For the purpose of congestion management, the effects of the rate will mainly depend on drivers’ opinions. Thus, this paper would discuss drivers’ route switching behavior under various congestion charging scenarios.

1.2 Motivation and objectives

Although there have been numerous studies discussing the effects of congestion charging on drivers’ route switching behavior, in most of these linear models, such as discrete choice models (i.e., multinomial logit model or nested logit model) have been applied to construct the behavioral model. There have been no previous studies using non-linear model (such as a catastrophe model) to explain their choice behavior. In the relevant behavioral research, most consumer choice behavior can be regarded as a discontinuous catastrophe phenomenon which is nonlinear and complex (Zeeman, 1976; Stewart, 1982; Oliva et al., 1992; Rense and James, 2000; Byrne et al., 2001; Lange et al., 2001; Guastello, 1982). The catastrophe theory is a powerful mathematical tool, first developed by the French mathematician René Thom (1923-2002) in the 1960s and 1970s, which has been adapted to analyze nonlinear systems with respect to transitional and discontinuous behavior. The catastrophe theory has been applied in many fields and the method has matured in consumer choice behavior studies (Zeeman, 1974; Zeeman, 1977; Oliva et al., 1992; Lange et al., 2001; Byrne et al., 2001).

In past route switching behavior studies, discrete choice models have been used to predict the probabilities for the choice of alternative routes and to explore related significant factors. However, these linear models do not really explain the sudden changes in behavior and route switching of drivers in response to traffic jams and congestion charging rates on the freeway. Therefore, it may be more appropriate to apply the catastrophe theory to explain the catastrophe characteristics of drivers’ switching behaviors and discuss the diversion threshold for congestion charging rates. Numerous nonlinear phenomena that exhibit discontinuous jumps in behavior have been modeled using the catastrophe theory. The purpose of this study is the application of catastrophe theory to analyze changes in freeway drivers’ route switching behaviors in response to various congestion charging rates, so as to realize the nonlinear phenomenon of driver route switching behavior more explicitly. The empirical study can provide an understanding of freeway drivers’ behaviors in the Taipei metropolitan area and suggest policy implications for a better congestion charging strategy.
1.3 Research Framework

Based on the research objectives, a research framework was developed for exploring the effects of the congestion charging strategy on freeway drivers’ route switching behavior as shown in Figure 1. Data were collected using a questionnaire survey which was used to fully analyze freeway drivers’ route switching behavior under various stated congestion charging rates in the Taipei metropolitan area. By means of quantitative and qualitative analysis, the catastrophe theory is applied to establish drivers’ route switching behavior model and interpret their catastrophe phenomena.

![Figure 1 Research framework](image)

2. RESEARCH METHODOLOGY

2.1 Catastrophe theory

Catastrophe theory is a mathematical model developed and revised by mathematician René Thom in the 1960s and 1970s (Thom, 1975) and then popularized by Zeeman (Zeeman, 1974; Zeeman, 1976; Zeeman, 1977). It has been popular applied to many fields, such as chemistry (Wales, 2001), physics (Aerts et al., 2003), finance (Ho et al., 1980), psychology (Stewart et al., 1983), and social sciences (Holyst et al., 2000; Oliva et al., 1992). In dynamic systems, catastrophe theory describes how small and continuous changes in independent variables can have sudden, discontinuous effects on dependent variables (Kenneth, 2006). The main advantage of catastrophe models is that more complex behaviors can be captured by using significantly fewer numbers of nonlinear equations than linear equations describing the same phenomena.

Catastrophe theory describes the relationship between two sets of variables, the control variables (i.e., independent variables) and state variables (i.e., dependent variables). It is a so-called gradient system. Thom (1975) has demonstrated seven elementary catastrophes through his classification theorem, finding that all discontinuous phenomena can be expressed in terms of four or fewer control variables and two or fewer state variables. Table 1 gives prototypical examples for equations showing each type of catastrophe. In general, the cusp catastrophe model (CCM) is specifically applied due to its simplified function and all catastrophe characteristics can be found in its model. Thus, in this work we also apply the cusp catastrophe model to discuss drivers’ route switching behavior while encountering congestion charging on the freeway.
Table 1: Rene Thom’s seven elementary catastrophes

<table>
<thead>
<tr>
<th>Catastrophes</th>
<th>Control variable</th>
<th>State variable</th>
<th>Potential function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fold</td>
<td>1</td>
<td>1</td>
<td>$\frac{1}{3}z^3 - xz$</td>
</tr>
<tr>
<td>Cusp</td>
<td>2</td>
<td>1</td>
<td>$\frac{1}{4}z^4 - xz - \frac{1}{2}yz^2$</td>
</tr>
<tr>
<td>Swallowtail</td>
<td>3</td>
<td>1</td>
<td>$\frac{1}{5}z^5 - xz - \frac{1}{2}yz^2 - \frac{1}{2}vz^3$</td>
</tr>
<tr>
<td>Butterfly</td>
<td>4</td>
<td>1</td>
<td>$\frac{1}{6}z^6 - xz - \frac{1}{2}yz^2 - \frac{1}{3}vz^3 - \frac{1}{4}uz^4$</td>
</tr>
<tr>
<td>Hyperbolic</td>
<td>3</td>
<td>2</td>
<td>$z^3 + w^3 + xz + yw + vz$</td>
</tr>
<tr>
<td>Elliptic</td>
<td>3</td>
<td>2</td>
<td>$z^3 - zw^2 + xz + yw + vz^2 + vw^2$</td>
</tr>
<tr>
<td>Parabolic</td>
<td>4</td>
<td>2</td>
<td>$z^2w + w^3 + xz + yw + vz^2 + uw^2$</td>
</tr>
</tbody>
</table>


2.2 Cusp catastrophe model

The cusp catastrophe model is in its simplest form includes a state variable ($x$) and two control variables ($c$). The two control variables have different qualitative meanings. One is called the normal factor ($v$) and the other is called the splitting factor ($u$) (Baack et al., 1992). The parameter space consisting of a normal factor and a splitting factor is defined as the control space. The normal factor is related to the state variable in a consistent pattern. The splitting factor is the key variable and is a moderator variable which specifies conditions under which the normal factor will affect the state variable in a continuous fashion. In other circumstances the normal factor will produce discontinuous changes in the state variable. It determines the “breaking point” or threshold of change in the state variable (Baack et al., 1992). In the catastrophe framework, when the intensities of the normal factor and splitting factor reach a critical point, the state variable will change suddenly and radically (Kenneth, 2006).

The potential function of the cusp catastrophe model can be expressed by Eq. (1),

$$F(u, v, x) = -\frac{1}{4}x^4 + \frac{1}{2}ux^2 + vx,$$  \hspace{1cm} (1)

where the state variable $x$ is controllability, and $u$ and $v$ are the environmental control parameters. As a stable equilibrium state $x$ for this potential gives relative value $x$ of a function $F(u, v, x)$, a set of point $(u, v, x)$ is defined as the equilibrium surface $M_F$,

$$\frac{\partial F}{\partial x} = -x^3 + ux + v = 0$$

$$M_F : \{(u, v, x) \mid -x^3 + ux + v = 0\},$$  \hspace{1cm} (2)

where $M_F$ is said to be cusp catastrophe manifold. The values of $x$ which correspond to attaining a local maximum or minimum satisfy the conditions expressed in Eq. (3),

$$3x^2 + u = 0.$$  \hspace{1cm} (3)
Eliminating $x$ from Eq. (2) and Eq. (3), the bifurcation set can be expressed by Eq. (4). It also called Cardan’s discriminant (denoted by using $\Delta$)

$$
4u^3 = 27v^2 \\
\Delta = 27v^2 - 4u^3
$$

(4)

A switch in topology takes place when the values of $u$ and $v$ satisfy Eq. (4), which constitutes the catastrophe set. The splitting factor $u$ determines whether the system has one or two stable equilibrium. When $u < 0$, one stable equilibrium exists whatever the value of $v$. When $u > 0$, whether the system has a single low level of stable equilibrium, or a low level and a high level equilibrium, or a single high level of equilibrium depends upon the value of $v$.

2.3 Catastrophe characteristics

Changes in the control variables ($v$-right/left movement; $u$-back/front movement) cause changes in the state variable ($x$-vertical movement). If $u$ is low, smooth changes in $v$ would occur in proportion to a change in $x$ as shown by examining the travel of point a and point b in Figure 2. When $u$ is high, changes in $v$ produce relatively small changes in $x$ until a threshold is reached and then there is a sudden discontinuous shift in $x$. This is depicted by the path from point c to point d in Figure 2. However, it should be noted that a reversal in $v$ back to the point of the shift in $x$ would not cause $x$ to return back to its original position, because $v$ would have to move well past to cause $x$ to shift back. This is shown as the movement from point d to point c’. The various moves on the surface are characterized by five qualities that Thom (1975) described are follows:

![Figure 2 A cusp catastrophe model](image)

1. Divergence

The divergence indicates that small differences in the starting position may result in vastly different and even opposite ending positions. It means that small initial differences can eventually bring about very diverse behaviors. As shown in Figure 2, point a and point b
would travel through path A and path B when there are changes in the splitting factor \( u \), until finally being located at point c and point d which would result in different behavioral states.

2. Catastrophe
If changes in the normal and splitting factor produce a path (such as the movement of path C) which crosses the bifurcation set, an abrupt change in the state variable would occur. At that point g, a discontinuous transition would arise from the lower to the upper equilibrium surface.

3. Hysteresis
After the sudden transitions, although the path returns, the hysteresis phenomena show that the abrupt change from one mode of behavior to another would take place at different values of the control factors depending on the direction of change.

4. Bimodality
When the control variable falls into the bifurcation set (i.e., Cardan’s discriminant \( \Delta \leq 0 \)), the state variable is ambiguous. The bimodality phenomenon indicates that either two stable or distinctly different states would occur.

5. Inaccessibility
Over part of this phenomenon, there is a middle region between the two types of behaviors that are inaccessible.

2.4 Estimating approach

In the 1980’s three different approaches were developed for estimating statistical catastrophe models. In general, estimation of the catastrophe model is particularly difficult because of its nonlinear dynamic characteristics. Several cusp-fitting procedures have been proposed, but none is completely satisfactory. There are three techniques for fitting the cusp catastrophe models including GEMCAT of Oliva et al. (1987), the maximum likelihood method of Cobb (1978), and the regression method of Guastello (1982). GEMCAT and Cobb’s method can be applied to cross sectional data; Guastello’s method can only be applied to time series data.

The limitation of Cobb’s and Guastello’s methods is that they do not allow researchers to specify models in terms of specific combinations of multiple indicator variables. Rather the techniques find a catastrophe if it exists, and to identify which independent variables are associated with the control factor and which independent variables are associated with the splitting factor. Cleary, this is a problem when the researchers are trying to develop a confirmatory method that estimates a specific catastrophe model. Additionally, the dependent variable is required to be univariate. Consequently, its usefulness is limited when a multivariate dependent construct is required for the catastrophe model. GEMCAT approaches have been successfully applied in a number of different organizational research contexts (e.g., Oliva, 1992; Gresov et al., 1993).

Oliva et al.’s (1987) GEMCAT approach allows all variables in a catastrophe to be latent composites. To accomplish this, the variables \( X, Y \) and \( Z \) in the canonical cusp are presented by Eq. (5),

\[
F(Y_{\mu}, X_{\kappa}, Z_{\mu}) = -\frac{1}{4}Z_{\mu}^4 + \frac{1}{2}Y_{\mu}X_{\kappa}^2 + X_{\kappa}Z_{\mu}.
\] (5)
Let:

\[ i = 1 \ldots I \] state variables;
\[ j = 1 \ldots J \] splitting factors;
\[ k = 1 \ldots K \] normal factors;
\[ t = 1 \ldots T \] observations;
\[ Z_{it} = \text{the value of the } i\text{-th state variable on observation } t; \]
\[ Y_{jt} = \text{the value of the } j\text{-th splitting factor on observation } t; \]
\[ X_{kt} = \text{the value of the } k\text{-th normal factor on observation } t. \]

Now, we define three unobservable variables:

\[ Z_i^* = \sum_{i=1}^{I} \alpha_i Z_{it}; \]  
\[ Y_j^* = \sum_{j=1}^{J} \beta_j Y_{jt}; \]  
\[ X_k^* = \sum_{k=1}^{K} \gamma_k X_{kt}; \]

where:
\[ \alpha_i = \text{the estimated coefficient for the } i\text{-th state variable in } Z = Z_{it}; \]
\[ \beta_j = \text{the estimated coefficient for the } j\text{-th splitting factor in } Y = Y_{jt}; \]
\[ \gamma_k = \text{the estimated coefficient for the } k\text{-th normal factor in } X = X_{kt}; \]
\[ Z_t^* = Z = Z_{it} = \text{the value of the performance variable on observation } t; \]
\[ Y_t^* = Y = Y_{jt} = \text{the value of the splitting variable on observation}; \]
\[ X_t^* = X = X_{kt} = \text{the value of the normal variable on observation } t. \]

Thus, equation (5) can be redefined as these three unobservable constructs, which can thus accommodate univariate or multivariate measurements for each type of variable. This allows the cusp catastrophe model to be rewritten as shown in Eq. (9)

\[ f(X_i^*, Y_j^*, Z_k^*) = \frac{1}{4} Z_t^{*4} - X_t^* Z_t^* - \frac{1}{2} Y_t^* Z_t^*. \]  
\[ \text{(9)} \]

In these terms, the estimation problem, given \( X = X_{kt}, Y = Y_{jt} \) and \( Z = Z_{it} \), and its derivative set equal to zero can be stated as:

\[ \frac{\partial f(X_i^*, Y_j^*, Z_k^*)}{\partial Z_t} = 0 \]

\[ = Z_t^{*4} - X_t^* - Y_t^* Z_t^*. \]  
\[ \text{(10)} \]

From equation Eq. (10) the estimating goal is to minimize Eq. (11):

\[ \min_{\alpha_i, \beta_j, \gamma_k} \Phi = \| e_t^2 \| = \sum_{t=1}^{T} \left[Z_t^{*4} - X_t^* - Y_t^* Z_t^* \right]^2, \]  
\[ \text{(11)} \]

where the \( e_t \) = error. That is, for a given set of empirical data on the various specified state variable, splitting factor, and normal factor, one wishes to estimate the impact coefficients that define their respective latent variables, which make \( \Phi \) as close to zero as possible. Minimizing \( \Phi \) is equivalent to find the best fitting cusp catastrophe surface for the empirical data.
3. CATASTROPHE MODELING

3.1 Data collection

This paper focuses on the route switching behavior of freeway drivers who regularly commute in the Taipei metropolitan areas while congestion charging strategy is implemented. Thus, a questionnaire with stated preferences was conducted to collect data on their socio-demographic, travel characteristics and choice preferences under various congestion charging scenarios. A sample questionnaire is showed in appendix. Five congestion charging rates were set: NT$0.8/km, NT$1.0/km, NT$1.2/km, NT$1.5/km, and NT$1.8/km. In addition, to realize the effects of distinct traffic situation on drivers’ choice preferences, two scenarios (Scenario I and Scenario II) were simulated. In Scenario I the speed of travel on the freeway is 60 km/hr which represents slight congestion, whereas Scenario II refers to worse congestion in which the travel speed decreases to 40 km/hr.

The data were collected using on-line questionnaires and interview surveys in the Taipei metropolitan area of Taiwan. A total of 461 valid questionnaires were returned during December, 2009. The survey characteristics are given in Table 2. The percentage of trip purposes are for work (20.61%), business (17.57%), social (24.51%), recreational (32.97%), and other (4.34%). According to the socioeconomic characteristics of respondents, nearly 70% of the respondents are males (70.28%), with a college or graduate school (70.07%) educational level. Most are between the ages of 25 and 54 (85.25%) with personal incomes between 20-70 thousand NT dollars per month (80.91%).

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Samples</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trip purpose</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Working</td>
<td>95</td>
<td>20.61%</td>
</tr>
<tr>
<td>Business</td>
<td>81</td>
<td>17.57%</td>
</tr>
<tr>
<td>Social</td>
<td>113</td>
<td>24.51%</td>
</tr>
<tr>
<td>Recreational</td>
<td>152</td>
<td>32.97%</td>
</tr>
<tr>
<td>Other</td>
<td>20</td>
<td>4.34%</td>
</tr>
<tr>
<td>Gender</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td>324</td>
<td>70.28%</td>
</tr>
<tr>
<td>Female</td>
<td>137</td>
<td>29.72%</td>
</tr>
<tr>
<td>Level of education</td>
<td></td>
<td></td>
</tr>
<tr>
<td>High-school</td>
<td>138</td>
<td>29.93%</td>
</tr>
<tr>
<td>College</td>
<td>147</td>
<td>31.89%</td>
</tr>
<tr>
<td>Graduate school</td>
<td>176</td>
<td>38.18%</td>
</tr>
</tbody>
</table>

Table 2 Statistical summary

3.2 Operation of variables

According to the research by Feng and Kuo (2007), the switching barrier would negatively impede drivers’ route switching behavior. Moreover, several consumer behavioral studies (Feng and Huang, 2006; Oliva et al., 1992; Vikram et al., 1998) have considered the related concept “switching cost” as the splitting factor in the cusp catastrophe model. Thus, this study also defines the “switching barrier” as splitting factor $u$ in the route switching behavior model due to its bifurcation identity. Based on the purpose of this paper, to explore the effect of
implementation of congestion charging strategy, the “congestion charging rate” is taken as normal factor \( v \). With these two control variables, the dynamics of route switching behavior may be conceptualized in terms of the cusp catastrophe model shown in Figure 3. In this hypothesized model, “switching intention” is defined as the state variable \( x \). The behavioral variable, switching intention, which categorizes into five probable level using five-point Likert scale as strongly unlikely, unlikely, neutral, likely, and strongly likely for switching.

The cusp catastrophe theory uses a continuous parameter to describe discrete morphology behavior. In a cusp catastrophe model framework, after the intensities of the normal factor and splitting factor are determined, model fitting and dynamical analyses are then performed. GEMCAT allows \( x, u, v \) in the cusp catastrophe model to represent “latent” variables consisting of arbitrary linear combinations of the more elementary “indicator” variable. The operations of the dependent and independent indicator are measured as follows:

1. Switching intention \( (X^*) \):
   \( x \): the likelihood of a switch to an alternative route when the driver encounters various congestion charging rates on the freeway

2. Congestion charging rate \( (V^*) \):
   \( v \): five congestion charging rates, NT$0.8/km, NT$1.0/km, NT$1.2/km, NT$1.5/km, and NT$1.8/km are set during peak hours on the freeway

3. Switching barrier \( (U^*) \): its value would be calculated by the average of the sum for following indicators \( u_1 \sim u_4 \). And, the four indicators of splitting factor, switching barrier, have also been measured by five-point Likert scale.
   \( u_1 \): used to driving the usual route instead of switching routes
   \( u_2 \): switching barrier concerning time saving
   \( u_3 \): feel it is troublesome to search for other route information
   \( u_4 \): willingness to pay more to drive the usual route

Figure 3 Hypothesized cusp catastrophe model of route switching behavior
The hypothesized cusp catastrophe model is tested by using the GEMCAT II software. All operational variables entered into the behavioral model should be standardized to transform into z-scores \( (i.e., \ M=0, \ SD=1) \), such as \( Z_{x_i} \), \( Z_{u_j} \) and \( Z_{v_k} \) respectively. Thus, the cusp catastrophe model can be expressed in the following general form:

\[
X^* = \sum_{i=1}^{l} \alpha_i \cdot Z_{x_i} = \alpha \cdot Zx; \tag{12}
\]

\[
V^* = \sum_{k=1}^{p} \gamma_k \cdot Z_{v_k} = \gamma \cdot Zv; \tag{13}
\]

\[
U^* = \sum_{j=1}^{q} \beta_j \cdot Z_{u_j} = \beta_1 \cdot Zu_1 + \beta_2 \cdot Zu_2 + \beta_3 \cdot Zu_3 + \beta_4 \cdot Zu_4. \tag{14}
\]

### 3.3 Estimation results

The estimation results produced by the route switching behavioral models for Scenario I and Scenario II are presented in Table 3. In these cusp catastrophe models, the significant indicator influence switching barrier is \( u_2 \), which is concerned with time saving with the maximum value of the parameter being \( \beta_2 \). This means that time will be the key concern in drivers’ diversion decisions after implementation of the congestion charging strategy. The minor importance indicator is \( u_3 \) which is an indication of how troublesome the driver feels it is to search for information about other routes. When the sign of parameter \( \beta_1 \) is negative, it indicates that drivers living in the Taipei metropolitan are not committed driving the usual route but will switch routes if necessary. In contrast it is important to note that they would accept to pay more for driving usual route if parameter \( \beta_4 \) is positive.

<table>
<thead>
<tr>
<th>Variable</th>
<th>State variable (x)</th>
<th>Control variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario</td>
<td></td>
<td>Normal factor (v)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Congestion charging rate</td>
</tr>
<tr>
<td><strong>Scenario I</strong></td>
<td>( X^* = Zx )</td>
<td>( V^* = 0.5014 \cdot Zv )</td>
</tr>
<tr>
<td><strong>Scenario II</strong></td>
<td>( X^* = Zx )</td>
<td>( V^* = 0.5354 \cdot Zv )</td>
</tr>
</tbody>
</table>

The distribution in the study sample within the control space (consisting of the x-axis and y-axis) is depicted in Figure 4. The x-axis shows five congestion charging rates, while the y-axis expresses the switching barrier for switching route. The five data sets arranged from left to right indicate the situation for rates of NT$0.8/km, NT$1.0/km, NT$1.2/km, NT$1.5/km and NT$1.8/km, respectively. Toward the bottom higher switching barriers exist. Depending on the rate, switching barrier and bifurcation set \( (i.e., \ Cardan’s \ discriminant \ \Delta \leq 0) \), the control space can be divided into six sub-areas, represented by \( A-F \). The percentage in each sub-area represents the proportion of samples located. For example, there are 2.6% samples located within \( E \) area in Scenario I.

Samples located in area \( A \) with the lower switching barrier and in area \( C \) with the higher switching barrier would continue to drive on the freeway when encountering lower congestion charging rates \( (i.e., \ NT$0.8/km, \ NT$1.0/km \ and \ NT$1.2/km) \), whereas samples located in area \( B \) with the lower switching barrier would prefer to switch to a free alternative route when
The congestion charging rates are higher (i.e., NT$1.5/km and NT$1.8/km). Even though samples located in area D are more concerned about switching barriers, they will probably decide to switch to an alternative route due to the higher rate on the freeway.

Samples located in area E, Cardan’s discriminant $\Delta \leq 0$, with a higher switching barrier have two stable solutions in the behavioral system. Meanwhile, information of parameters unable to confirm which route would be chosen in these cusp catastrophe models. Two route choices are acceptable whether driving on the freeway or switching to an alternative route. This bifurcation situation happens most easily when the rate is NT$1.2/km.

The dimension of six sub-areas in the control space for Scenario I and Scenario II are all the same. But the percentages of samples located in each sub-area are different compared Scenario I with Scenario II. Comparison of the two congestion scenarios shows that when drivers encounter more serious traffic congestion, i.e., Scenario I transferred to Scenario II, the probability of switching to an alternative route in areas B and D is slightly enhanced. The uncertainty in area E will diminish with the higher level of congestion. This indicates that congestion would encourage drivers to confirm their route decision.

4. DYNAMIC ANALYSIS

The conceptual switching behavior model developed by the cusp catastrophe method is shown in Figure 5 where the control variables are mapped onto the catastrophe surface structure. The upper behavioral surface represents drivers’ switching intention which categorizes into five probable levels. The projection of the fold curve of behavioral surface into parameter space C yields the bifurcation set. System dynamics occur on the surface of the behavioral model. Changes in position result from changes in the splitting factor $u$ (switching barrier) and normal factor $v$ (congestion charging rate) which would cause changes in the state variable $x$ (switching intention).

Therefore, the projection of behavioral surface into control space which divided into six sub-areas $A$–$F$ would define the probabilities of drivers’ route switching intention. For example, while the sample on behavioral surface has been projected into area $A$ and $C$, it would prefer to continue to drive on the freeway. On the contrary, if the sample has been projected to area $B$
and $D$, it means that the driver would more likely to switch to the alternative route. Besides, we may not sure drivers’ switching intentions while the samples projected into area $E$ and $F$.

Significant catastrophe characteristics can be found in Figure 5 and are described below. If the magnitude of change in $u$ (switching barrier) is small, then a smooth change in $x$ (switching intention) would occur, directly proportional to the change in $x$ (switching intention), as depicted by path D in Figure 5. A small difference in the initial starting positions (e.g., point $a$ and point $b$) can result in vastly different values for $x$ (switching intention) when the magnitude of $u$ (switching barrier) increases beyond the point where the pleat starts. This phenomenon is illustrated by path A and path B in Figure 5, where point $a$ is driven downward to point $c$, and point $b$ is driven upward to point $d$. At higher values of $u$ (switching barrier), however, large changes in $v$ (congestion charging rate) will produce a sudden discontinuous shift in $x$ (switching intention) as shown in path C, which then contributes to the reverse behavioral result.

It should be noted that once a sudden shift has happened, reversing the values of $v$ (congestion charging rate) may not cause a substantial downward change in $x$ (switching intention) namely hysteresis. There must be a significant reversal in $v$ (congestion charging rate) before a shift down to point $f$ would occur. These lags in response are aggravated or mitigated by the size of the $u$ (switching barrier). Within the cusp area, the state variable $x$ can take on two possible values for a given $(v, u)$ pair. This characteristic allows for the modeling of lag effects (hysteresis).

![Figure 5 Behavior Manifold for the Cusp Catastrophe Model](image-url)
5. CONCLUSIONS AND RECOMMENDATIONS

For congestion management, the congestion charging strategy is used to encourage drivers to shift unnecessary trips during peak hours, especially in metropolitan area. A questionnaire survey was conducted in Taiwan’s Taipei metropolitan area to explore drivers’ route switching behavior under various congestion charging rates. Two congestion scenarios with five charging rates were designed. We collected freeway drivers’ opinions and preference for switching routes. Unlike previous studies, the cusp catastrophe model was applied to explain drivers’ non-linear switching behavior. In the hypothesized model “switching intention” is taken as the state variable $x$, “congestion charging rate” as the normal factor $v$, and the “switching barrier” is a splitting factor $u$.

This empirical study demonstrates that higher rates (i.e., NT$1.5/km and NT$1.8/km) would lead drivers to increase the tendency to switch to alternative routes, even though the switching barrier is higher. When the congestion charging rate is set at a lower level, such as NT$0.8/km and NT$1.0/km, drivers tend to continue driving on the freeway. This means that with the congestion charging strategy we are unable to reach the anticipated outcome. If the congestion charging rate set to be NT$1.2/km, the decision of drivers in the behavioral system is ambiguous with a higher switching barrier. Several significant catastrophe characteristics, such as divergence, catastrophe, hysteresis, and bimodality can be found in the behavioral models. In addition, traffic managers should consider how to lower the drivers’ switching barrier. For example, more information about alternative routes should be provided when drivers’ encounter worse congestion on the freeway.

The results of this paper should assist traffic managers to judge the effects of the congestion charging strategy and realize drivers’ discontinuous switching behavior. We initially proposed the catastrophe model to describe drivers’ nonlinear and complex switching behaviors. Future research should be conducted to collect more relevant factors and to discuss their effects on behavioral decisions in order to more explicitly explain real route switching behavior. Other catastrophe models may be applied to construct drivers’ route switching behaviors as well.

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REFERENCES


APPENDIX

A sample questionnaire

I. Socio-demographic

1. Gender: □ Female □ Male
2. Level of education: □ High-school □ College □ Graduate school
3. Age: □ 18-24 years old □ 25-34 years old □ 35-44 years old □ 45-54 years old □ ≥ 55 years old
4. Personal monthly income: □ < NT$ 20 thousand □ NT$ 20-40 thousand □ NT$ 40-60 thousand □ NT$ 60-80 thousand □ ≥ NT$ 80 thousand

II. Travel characteristics

1. What is your most frequent trip purpose while driving on freeway?
   □ Working □ Business □ Social □ Recreational □ Other
2. You are used to driving the usual route instead of switching routes.
   □ strongly disagreeable □ disagreeable □ neutral □ agreeable □ strongly agreeable
3. You always concern time saving while choosing the driving route.
   □ strongly disagreeable □ disagreeable □ neutral □ agreeable □ strongly agreeable
4. It is troublesome to search for other route information.
   □ strongly disagreeable □ disagreeable □ neutral □ agreeable □ strongly agreeable
5. You prefer to pay more to drive the usual route if necessary.

☐ strongly disagreeable  ☐ disagreeable  ☐ neutral  ☐ agreeable  ☐ strongly agreeable

III. Choice preferences

If you would encounter traffic congestion and then be charged different levels of congestion rate on freeway, you can choose to keep driving on freeway or switching to alternative route. Please count how much you should pay for congestion charging depending on your frequent driving distances on freeway. What is your preferable choice?

1. (1) If you would drive on freeway which travel speed is 60 km/hr and then should be charged NT$0.8/km, would you like to switch to the free alternative route?
☐ strongly unlikely  ☐ unlikely  ☐ neutral  ☐ likely  ☐ strongly likely

(2) If you would drive on freeway which travel speed is 60 km/hr and then should be charged NT$1.0/km, would you like to switch to the free alternative route?
☐ strongly unlikely  ☐ unlikely  ☐ neutral  ☐ likely  ☐ strongly likely

(3) If you would drive on freeway which travel speed is 60 km/hr and then should be charged NT$1.2/km, would you like to switch to the free alternative route?
☐ strongly unlikely  ☐ unlikely  ☐ neutral  ☐ likely  ☐ strongly likely

(4) If you would drive on freeway which travel speed is 60 km/hr and then should be charged NT$1.5/km, would you like to switch to the free alternative route?
☐ strongly unlikely  ☐ unlikely  ☐ neutral  ☐ likely  ☐ strongly likely

(5) If you would drive on freeway which travel speed is 60 km/hr and then should be charged NT$1.8/km, would you like to switch to the free alternative route?
☐ strongly unlikely  ☐ unlikely  ☐ neutral  ☐ likely  ☐ strongly likely

2. (1) If you would drive on freeway which travel speed is 40 km/hr and then should be charged NT$0.8/km, would you like to switch to the free alternative route?
☐ strongly unlikely  ☐ unlikely  ☐ neutral  ☐ likely  ☐ strongly likely

(2) If you would drive on freeway which travel speed is 40 km/hr and then should be charged NT$1.0/km, would you like to switch to the free alternative route?
☐ strongly unlikely  ☐ unlikely  ☐ neutral  ☐ likely  ☐ strongly likely

(3) If you would drive on freeway which travel speed is 40 km/hr and then should be charged NT$1.2/km, would you like to switch to the free alternative route?
☐ strongly unlikely  ☐ unlikely  ☐ neutral  ☐ likely  ☐ strongly likely

(4) If you would drive on freeway which travel speed is 40 km/hr and then should be charged NT$1.5/km, would you like to switch to the free alternative route?
☐ strongly unlikely  ☐ unlikely  ☐ neutral  ☐ likely  ☐ strongly likely

(5) If you would drive on freeway which travel speed is 40 km/hr and then should be charged NT$1.8/km, would you like to switch to the free alternative route?
☐ strongly unlikely  ☐ unlikely  ☐ neutral  ☐ likely  ☐ strongly likely