Hierarchical Shortest Path Finding Algorithm with Probabilistic Travel Time

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Abstract

With the development of Route Guidance System, travelers’ route choice behaviors change rapidly yet the existing modeling methods are not satisfactory. In addition, most existing models were formulated without integrating the driver’s attitude and preference in detail. In this paper, a new route choice model which can deal with the mentioned topics is proposed. With the combined concepts of hierarchical road classification and resource constraints, the paths they are not seriously overlapped are enumerated. After then, the model calculates the path choice probabilities with respect to the given preferred arrival time. Using a real-size network, the proposed model calculated path choice probabilities with Monte Carlo simulation.

Keywords: hierarchical shortest path algorithm, route choice problem, probabilistic travel time, multinomial logit model.

1. INTRODUCTION

A route choice problem has been one of the core study topics in transportation network analysis. In traffic assignment, forecasting the route choices of drivers is a fundamental requisite. In addition, a route choice problem is a rising issue in Intelligent Transportation System (ITS). In the last decade, a car navigation system (CNS) becomes prevalent and many commercial companies are devoting themselves to develop a better route finding algorithm. Although there have been significant developments on this topic, the existing models for the route choice problem are not so satisfactory and still have numerous flaws to be improved.

The main difficulty in modeling the route choice problem is that it consists of various influencing factors. The factors included in the route choice problem can be classified into two groups: 1) user behavior and 2) system uncertainty.

First, each driver in the real world has heterogeneous characteristics. For example, the attitudes of drivers to uncertainty or danger are different from one another. Some drivers are risk-averse; while others are risk-taking. In addition, the travel time budgets of travelers differ even if their origin and destination are the same. Second, the supply side of road system also has some uncertainties. As we can observe in the real world, travel time between two locations is ever-fluctuating. It varies depending on the time of day and day of week. There would be various recurrent and non-recurrent sources of the fluctuation. As a result, the variables included in the route choice problem are not deterministic but probabilistic ones.

In this paper, the authors try to solve these problems. The way for considering the travel time budget of drivers is proposed using the path reference travel time. The need for a better model which incorporates the behavioral aspects into the route choice model is among the highlights of recent ITS
applications (Beckhor et al, 2006). In the context, an important behavioral aspect will be taken into account. The uncertainty in travel time will be considered in the route choice process. Travel time is denoted as a probabilistic distribution instead of a constant value. With the new definition of travel time, the route choice probability is formulated as a conditional one. In addition, the cognitive process of human beings for coding the topology of road network onto their minds is taken into account.

One of the most important advantages of using the new model is that it does not require parameter estimation. If a discrete choice method such as an MNL model is employed, then the logit model should be calibrated with the samples’ choice results of survey participants. Conversely, only traffic surveillance data such as link travel time samples are required for executing the developed model. Nowadays, travel time data can be collected in real-time everyday under ITS environment so the developed model is an ITS-friendly approach for a real-time route guidance system.

2. LITERATURE REVIEW

2.1 Shortest path problems and algorithms
A shortest path algorithm plays an important role in route choice problems. Generally, the shortest path algorithms are based on Bellman’s principle of optimality which states that: “Any optimal policy has a property that whatever the initial state and initial decisions are; the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.”

Among the well-known algorithms for finding the shortest path (Dijkstra’s, 1959; Floyd, 1962), Dijkstra algorithm is employed as a path finding algorithm. In terms of the randomness of link attributes, shortest path algorithms can be classified into two categories: deterministic and stochastic. The deterministic shortest path (Dijkstra, 1959; Sharma, 2001; Nepal, 2002) uses one exact value, usually the mean travel time for transportation networks, in determining the shortest path. On the other hand, the stochastic shortest path (Frank, 1969; Mirchandani, 1976) uses random variables, the link travel time probability distribution (PDF), in determining the shortest path. For real transportation networks, the stochastic approach is recommended since it is similar to how drivers perceive their path travel time. However, stochastic problem is more complex and more difficult.

2.2 Path enumeration algorithms
In the route choice problem, the preparation of choice alternatives (i.e. routes) is the most important work since a false definition of alternative set brings out false choice probabilities. Therefore, it is important to enumerate the plausible alternative paths for a given OD pair. In this study, k-shortest path algorithm is used for enumerating paths. The first k-shortest path algorithms was introduced by Bock et al (1957). All possible paths from an origin to a destination are enumerated and sorted based on the travel cost. Pollack (1961) calculated the k-shortest path set by subjecting the links of the k-1 links to infinity. Eppstein (1998) formulated a k-shortest path algorithm using heap-ordered tree. To overcome the excessive calculation burden of the aforementioned k-shortest path algorithms, Yen (1971) introduced an algorithm that eliminates the nodes used by the enumerated shortest path to find the next shortest path. In this algorithm, overlapping among enumerated paths, well-known as IIA (Independence of Irrelevant Alternatives) problem, arises. Lim and Hydecker (2006) illustrated a full path enumeration method but it cannot be applied to a practical problem without constraints since the required memory for storing the full path set is tremendous. Lim and Kim (2006) introduced a link-based algorithm that considers path overlap and turn prohibitions in enumerating reasonable paths.

A hierarchical approach is also used with k-shortest path algorithm. Park and Rillet (1997) presented an example. At first, a link penalty is used for minimizing the link overlaps. After
then, a network is categorized according to road classification function or hierarchy. Both algorithms use a node-based Dijkstra algorithm in finding the shortest path.

2.3 Route choice problem

In this paper, the way for incorporating the uncertainty of travel time into the route choice problem is the main concern. In transportation studies, a prevalent method is to use a random utility function (Sheffi, 1985). Analytically the perceived travel time, $C^r_s$, on route $k$ between origin $r$ and destination $s$, is computed as:

$$C^r_s = c^r_k + \xi^r_s \quad \forall k, r, s, k \in K^r$$

where $c^r_k$ is the measured or deterministic travel time and $\xi^r_s$ is the random error. $K^r$ is the set of paths for OD pair $rs$. Assuming that the travel time is the most important attribute of concern to travelers in their choices of travel routes, the probability of being the shortest path, $P^r_s$, is given by

$$P^r_s = \Pr(C^r_s \leq C^r_l, \forall l \in K^r) \quad \forall k, r, s; l \neq k \in K^r$$

Note that we assume a reasonable traveler for route choice. It means a traveler always selects the minimum travel cost or time path. With this assumption, the probability of being the shortest path calculated by Eq. (2) can be equal to the path choice probability of traveler. For this problem, the most common models are the multinomial logit (MNL) and the multinomial probit (MNP) models. The former is based on the assumption that the utilities of the alternatives in the choice set are identically and independently distributed (i.i.d.) $Gumbel$ variates and can be derived (assuming the utility of using the $k^{th}$ path between origin $r$ and destination $s, U^r_s$) as follows.

$$U^r_s = -\theta \cdot c^r_k + \xi \quad \forall k, r, s$$

The logit choice probability is then computed as

$$P^r_s = \frac{e^{-\theta c^r_k}}{\sum_l e^{-\theta c^r_l}} \quad \forall k, r, s$$

where $\theta$ is a coefficient that scales the perceived travel time.

2.4 Travel Time Reliability

In the real world, the drivers may consider travel time variations due to the variation of travel demand and supply, accidents, and poor weather. In this regard, the reliability of path travel time is an important decision factor in the real route choice situation. Asakura (1998) proposed a model that evaluates the travel time reliability between an OD pair in the case of natural disasters. Using a stochastic travel time, a reliability measure is defined as a probability that one can travel between an OD pair within an acceptable level of travel time. Lee et al. (2000) formulated a reliability traffic assignment model where travel time reliability is determined by the degree of travel time variation on the paths chosen by the motorists. Asakura and Hato (2000) formulated a behavioral model in a deteriorated network focusing on the difference between the recognized network and the actual network for non-informed and informed drivers. Fan and Nie (2006) proved the monotonic property of successive approximation sequences for routing problems with recourse. The study further showed that the deterministic shortest path and $k$-shortest path problems are equivalent to the special case of the stochastic on-time arrival (SOTA) problem where link travel time probability densities are delta functions. Kim (2008) used PDFs of OD and route travel time for an activity scheduling and route choice problem, respectively. In his study, reliable travel time and travel time margin were introduced in order to model the risk-averse behavior of
travelers. A traveler who has a preferred arrival time at a destination would consider it for deciding one’s departure time. If necessary, the preferred arrival time can be reflected in one’s route choice.

3. MODELING FRAMEWORK

3.1 Path enumeration problem and Hierarchical Road Network Classification
The concept of the hierarchical algorithm proposed in this study is based on Park and Rillet’s (1997). The algorithm identifies $k$ reasonable shortest paths in the hierarchical network in which arterial and highway networks are denoted as major network and drivers find independent corridors on it. In reality, human beings recall major corridors first and only consider them when choosing and changing their routes since the memory capacity of human beings are not sufficient for storing a whole minor network topology. Therefore, the first process of hierarchical path enumeration is an independent corridor finding problem. In order to find the corridors in the major network, a penalty method is employed (Lim and Kim, 2006) in which the links used by previously identified paths are penalized to minimize the chances of generating the $k$-shortest path set with similar links used. After enumerating all feasible paths, the enumerated paths were further subjected to the travel time constraint. Based on the driver’s perspective, paths with costs higher than a driver’s acceptable limit would not be included in the $k$-reasonable path set. In this study, the maximum travel time is 2.5 times higher than the minimum OD travel time. After finding a corridor, a driver has to connect one’s origin and destination to the closest points on the corridor because most original origins and destinations are not located on the highway or main corridor. The original origin and destination should be connected to the closest entering and exiting locations on the corridors. Dijkstra algorithm is used for finding the shortest connection.

To check the properties of the identified corridors, two measures of effectiveness (MOE) are proposed. The total travel time ratio (TTTR) is the first, which is the ratio of the travel time of alternative path $p$ to the travel time of the fastest path $k$. The TTTR shows a pair-wise comparison of relative efficiency between two routes $k$ and $p$ in terms of travel times.

![Figure 1 Hierarchical path building process](image-url)
second is the route similarity (RS), which is the ratio of the length of the route $k$ repeatedly used while traversing through the alternative path $p$.

$$TTTR_{kp} = \frac{TT_p}{TT_k}$$  \hspace{1cm} (5)

$$RS_{kp} = \sum_{a=\in A} d_a \cdot \delta_{a,kp} \div d_{rs}^{\min}$$  \hspace{1cm} (6)

where $TTTR_{kp}$ = TTTR between route $k$ and $p$; $TT_k$ = travel time on route $k$; $RS_{kp}$ = RS of route $k$ to route $p$; $d_a = \text{distance on link } a$; $d_{rs}^{\min} = \text{distance of the shortest route for OD pair } rs$; $\delta_{a,kp} = 1$ if link $a$ is used by route $k$ and route $p$, 0 otherwise; and $n = \text{number of links in the network}$.

### 3.2 Path choice problem with probabilistic travel time distribution

#### 3.2.1 Travel time modeling

In the developed model, path travel time is defined as a random variable following a probabilistic distribution. The first step of the modeling procedure for the probabilistic path travel time is the generation of random link travel time. Generally, link travel time is dependent on the traffic flow on the link. However, incorporating traffic flows to the route choice model requires a huge calculation burden. To simplify the problem while considering the link travel time variation, link travel time $t_a$ is assumed as a random variable with a normal probability distribution. This is not a critical flaw of the model because the probability distribution of link travel time can be estimated with daily surveillance data for a network.

In randomly generating the sample of actual link travel time ($t_a$), Monte Carlo simulation is used. It is a technique in which an output of random numbers is related to an assumed probability distribution so that a set of probable values for the basic variables of the function is obtained (Smith, 1986).

After generating random link travel times with the assumed average link travel time and the standard deviation, the path travel time distribution should be estimated. In this study, the randomness of travel time is considered in terms of path instead of link as in Mirchandani and Soroush’s (1987) formulation. Path travel time $C_{k}^p$ is calculated by summing sample link travel times $t_a$ along path $k$ as shown in Eq. (7).

$$C_{k}^p = \sum_{a=1}^{n} t_a \cdot \delta_{a,k} \quad \text{where } \delta_{a,k} = \begin{cases} \frac{1}{d_{rs}} & \text{if } a \in k \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (7)

The addition of the randomly generated sample link travel times might be questionable because it does not consider the correlation of subsequent links. Due to computational complexities, however, the inclusion of link correlation is left for the future study. After a number of simulation trials $m$, the distribution of path travel time is estimated. With the calculated path mean travel time and standard deviation, the path distribution function is assumed as a normal distribution. A statistical test for a normal distribution of a path travel time would be unnecessary because the path travel time is the sum of normal link travel times. If link travel time is provided with detectors in the field, the sum of the travel time would not
follow a normal distribution. Therefore, in that case, a statistical test for other probability distributions is required.

### 3.2 Formulation of a new route choice model

Two cases are considered in the calculation of the route choice probability; namely, 1) with travel time budget and 2) without travel time budget. For the with travel time budget case, it is assumed that there is a known preference arrival time at the destination; while for the without travel time budget case, the route choice probabilities of a typical driver is calculated.

#### 3.2.1 Route choice with travel time budget

Some drivers demand a specific arrival time or maximum acceptable travel time for their travels but existing route choice models have not reflected this demand. In this study, the path choice probability is calculated based on the user’s maximum acceptable travel time and it is denoted as “reference time.” As hypothesized by Fuji and Kitamura (2004), the path choice probabilities can be calculated considering the reference travel time within the users’ time frame. Usually, the drivers recall the route travel time in ranges; that is, from a minimum to a maximum value. To imitate the perception process of path travel time, in their study, it is assumed that the drivers believe that the path travel time of path \( k \) would not be less than \( t_{\text{min}} \) or greater than \( t_{\text{max}} \).

In calculating the path choice probability, this study accounts for the PDF function of the path travel time. As shown in Figure 2, the probability that the traveler arrives at a destination within a reference travel time \( t \) is determined by the shaded area. The greater the shaded area, the higher is the probability of arriving on time using the path. In addition, in reality, the reference time \( t \) differs from one person to the other. The path choice calculation is taken as the probability that a given path \( k \) is the shortest path at a particular reference time \( t \). Eq. (8) shows the probability that path \( k \) becomes the shortest path given a minimum OD travel time \( \pi^{rs} \). That is, when path \( k \)’s travel time is shorter than those of the other alternative’s and mathematically represented by

\[
pr(C_k^r = \pi^r) = pr(c_k^r < c_1^r \text{ and } c_k^r < c_2^r \text{ and } \ldots \ldots \ldots c_k^r < c_n^r) \quad \forall 1,2,\ldots, k,\ldots, n \in P^r \quad (8)
\]

where \( P^r \) is the path set for OD pair \( rs \). Note that all paths found by a hierarchical finding algorithm do not have seriously overlapping links, so the paths can be assumed to be independent from one another. Accordingly, the choice probability is further simplified as follows:

\[
pr(c_k^r = \pi^r) = pr(c_k^r < c_1^r) \cdot pr(c_k^r < c_2^r) \cdot \ldots \ldots \cdot pr(c_k^r < c_n^r) \quad \forall 1,2,\ldots, n \in P^r \quad (9)
\]

Analytically, the calculation of choice probability \( pr(c_k^r = \pi^r) \) for path \( k \) by Eq. (8) and (9) requires a big calculation burden especially in the real-size network since all cases have to be compared pair by pair. Let’s assume that there are only two independent paths. In order to calculate the choice probabilities of the two paths, the probability distributions of the two paths should be considered simultaneously. In addition, the cases in which the minimum travel time is shorter than the reference time \( (c_{\text{min}}^r) \) are only taken into account because travelers do not consider the case in which they arrive later than their maximum acceptable time. If both paths cannot give a travel time shorter than the reference time, then it is regarded as a fail. The choice probability of path 1 for a success cases can be calculated as Eq. (10).
The calculation of Eq. (10) is not so difficult in the case of two paths. However, a calculation burden increases sharply as the number of paths in the network increases. In addition, the calculation of \( pr(c''_1 < c''_2) \) requires analytical manipulation. Figure 3 shows the concept of the probability calculation. In this study, a simple approach is presented. Using the Monte Carlo simulation technique, the travel time of each path is generated and the feasibility of the minimum travel time under the reference time is checked. The number of events, \( N''_k \), that a path travel time \( c''_k \) is the shortest where \( c''_k \leq \pi''_k \), is counted and the choice probability of path \( k \) is estimated as follows:

\[
pr(c''_k = \pi''_k) = \frac{N''_k}{N_{\text{total-success}}}
\]  

(11)

where \( N_{\text{total-success}} \) is the number of trials where the shortest path has a path travel time less than or equal to the reference time \( c''_k \). The feasibility of arriving at the destination within \( c''_j \) is also calculated as

\[
pr(C''_i \leq t) = \frac{pr(c''_1 < c''_2)}{pr(\min(c''_1, c''_2) < c''_j)} = \frac{pr(c''_1 < c''_2)}{pr(c''_1 < c''_j) + pr(c''_2 < c''_j)}
\]

(10)
\[ \text{SR}(c_{rs}^{ref}) = \frac{N_{\text{total-success}}}{\text{Total Trials}} \]  

where \( \text{SR}(c_{rs}^{ref}) \) is the success rate where the shortest path has a travel time less than \( c_{rs}^{ref} \) (also known as Expected Satisfaction Rate) and Total Trials is the total number of Monte Carlo trials.

### 3.2.2 Route choice without travel time budget

In some cases, the traveler does not declare a specific arrival time or it is unknown to a route choice model. In this condition, the reference travel time cannot be defined as a deterministic value. If the reference time is uncertain, the driver’s reference travel time should be assumed as a random variable. As in the previous formulation, the probability that the driver arrives at the destination on time is calculated as

\[ p(r_{rs}^{ref} = \pi^{rs}) = p(r_{rs}^{ref} = \pi^{rs} | c_{rs}^{ref} = t) \]  

In this case, \( t \) is unknown but has a probability distribution. In this study, an interview survey was conducted for finding it.

Analytically, over the continuous time, equation (13) could be elaborated as follows:

\[ p(r_{rs}^{ref} = \pi^{rs}) = \int_{t_{\min}}^{t_{\max}} \frac{p(r_{rs}^{ref} = \pi^{rs}) \cdot p(c_{rs}^{ref} = t)}{\int_{t_{\min}}^{t_{\max}} p(c_{rs}^{ref} = t)} \]

For simplicity, the path choice probability by Eq. (14) can be approximated as a discrete form.

\[ p(r_{rs}^{ref} = \pi^{rs}) = \sum_{t_{\min}}^{t_{\max}} \frac{p(r_{rs}^{ref} = \pi^{rs}) \cdot p(c_{rs}^{ref} = t)}{\sum_{t_{\min}}^{t_{\max}} p(c_{rs}^{ref} = t)} \]

where \( t_{\min} \) and \( t_{\max} \) represent a temporal framework for route choice problem and they are set as \( t_{\min} = 5 \) minutes and \( t_{\max} = 120 \) minutes, respectively. In other words, drivers only consider their reference travel time within this time frame. The calculation of the term \( p(r_{rs}^{ref} = \pi^{rs}) \) is the same to the solution in the previous section; while the value of the term \( p(c_{rs}^{ref} = t) \) is taken from the probability distribution function of the perceived OD travel time found in the survey, which is an Erlang distribution and is given by

\[ p(c_{rs}^{ref} = t) = \frac{v(xk)^{k-1}e^{-vx}}{(k-1)!} \]

where \( v \) and \( k \) are parameters.

### 3.3 Measure of the Model Performance

For the performance evaluation, the proposed model was compared to the true success or hit ratio (TSR) shown in Eq. (17) and the Multinomial Logit (MNL) model. TSR has the nearly same concept with Eq. (11) but the success and the fail of arrival by the reference time is not considered in Eq. (11).

\[ \text{TSR}_k = \frac{N_{rs}^{total}}{N_{total}} \]

where, \( N_{total} \) is the total number of events for a comparison.
3.4 Survey design for perceived travel time

In order to identify the probability distribution function of the reference time, a survey was conducted on the test route (Suvarnabhumi Airport to Victory Monument in Bangkok, Thailand). Respondents were asked regarding their expected travel time for the OD pair and they gave answers as an interval. The interval of travel time was standardized and plotted in a histogram. The number of bins, $k$ is assumed based on a criterion in Hahn and Shapiro (1967). The chi-square goodness-of-fit test is used for a statistical test. The candidates for the perceived OD travel time distribution are normal, lognormal, gamma, Erlang and beta. The appropriate probability distribution for the perceived OD travel time is used in the calculation of the route choice probability. The perceived OD pair travel time was taken from the survey conducted. In the interview, the perceived OD travel time responses were in ranges. In increments of 1 minute; the data were plotted and the histogram of the OD travel time was obtained as shown in Figure 4 (a). The length of range can be different from one person to another, so the weight of each response for the range is normalized. From the data gathered, a chi-square goodness-of-fit test was conducted. From the test results, the Erlang distribution best represents the perceived travel time distribution function, as shown in Figure 4 (b).

![Histogram](a)  ![Erlang Distribution](b)

Figure 4 Perceived OD Travel Time Distribution

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Parameters</th>
<th>Chi-squared value (w)</th>
</tr>
</thead>
</table>
| 1. Normal    | $\mu = 61.149$  
               | $\sigma = 11.343$ | 158.185 > 75.624; Rejected |
| 2. Lognormal | $\lambda = 3.596$  
               | $\zeta = 0.967$   | 50.808 < 75.624; OK          |
| 3. Gamma     | $\alpha = 29.059$ 
               | $\beta = 0.475$; $1/\beta = 2.104$ | 660.701 > 75.624; Rejected |
| 4. Erlang    | $\alpha = 15$   
               | $\beta = 0.245$; $1/\beta = 4.077$ | 31.526 < 75.624; OK          |
| 5. Beta      | $\alpha = 2.0$  
               | $\beta = 3.5$    | 44.224 < 75.624; OK          |
4. TEST RESULTS

4.1 Test area and path enumeration
An OD pair in Bangkok, Thailand (see Figure 5) is selected as a test network. The origin-destination (OD) pair chosen is from the Suvarnabhumi Airport to Victory Monument. The network consists of 314 links and 102 nodes. The travel cost is taken from the average travel speed and length of each link. Using the hierarchical path enumeration model, five sufficiently independent paths are found as shown in Figure 5.

In the final path set, TTTR of the longest path is 2.003 with respect to the shortest path; which means that the worst path is 200.3% longer than the shortest one. Similarly, TTTR of the second, third, and fourth paths are 1.191, 1.284, and 1.448, respectively. In the case of the RS, most path pairs have a trivial value. The biggest one is between path 3 and 1 in which the RS value is 0.041; which implies that only 4.1% of the total distance of path 3 overlaps with path 1. The second biggest RS is 0.039 which occurred between paths 5 and 4. From these results, all five paths are sufficiently independent.

4.2 Travel time generation for paths
Using Monte Carlo simulation technique, the travel time of the enumerated path links are generated. In the simulation, the standard deviation of link travel time is assumed according to the hierarchy of links. The standard deviation is defined as 0.75 and 1.5 times the travel time for major arterial highways and minor network links, respectively. The standard deviation values are arbitrarily assumed but the travel time on highway is assumed more stable than that on the minor links. In every trial, the sample path travel time is taken as the summation of the link travel times along the path. After then, the probability distribution of path travel time is assumed to be normal and a chi-square goodness-of-fit test was conducted. The Chi-square value ($w$) for the test is ($Chi_{(0.05,6)} \leq 23.685$). In the test, a normal distribution fits well for the five paths. The $w$ value of path 2 is the lowest (6.175) and the biggest occurs on path 3 (15.514). Figure 6 depicts the normal distribution of path travel time of the five paths. Path 1 shows the smallest average travel time (22.66 minutes). In addition, the SD of travel time on path 1 is also the smallest (6.831). The second best path is path 3. The average and SD of travel time on path 3 are 28.650 (minutes) and 8.758, respectively. Contrarily, Path 5 gives the worst travel time with an average travel time of 60.842 and SD of 13.23 minutes. Therefore, path 5 has the least possibility to become the shortest path.
4.3 Calculation of path choice probability

4.3.1 Route choice probability with travel time budget
If a traveler has a travel time budget, then it defines the maximum acceptable arrival time of the traveler. In this study, the maximum acceptable value is denoted as “reference time.” If there is a specific reference time, the probability that the traveler arrives at the destination on time is determined by the cumulative distribution function (CDF) of the path actual travel time. As expectedly, the probability of arriving on time increases as the reference time increases. If the on-time arrival probability is very low, the driver has to delay his arrival time target so as to have sufficient probability for on-time arrival.
The total number of events where the shortest travel time is less than or equal to the given reference time is set to 500, also known as the total number of successful trials in this study. The total number of trials regardless of whether the shortest path is less than or equal to the reference time is also noted to determine the feasibility of arriving at the destination denoted as the Expected Satisfaction Rate (ESR) in Figure 7. If a traveler wants to arrive at the destination within 35 minutes or less, the probability that at least one path would have such travel time is less than 100%.

Figure 8 shows the probability of being the shortest path at a specific reference time. When the reference time is very short, such as 5 minutes, paths 1, 2, and 3 have similar probabilities. As shown in Figure 7, the on-time arrival probability of the three paths are very small, so the paths show similar performance. However, the superiority of path 1 increases as the reference time is extended. In Figure 7, path 1 provides more than 60% probability for arriving on time at 25 minutes reference time. Oppositely, those of path 2 and 3 are less than 35%. The superiority keeps increasing until the reference time reaches 25~40 minutes. After then, path choice probabilities are stabilized.

![Figure 8 Path Choice Probabilities at Different Reference Time](image)

4.3.2 Route choice probability without travel time budget

In this case, we assume that the reference time is unknown but its probability distribution is available from the survey conducted. That is to say, the path choice procedure relies on a common sense for the OD travel time. Figure 9 shows the trend of the path choice probabilities with uncertain reference time. In the test, the maximum and the minimum reference time are assumed as 5 minutes and 120 minutes, respectively. From the survey, the maximum perceived travel time from Suvarnabhumi Airport to Victory Monument was known to be 120 minutes. Drastic changes in path choice probabilities are observed when the maximum acceptable travel time is relatively low; 5 to 30 minutes. After the interval, the path choice probabilities are stabilized.
Figure 9 Path choice probabilities with increasing maximum reference travel time

Figure 10 shows the path choice probability as well as the comparison to the TSR and Multinomial Logit (MNL) model results. In the case of MNL, utility coefficients are heuristically optimized based on TSR. As shown in the figure, the developed model and MNL give very close path choice probabilities to TSR. It means that the true probabilities for being the shortest path are nearly similar to the forecasted probabilities of the two models. Note that there is no IIA problem for MNL. It means the test condition is ideal for MNL model.

Therefore, the performance of the developed model is very promising. Even without any calibration, the developed model forecasts accurate probabilities for being the shortest path. It is very attractive when applying the developed model to the real-time route guidance system (RGS). If we can have daily link travel time data, then the travel time distribution can be constructed and updated automatically. After then, the probability of being the shortest path can be calculated without any calibration. The automatic process is especially attractive for on-line data collection and information provision environment.

Figure 10 Path Choice Probabilities between the Proposed Model, TSR and MNL
5. CONCLUSION

Route choice models have played an important role in solving the unmanageable urban congestion problem. This study presents a new route choice model which can consider path travel time and perceived OD travel time in a probabilistic way. Through the Monte Carlo simulation technique, the link travel times were generated and the mean and variance of path travel time are estimated. From the Chi-square test, a normal distribution is selected for path travel time distributions. Moreover, the perceived OD travel time of people is formulated by a PDF (Erlang distribution) based on the survey data. The developed model is tested with the two case scenarios: with and without travel time budget. The result was compared to the True Success Rate \( (TSR_k) \) and the deterministic Multinomial Logit (MNL) model.

There are several important academic contributions of this study. First, a route choice model for the PDF of path travel time is proposed. Previously, a discrete choice model assuming the uncertainty of path travel time has been used. For example, a logit model assumes that an error term in the utility function follows a Gumbel distribution. Oppositely, the developed model does not need any detail assumption on probabilistic characteristics of the travel time uncertainty. Instead, several probability density function are tested and the best one is selected based on the field data.

Second, the developed model can calculate route choice probabilities without calibration. Most route choice models have to be calibrated with real survey data. Conversely, the developed model does not have any coefficient to be calibrated. Even without calibration, however, the developed model gives the same route choice probabilities with MNL. It is a big advantage for practical applications.

Thirdly, the drivers’ travel time budget can be taken into account in the route choice problem. Existing route choice models and route guidance systems does not consider the arrival time demanded by drivers. In reality, however, many travelers have preferred arrival time in their travels. For example, if a driver has an appointment 50 minutes later, the driver’s reference time is 50 minutes. Hence, his request for route choice is a conditional problem in which he wants to find the shortest path for arriving at his destination within 50 minutes. The developed model also shows that the route choice probability is dependent on the reference time when the OD and path travel time is uncertain. In addition, the developed model can calculate the success rate for on time arrival given the reference time. If this methodology is embedded into the car navigation system, drivers can adjust their scheduled appointments with the information.

Fourthly, a hierarchical multi-path enumeration method is presented. All paths found by the model are sufficiently independent so the method is useful for applying MNL to a real-size network. The concept of a building two-layer network is also consistent with the cognitive process of human beings.

To further improve the proposed route choice model, it is suggested to extend the application to dynamic and stochastic traffic assignment problem. It is further recommended to apply the model to integrated transportation networks where turn prohibitions and/or multimodal conditions exist. An analytical approach in calculating the path utility choice function should also be developed in order to eliminate the calculation burden of Monte Carlo simulation.

REFERENCES


