An Integrated Scenario Tree Model for Stochastic Degradable Road Network Design Against Recurrent Congestions and Sporadic Disasters

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Abstract: This paper proposes an integrated scenario tree model that incorporates recurrent congestions and sporadic disasters into a stochastic degradable road network design problem (SDNDP). The traffic pattern of the stochastic degradable network (SDN) under the recurrent congestion condition is evaluated by probit-based stochastic user equilibrium (called SDN-SUE), whereas the system optimum is used to assess the traffic pattern of the SDN under the sporadic disaster (called SDN-SO). The proposed model determines optimal link capacity expansions that minimize the sum of the total network travel time costs of all recurrent congestion conditions plus the total expansion cost subject to the desired total network travel time constraints for evacuation purposes and the SDN-SUE and SDN-SO conditions. A solution algorithm is also developed for solving the SDNDP. Numerical examples are given to demonstrate the potential pitfall in considering the network improvement policies separately and to show the benefit from the integrated model.

Key Words: scenario tree, network design problem, stochastic degradable road network, uncertainty

1. INTRODUCTION

The major roles of any transport network, especially a road network, are to serve basic individual and community needs, to connect urban centers with non-urban areas, and to provide sufficient regional coverage. However, travel demands on a day-to-day basis (normal conditions) in the road network continually fluctuate due to various sources of uncertainty, such as traveler characteristics (e.g., occupation, income, car ownership) and temporal factors (e.g., day of week). These recurrent demand fluctuations can lead to recurrent congestion (Clark and Watling, 2005). The same network may also be exposed to non-recurrent congestion caused by sporadic events/disasters, such as traffic incidents, adverse weather conditions (e.g., rain, snow, fog), work zones (e.g., road maintenance/construction), and man-made (e.g., civil disorder, terrorism) or natural (e.g., earthquakes, tsunamis, floods) disasters. These events can cause a variation in the roadway capacities (Lo et al., 2006). Thus, apart from recurrent congestion, the road network should also be able to efficiently mitigate the impacts of such sporadic and largely unpredictable events/disasters.

In transport modeling, travel demand and roadway capacity (supply) uncertainties, which can contribute to variable network service states (from fully serviced to totally disrupted), are commonly represented by scenario tree models (see, e.g., Liu et al., 2008; Sumalee and Watling, 2003, 2008; Ukkusuri and Patil, 2009). Under supply uncertainty, Sumalee and Watling (2003, 2008) proposed the scenario tree model (called the cause-based failure tree...
model) to represent possible dependent link degradations due to several causal factors. Liu et al. (2008) developed a two-stage stochastic programming model for allocating limited retrofit resources across multiple highway bridges to improve the network robustness against natural and human-caused hazards. On the other hand, under demand uncertainty, Ukkusuri and Patil (2009) formulated a multi-period transport network design problem (NDP) that includes endogenous demand uncertainty and elasticity in the scenario tree model. Their model allows planners to delay, change, and abandon future network improvement policies. To the best of the authors’ knowledge, the scenario tree models investigated in most studies consider either demand or supply uncertainty. Few studies have explored the scenario tree model for NDP that includes both demand and supply uncertainties.

This paper proposes an integrated scenario tree model for stochastic degradable road network design problem (SDNDP) that incorporates both recurrent congested network conditions due to different levels of uncertain future demand growth and sporadic degraded network conditions (i.e. link capacity degradations) caused by sporadic events/disasters in the future. A scenario tree is used to represent possible recurrent and sporadic network conditions. The framework of stochastic road network (Luathep et al., 2010) is adapted to represent the stochastic degradable road network (SDN), in which link capacity, network state, and network performance can be degraded. The traffic pattern of the SDN under recurrent congestion is evaluated by probit-based stochastic user equilibrium (SUE) (Daganzo and Sheffi, 1977), whereas the system optimum (SO) (Wardrop, 1952) is used to assess the traffic pattern of the SDN under sporadic disaster. For brevity, these two models are called SDN-SUE and SDN-SO, respectively. Note that, under the SDN-SO condition, the paper assumes that there is an overall network manager who decides on the route that individuals should take. In addition, all travelers are well informed about network failures, traffic conditions, and alternative evacuation route(s). These two assumptions thus allow the network to be considered under full control (e.g., evacuation operations).

The proposed model determines optimal link capacity expansions that minimize the sum of the total network travel time costs of all recurrent congestions plus the total expansion cost subject to the desired total network travel time constraints for evacuation purposes and the SDN-SUE and SDN-SO conditions. The remainder of the paper is outlined as follows. Section 2 presents the framework of SDN model. Section 3 explains two travelers’ routing rationales under recurrent congestion and sporadic disaster. Section 4 formulates the SDNDP model and explains the solution algorithm. Section 5 presents two numerical examples. Finally, Section 6 concludes the paper and discusses the future research issues.

2. FRAMEWORK OF STOCHASTIC DEGRADABLE ROAD NETWORK MODEL

This section presents notions and assumptions made in this paper. The models of stochastic flow conservation and travel time-flow relationship are also given in this section.

2.1 Notations and Assumptions

A road network is represented by a directed graph $G(N,A)$ where $N$ is the set of all nodes and $A$ is the set of all $A$ links (i.e. road segments) within the network. A subset of nodes form a number of origin-destination (O-D) movements. The set of all $W$ O-D movements within the network is denoted by $W$. Let $K^w$ be the set of all non-cyclic $K^w$ paths connecting the O-D movement $w \in W$, then the path set of the network is $K = \bigcup_{w \in W} K^w$. 
The following notations are used throughout the paper unless otherwise specified. For consistency, random variables are expressed in italic capital letters whereas mean values are represented in lower-case letters.

\[ Q^w \] travel demand between O-D movement \( w \in W \);
\[ q^w \] mean travel demand between O-D movement \( w \in W \);
\[ \sigma_Q^w \] standard deviation (SD) of travel demand between O-D movement \( w \in W \);
\[ F_k^w \] traffic flow on path \( k \in K^w \);
\[ f_{k}^w \] mean traffic flow on path \( k \in K^w \);
\[ \sigma_{F,k}^w \] SD of traffic flow on path \( k \in K^w \);
\[ f \] vector of mean path flows, where
\[
 f = [f_{1,1}, \ldots, f_{K',1}, f_{1,2}, \ldots, f_{K',w}, \ldots, f_{K'',w}]^T
\];
\[ \Sigma^f \] variance-covariance matrix of path flows;
\[ X_a \] traffic flow on link \( a \in A \);
\[ x_a \] mean traffic flow on link \( a \in A \);
\[ \sigma_{X,a} \] SD of traffic flow on link \( a \in A \);
\[ x \] vector of mean link flows, where
\[
 x = [x_1, x_2, \ldots, x_a]^T
\];
\[ \Sigma^x \] variance-covariance matrix of link flows;
\[ y_a^0 \] mean of existing capacity on link \( a \in A \);
\[ y_a \] additional capacity on link \( a \in A \);
\[ \rho_a \] the expected degree of capacity degradation on link \( a \in A \);
\[ p_k^w \] probit choice probability on path \( k \in K^w \);
\[ p^w \] vector of probit choice probabilities on the paths connecting O-D movement \( w \in W \), where
\[
 p^w = [p_1^w, p_2^w, \ldots, p_k^w, \ldots, p_{K^w}]^T
\];
\[ \Delta^w \] link-path incidence matrix between the O-D movement \( w \in W \), whose components are \( \delta_{a,k}^w \), \( \delta_{a,k}^w = 1 \) if link \( a \) is used by path \( k \) connecting O-D movement \( w \in W \) and \( \delta_{a,k}^w = 0 \) otherwise;
\[ \Delta \] all link-path incidence matrices in the network, where
\[
 \Delta = [\Delta_1, \Delta_2, \ldots, \Delta^w]
\];
\[ T_a \] travel time on link \( a \in A \);
\[ t_a \] mean travel time on link \( a \in A \);
\[ t \] vector of mean link travel times, where
\[
 t = [t_1, t_2, \ldots, t_a]^T
\];
\[ c^w \] \( [K^w \times 1] \) vector of mean travel costs on the paths connecting O-D movement \( w \in W \), where
\[
 c^w = [c_1^w, c_2^w, \ldots, c_k^w, \ldots, c_{K^w}]^T
\];
\[ TT \] total travel time of the stochastic road network,
\[
 TT = \sum_{a \in A} X_a T_a
\].

The following assumptions are made throughout the paper.

**Assumption 1.** O-D travel demand \( (Q^w) \) is an independent random variable with a mean of \( q^w \) and a variance of \( \sigma_Q^w \).

**Assumption 2.** Stochastic O-D travel demand follows a normal distribution, i.e. \( Q^w \sim N\left(q^w, (\sigma_Q^w)^2\right) \). This assumption has been used in other studies (see, e.g., Asakura and Kashiwadani, 1991; Chen et al., 2003; Lam et al., 2008; Luathep et al., 2010). In addition, all O-D travel demands are positive.
2.2 Stochastic Flow Conservation

As the O-D travel demand is a normally distributed random variable (Assumption 2), the stochastic path flow, i.e. \( F^w_k = p^w_k Q^w \), also follows a normal distribution. The mean and variance of the stochastic path flow, respectively, are

\[
E\left( p^w_k Q^w \right) = p^w_k \mu^w, \quad \forall k \in K, \ w \in W, \\
\left( \sigma^w_{F,k} \right)^2 = Var\left( p^w_k Q^w \right) = \left( p^w_k \sigma^w_Q \right)^2, \quad \forall k \in K, \ w \in W,
\]

where \( E(.) \) and \( Var(.) \) denote the expected and variance operators, respectively. The covariance between two arbitrary path flows (say \( F^w_k \) and \( F^w_j \)) joining the same O-D movement \( w \in W \) is formulated by following Lam et al. (2008) as

\[
Cov\left( F^w_k, F^w_j \right) = \left( \sigma^w_Q \right)^2 p^w_k p^w_j, \quad k \neq j, \ \forall k, j \in K, \ w \in W,
\]

while the covariance of traffic flows on paths connecting different O-D pairs is zero.

Let \( \Omega_X = \{ X | X = \Delta \cdot \mathbf{F}, \mathbf{F} \geq 0, Q^w = \sum_{k \in K} F^w_k, \forall w \in W \} \) denote the closed and convex set of feasible stochastic link flow region of SDN. As the stochastic link flow is a sum of stochastic path flows \( \mathbf{X} = \Delta \cdot \mathbf{F} \), \( \mathbf{X} \) follows a multivariate normal distribution \( \mathbf{X} \sim \mathcal{N}(\mathbf{x}, \Sigma_X) \) with the mean link flow vector \( \mathbf{x} \) and the variance-covariance matrix \( \Sigma_X \), i.e.

\[
\mathbf{x} = E(\Delta \cdot \mathbf{F}) = \Delta \cdot \mathbf{f} = \sum_{w \in W} \Delta^w \cdot \mathbf{q}^w \cdot \mathbf{p}^w, \\
\Sigma_X = Var(\Delta \cdot \mathbf{F}) = \Delta \cdot \Sigma_F \cdot \Delta^T = \sum_{w \in W} \Delta^w \cdot \Sigma_F^w \cdot \Delta^w.
\]

2.3 Stochastic Travel Time-Flow Relationship

This paper assumes that a link travel time follows the Bureau of Public Roads (BPR) function \( T_a \left( x_a, y_a \right) = t^0_a + b_a \left[ x_a / \left( 1 - \rho_a \right) \left( y_a + \rho_a \right) \right]^\mu \) where \( t^0_a \) is the free-flow travel time, \( b_a \) and \( n_a \) are the parameters of the BPR function, \( y_a + \rho_a \) is the effective capacity after adding the link capacity expansion \( y_a \) (decision variable), and \( \rho_a \) denotes the expected degree of link capacity degradation \( 0 \leq \rho_a < 1 \). Following Luathep et al. (2010), if \( n_a = 4, \ \forall a \in A \) then the mean link travel time can be calculated from

\[
t_a \left( x_a, \sigma_{X,a} \right) = t^0_a + \frac{b_a}{(1 - \rho_a)^4} \left( y_a + x_a \right)^4 \left( x^4_a + 6x^2_a \sigma^2_{X,a} + 3 \sigma^4_{X,a} \right), \quad \forall a \in A.
\]

The vector of mean path travel costs is

\[
e^w = \omega \cdot (\Delta^w)^T \cdot \mathbf{t}, \ w \in W.
\]
3. TRAVELER ROUTE CHOICE MODELS

This section formulates the SDN-SUE and SDN-SO models used to evaluate the traffic pattern of the networks under recurrent and sporadic conditions, respectively.

3.1 SDN-SUE Model

In this study, the travel time (or cost) variation due to travel demand uncertainty and the travel time (or cost) perception error due to imperfect knowledge are considered separately. The study assumes that travelers in the network consider only personal travel costs as the disutility of their trips. Then, the perceived travel cost of using path $k$ is defined as

$$
\tilde{c}^w_k = c^w_k + \varepsilon^w_k, \forall k \in K^w, \ w \in W,
$$

where $c^w_k$ is the mean path travel cost, which is a component of $c^w$, and $\varepsilon^w_k$ is the perception error of the stochastic path travel cost, which is a member of the vector of travel cost perception errors on the paths connecting OD movement $w \in W$, denoted by $\varepsilon^w$.

In the probit-based SUE model, $\varepsilon^w$ is assumed to follow a multivariate normal distribution with zero mean and variance-covariance matrix $\Sigma^w$. Each stochastic component of $\Sigma^w$ is assumed to have a non-degenerate joint probability density function that is continuous and strictly positive definite. If $\Sigma^w$ is singular ($\Sigma^w$ is not strictly positive definite), then $\Sigma^w$ is non-invertible. To avoid this problem, a certain path set should be constructed in advance (Connors et al., 2007). $\Sigma^w$ is commonly calculated by using the joint distribution of link travel cost perception errors. This condition allows the correlation of travel cost perception errors on different links in the probit-based SUE model. For simplicity, the correlation is neglected and the travel cost perception error on each link is assumed to follow a normal distribution with zero mean and variance $\sigma_a^2$. Thus, each component of $\Sigma^w$ is calculated from

$$
\varepsilon^w_{k,j} = \sum_{a \in A} \delta^w_{a,k} \delta^w_{a,j} \sigma_a^2, \forall k, j \in K^w, \ w \in W,
$$

where $\varepsilon^w_{k,j}$ is the covariance of travel cost perception error between paths $k$ and $j$. If $k = j$, then $\varepsilon^w_{k,j}$ is equivalent to $\varepsilon^w_k$, as defined in Equation (8), hence $\varepsilon^w_{k} = \sum_{a \in A} \delta^w_{a,k} \sigma_a^2, \forall k \in K^w, \ w \in W$. In other words, $\varepsilon^w_k$ is the diagonal element of $\Sigma^w$.

Travelers on the $w^{th}$ O-D movement choosing the $k^{th}$ path are those who perceive the travel cost on this path as the minimum from all possible paths on this O-D movement, given the current mean path cost vector $c^w$. The corresponding path choice probability is

$$
p^w_k = \Pr[\tilde{c}^w_k \leq \tilde{c}^w_j, \forall j \in K^w | c^w] = \Pr\left[\begin{array}{l}
\tilde{c}^w_k \left( t_a(x_a, \sigma_{x,a}, y_a) \right) + \varepsilon^w_k \leq \\
\tilde{c}^w_j \left( t_a(x_a, \sigma_{x,a}, y_a) \right) + \varepsilon^w_j, \ \forall j \in K^w
\end{array}\right],
$$

where $\Pr[.]$ denotes the probability.
There is no closed form for solving the probit path choice probability as defined in Equation (10). However, several methods can be used to calculate this probability, e.g., numerical integration, simulation technique, or analytical approximation.

At the SDN-SUE condition, no traveler can reduce the perceived travel cost by unilaterally changing his/her path. Following Sheffi (1985), the SDN-SUE condition is formulated as

$$\mathbf{x}^{SUE} = \sum_{w \in \mathcal{W}} \Delta^w \cdot q^w \cdot p^w \left( \mathbf{e}^w \left( t(\mathbf{x}^{SUE}, \sigma^{SUE}_x, y) \right) \right). \quad (11)$$

In Equation (11), $\mathbf{e}^w$ can be derived from the link travel time vector $\mathbf{t}$ and further expressed as a function of the link flow vector $\mathbf{x}^{SUE} \in \Omega_s = \{ \mathbf{x} \mid \mathbf{x} = \Delta \cdot \mathbf{f}, \mathbf{f} \geq \mathbf{0}, q^w = \sum_{k \in \mathcal{K}} f^w_k, \forall w \in \mathcal{W} \}$. Thus, Equation (11) is the fixed-point (FP) problem. Several methods for solving the FP problem can be found, e.g., in Sheffi (1985).

### 3.2 SDN-SO Model

Under a sporadic condition, several road network managers with different risk-taking attitudes can be considered. For illustrative purposes, this study focuses on the risk-neutral manager. However, the proposed model can be relaxed to other attitudes (e.g. risk-prone or risk-averse). For the degraded road network due to such a event/disaster, the network manager aims to assign the traffic so as to minimize the expected total network travel time (or cost). Following Luathep et al. (2010), the SDN-SO condition can be formulated as the variational inequality (VI) problem. The VI problem is to find $\mathbf{x}^{SO} \in \Omega_x$ and $\mathbf{x}^{SO} \in \Omega_s$ such that

$$\nabla_x \mathbb{E}\left[ TT\left( \mathbf{x}^{SO} \right) \right] (\mathbf{x}^{SO} - \mathbf{x}^*)^T \leq 0, \forall s \in \mathbf{S}, \quad (12)$$

where $\nabla_x \mathbb{E}\left[ TT\left( \mathbf{x}^{SO} \right) \right] = \left[ \partial \mathbb{E}\left( TT\left( \mathbf{x} \right) \right) / \partial x_1, \ldots, \partial \mathbb{E}\left( TT\left( \mathbf{x} \right) \right) / \partial x_n \right]^T$ is the expected marginal total travel time vector whose elements are the partial derivatives of the expected total network travel time with respect to the mean of SDN-SO link flows, $\mathbf{x}^*$ is the extreme point vector whose elements denoted by $x^*$ are the $s^{th}$ extreme points (corner points) of the polyhedron $\Omega_x$, and $\mathbf{S}$ is the finite set of extreme points of $\Omega_x$. Following Luathep et al. (2010), for $n_a = 4$, the expected system travel time, $\mathbb{E}\left( TT\left( \mathbf{x} \right) \right)$, is defined as

$$\mathbb{E}\left( TT\left( \mathbf{x} \right) \right) = \sum_{a \in \mathcal{A}} x_a^t \left( \frac{b_a}{(1 - \rho_a)^4 (y_a^s + y_a^d)^4} \left( x_a^s + 10 x_a^3 \sigma_x^2 + 15 x_a^4 \sigma_x^4 \right) \right). \quad (13)$$

Hence, each component of $\nabla_x \mathbb{E}\left[ TT\left( \mathbf{x}^{SO} \right) \right]$ is

$$\frac{\partial \mathbb{E}\left( TT\left( \mathbf{x}^{SO} \right) \right)}{\partial x_a^t} = x_a^t + \frac{b_a}{(1 - \rho_a)^4 (y_a^s + y_a^d)^4} \left[ 5 \left( x_a^{SO} \right)^4 + 30 \left( x_a^{SO} \right)^2 \left( \sigma_x^{SO} \right)^2 + 15 \left( \sigma_x^{SO} \right)^2 \right], \quad \forall a \in \mathcal{A}. \quad (14)$$
4. PROBLEM FORMULATION AND SOLUTION ALGORITHM

This section proposes the model of stochastic degradable road network design problem (SDNDP) and develops the solution algorithm for solving the proposed model.

4.1 SDNDP Model

The scenario tree, as shown in Figure 1, is used to represent possible network states (conditions) under different levels of uncertain demand growth, causing recurrent congestion, and sporadic disasters in the future. The stochastic degradable road network is designed to cope with different recurrent congestions, denoted by $\text{SUE} = \{SUE_1, SUE_2, \ldots, SUE_g\}$ where $g$ is the index of demand growth level (e.g. low, medium, high) with the set of probabilities $\text{Pr} = \{\text{Pr}_1, \text{Pr}_2, \ldots, \text{Pr}_g\}$. The traffic pattern of the network under each recurrent congestion can be evaluated by using Equation (11). Apart from the recurrent congestions, the network should also be able to deal with future sporadic disasters that lead to different network disruptions, denoted by $\text{SO} = \{SO_1^1, SO_2^1, \ldots, SO_n^1, \ldots, SO_1^n, \ldots, SO_n^n\}$ where $n$ is the index of sporadic disaster, and $m$ is the index of network disruption (e.g., low, medium, and high). The traffic pattern of the network under each sporadic condition is assessed by Equation (12).

Figure 1 Scenario tree of the network under recurrent and sporadic conditions

The SDNDP determines the optimal link capacity expansions that minimize the sum of the total network travel time costs of all recurrent congestions plus the total expansion cost subject to the desired total network travel time constraints for evacuation purposes and the SDN-SUE and SDN-SO conditions. The SDNDP is formulated as the following mathematical programming with equilibrium constraints (MPEC) problem:
(SDNDP)
\[ \min Z = \omega \cdot \sum_{vg} \Pr_g \cdot E\left( TT\left( X^{SUE}_g \right) \right) + \gamma \cdot \mathbf{1}^T \cdot \mathbf{y} \] (15)
subject to
1) The desired total network travel time constraint
\[ E\left( TT\left( X^{SOC}_g \right) \right) \leq \text{desired total network travel time}_{m}. \] (16)
2) The SDN-SUE for each recurrent condition
\[ X^{SUE}_g = \sum_{w \in W} \Delta^w \cdot q^w \cdot p^w \left( w \left( X^{SUE}_g, \sigma^{SUE}_g, y \right) \right). \] (17)
3) The SND-SO for each sporadic condition
\[ \nabla_x E\left[ TT\left( X^{SOC}_g \right) \right] \cdot \left( X^{SOC}_g - X^g \right)^T \leq 0, \ s \in S. \] (18)
4) The feasible mean and stochastic link-flow regions
\[ \Omega = \left\{ \mathbf{x} | \mathbf{x} = \Delta \cdot \mathbf{f}, \mathbf{f} \geq 0, q^w = \sum_{k \in K^w} f^w_k, \forall w \in W \right\}, \] (19)
\[ \Omega = \left\{ \mathbf{X} | \mathbf{X} = \Delta \cdot \mathbf{F}, \mathbf{F} \geq 0, Q^w = \sum_{k \in K^w} F^w_k, \forall w \in W \right\}. \] (20)
5) The lower and upper bounds of link capacity expansion
\[ 0 \leq y_a \leq y_a^{\max}, \ s \in A, \] (21)

where \( \omega \) is the value of time, \( \gamma \) is the cost/unit link length/unit link capacity expansion, \( \mathbf{1} = [l_1, \ldots, l_s] \) is the vector of link lengths, and \( y_a^{\max} \) is the upper bound of link capacity expansion.

The objective function as defined in Equation (15) minimizes the sum of the total network travel time costs plus the total expansion cost. Constraint (16) ensures that the expected total network travel time under each sporadic condition is less than the desired value for evacuation purpose. Equation (17) determines the SDN-SUE condition whereas the SDN-SO condition is evaluated by using Equation (18). The feasible mean and stochastic link-flow regions are defined in Equations (19) and (20), respectively, whereas the bounds of \( y_a \) are defined in Equation (21).

4.2 Solution Algorithm
The proposed SDNDP model, which is classified as the MPEC problem, can be solved by various optimization methods including gradient-based and derivative-free (or meta-) heuristic algorithms. However, this study develops a solution algorithm based on the sequential quadratic program (SQP), which can be classified as the gradient-based method. The SQP solves a series of sub-problems designed to minimize a second-order (quadratic) approximation of the objective function subject to linearized constraints. In addition, the cutting constraint algorithm (CCA) proposed by Lawphongpanich and Hearn (2004) is used to avoid enumerating all extreme points in Equation (18) at once. The CCA will generate a set of necessary extreme points at each iteration. Then the generated extreme points will be used to formulate a new VI constraint (18). At each iteration, the method of successive average (MSA) (Sheffi, 1985) is used to solve the SDN-SUE condition (17) and SDN-SO condition (18). The solution algorithm can be summarized in Figure 2.
The details of solution algorithm can be described as follows:

**Step 0:** *Initialization.* Set the iteration index \( i = 0 \); find initial vectors of link capacity degradations \( y^0 \) and extreme points \( x^i(i) = [x^i_1(i), x^i_2(i), \ldots, x^i_n(i)]^T \) and set \( i = 0 \).

**Step 1:** *SDRND optimization.* Formulate the SDNDP model with \( y^0 \) and \( x^i(i) \) and solve it to obtain the current solution of link capacity expansion vector \( y(i) \), equilibrium link flow vectors \( X^{SUE}(i) \) and \( X^{SOC}(i) \).

**Step 2:** *Finding the shortest path.* Formulate the LP problem \( \min \nabla_x E\left[ TT\left( X^{SOC}\right)\right] \cdot (x^{SOC} - x^i)^T \) to obtain \( x^i \).

**Step 3:** *Convergence test.* Terminate the algorithm if \( \nabla_x E\left[ TT\left( X^{SOC}\right)\right] \cdot (x^{SOC} - x^i)^T \leq 0 \). The solution of \( y(i) \), \( X^{SUE}(i) \) and \( X^{SOC}(i) \) is optimal. Otherwise, go to Step 4.

**Step 4:** *Extreme point set updating.* Include the new set of extreme points \( x^i + 1 = x^i(i) \cup x^i \), set \( i = i + 1 \) and go to Step 1.

To reduce the computational effort in solving the SDNDP, the initial set of extreme points \( x^i(i) \) in Step 0 is obtained by solving the VI problem in Equation (18) with the equilibrium link flows of the existing network (without any expansion).
5. NUMERICAL RESULTS

In this paper, the 18-link network used in Sumalee et al. (2006) and Luathep et al. (2010) is adopted for all tests. The network is shown in Figure 3. The mean and the coefficient of variation (in brackets) of O-D travel demands are given in Table 1. The parameters of the BPR function and the link lengths are presented in Table 2. The value of time ($\omega$) is set at $1.00 per hour for the tests.

Figure 3 The test network

Table 1 Mean and coefficient of variation of OD travel demands

<table>
<thead>
<tr>
<th>Origin</th>
<th>Destination</th>
<th>1</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>1,200 (0.20)</td>
<td>-</td>
<td>800 (0.25)</td>
</tr>
<tr>
<td>5</td>
<td>1,000 (0.60)</td>
<td>-</td>
<td>-</td>
<td>1,200 (0.50)</td>
</tr>
<tr>
<td>7</td>
<td>700 (0.25)</td>
<td>1,600 (0.20)</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Note: The unit is in vehicles/hour

Table 2 Link travel time parameters for the test network

<table>
<thead>
<tr>
<th>Link</th>
<th>$t^0_a$ (hr)</th>
<th>$b_a$ (h)</th>
<th>$y^0_a$ (veh/h)</th>
<th>$l_a$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0125</td>
<td>0.0025</td>
<td>1800</td>
<td>1.0</td>
</tr>
<tr>
<td>2</td>
<td>0.0125</td>
<td>0.0025</td>
<td>1800</td>
<td>1.0</td>
</tr>
<tr>
<td>3</td>
<td>0.0917</td>
<td>0.0626</td>
<td>1100</td>
<td>5.5</td>
</tr>
<tr>
<td>4</td>
<td>0.0917</td>
<td>0.0626</td>
<td>1100</td>
<td>5.5</td>
</tr>
<tr>
<td>5</td>
<td>0.0917</td>
<td>0.0626</td>
<td>1100</td>
<td>5.5</td>
</tr>
<tr>
<td>6</td>
<td>0.0250</td>
<td>0.0171</td>
<td>1100</td>
<td>1.5</td>
</tr>
<tr>
<td>7</td>
<td>0.0750</td>
<td>0.0109</td>
<td>1100</td>
<td>6.0</td>
</tr>
<tr>
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<td>0.0626</td>
<td>1100</td>
<td>5.5</td>
</tr>
<tr>
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<td>0.0170</td>
<td>1100</td>
<td>1.5</td>
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<td>0.0108</td>
<td>1100</td>
<td>6.0</td>
</tr>
<tr>
<td>11</td>
<td>0.0250</td>
<td>0.0171</td>
<td>1100</td>
<td>1.5</td>
</tr>
<tr>
<td>12</td>
<td>0.0250</td>
<td>0.0171</td>
<td>1100</td>
<td>1.5</td>
</tr>
<tr>
<td>13</td>
<td>0.0200</td>
<td>0.0137</td>
<td>1100</td>
<td>1.2</td>
</tr>
<tr>
<td>14</td>
<td>0.0750</td>
<td>0.0109</td>
<td>1100</td>
<td>6.0</td>
</tr>
<tr>
<td>15</td>
<td>0.0750</td>
<td>0.0109</td>
<td>1100</td>
<td>6.0</td>
</tr>
<tr>
<td>16</td>
<td>0.0200</td>
<td>0.0137</td>
<td>1100</td>
<td>1.2</td>
</tr>
<tr>
<td>17</td>
<td>0.0125</td>
<td>0.0025</td>
<td>1800</td>
<td>1.0</td>
</tr>
<tr>
<td>18</td>
<td>0.0125</td>
<td>0.0025</td>
<td>1800</td>
<td>1.0</td>
</tr>
</tbody>
</table>
In the SDN-SUE model, the independent link travel time perception error is assumed as $\varepsilon_a \sim N(0, 0.3 (\sigma_a^2))$, $\forall a \in A$. The SDN-SUE and SDN-SO conditions are evaluated with the maximum number of iterations of 500 and the gap tolerance of 1E-6.

To demonstrate the applications of the proposed model and solution algorithm, two examples are investigated. The first example evaluates the performance of existing network (without any expansion) under recurrent congestions and sporadic disasters. The second example enhances the performance of the network by solving the SDNDPs under different conditions. The results of different SDNDPs are also compared and discussed.

### 5.1 Example 1: Network Evaluation

The expected total network travel time (ETT) is used to represent the network performance. For illustrative purposes, low and high levels of future demand growth are assumed with the same probabilities ($Pr_1, Pr_2 = 0.5$). The percentages of future O-D demand growth under the two levels are given in Table 3. In addition, low and high levels of network disruption (i.e. link capacity degradation) are assumed. The degrees of link capacity degradation ($\rho_a$, $\forall a \in A$) are presented in Table 4. Under these assumptions, the scenario tree for the tests can be depicted in Figure 4.

<table>
<thead>
<tr>
<th>O-D</th>
<th>Low increase</th>
<th>High increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>10%</td>
<td>50%</td>
</tr>
<tr>
<td>1-7</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>5-1</td>
<td>10%</td>
<td>50%</td>
</tr>
<tr>
<td>5-7</td>
<td>25%</td>
<td>20%</td>
</tr>
<tr>
<td>7-1</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>7-5</td>
<td>25%</td>
<td>20%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Link</th>
<th>Low capacity degradation</th>
<th>High capacity degradation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>8</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>9</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>10</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>11</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>12</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>13</td>
<td>0.4</td>
<td>0.8</td>
</tr>
<tr>
<td>14</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>15</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>16</td>
<td>0.4</td>
<td>0.8</td>
</tr>
<tr>
<td>17</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>18</td>
<td>0.1</td>
<td>0.5</td>
</tr>
</tbody>
</table>
As shown in Figure 4, the network performance (i.e. ETT) is evaluated under two conditions of demand growth (SDN-SUE1 and SDN-SUE2) and four combined conditions between two network disruptions and two demand growths (SDN-SO1, SDN-SO2, SDN-SO3, and SDN-SO4). The values of ETT under these conditions are shown in Table 5.

Table 5 Values of ETT under different network conditions

<table>
<thead>
<tr>
<th>Equilibrium condition</th>
<th>Existing network</th>
<th>SDN-SUE1</th>
<th>SDN-SUE2</th>
<th>SDN-SO1</th>
<th>SDN-SO2</th>
<th>SDN-SO3</th>
<th>SDN-SO4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUE</td>
<td>14,601</td>
<td>17,320</td>
<td>20,497</td>
<td>17,970</td>
<td>21,918</td>
<td>34,202</td>
<td>60,738</td>
</tr>
<tr>
<td>SO</td>
<td>9,938</td>
<td>11,722</td>
<td>12,679</td>
<td>13,566</td>
<td>14,185</td>
<td>26,716</td>
<td>52,214</td>
</tr>
</tbody>
</table>

Note: The unit is in vehicle hours

Table 5 shows that the values of ETT of the existing network evaluated under the SUE and SO conditions are 14,601 and 9,938 vehicle hours, respectively. The values of ETT become larger when the levels of demand growth and network disruption increase. As expected, the highest values of ETT under the two equilibrium conditions are found under SDN-SO4 condition (60,738 and 52,214 vehicle hours, respectively). In the next section, the network will be enhanced to reduce the value of ETT.

5.2 Example 2: Network Enhancement

In this section, the proposed SDNDP model is used to determine optimal link capacity expansion strategy. The desired values of total network travel time are set to be 12,000 vehicle hours for the cases of SDN-SO1 and SDN-SO2 (low level of network disruption) and 15,000 vehicle hours for the cases of SDN-SO3 and SDN-SO4 (low level of network disruption). \( \gamma \) is set at $1 million/km/(pcu/hr) and \( y_a^{\text{max}} = 3,600 - y_a^0 \), \( \forall a \in A \) is set for all tests.

The first test independently solves different SDNDPs under the six network conditions, as shown in Figure 4. These designs can be classified into the separated design approach. The test also considers the integrated design approach, which simultaneously incorporates all predefined network conditions in the SDNDP model. The results of optimal link capacity expansions and the ETT are presented in Table 6.
Table 6 shows that different SDNDPs provide different values of optimal link capacity expansion. For the case of separated design approach, the values of objective function (Z) increase when the levels of travel demand growth and network disruption increase. The values of Z are $19,051 \times 10^3$ and $22,171 \times 10^3$ for the cases of SDN-SUE$_1$ and SDN-SUE$_2$, respectively. The values of Z also increase for the cases of network disruptions, i.e. $23,971 \times 10^3$ (SDN-SO$_1$), $31,767 \times 10^3$ (SDN-SO$_2$), $34,013 \times 10^3$ (SDN-SO$_3$), and $53,678 \times 10^3$ (SDN-SO$_4$). This increasing trend of Z is similar to that of the expansion cost. The expansion costs increase from $1,861 \times 10^3$ (SDN-SUE$_1$) to $34,428 \times 10^3$ (SDN-SO$_4$). In contrast, the integrated design approach allocates the resources for the link capacity improvements to cope with all of the network conditions. The results show that the value of Z ($59,864 \times 10^3$) and the improvement cost ($42,438 \times 10^3$) are the highest as compared to the results under the separated design approach.

The second test uses the previous results of optimal link expansion from the seven design problems to calculate the values of ETT (evaluated under the SO condition) across the six network conditions again. The cross-comparison of the results is presented in Table 7.

In Table 7, the six SDNDPs under the separated design approach aim to find the optimal link expansions that satisfy the desired value of total network travel time under the condition considered only or a few conditions. For example, based on the result of SDN-SO$_4$, only the ETT value under the SDN-SO$_4$ condition satisfies the desired value (15,000 vehicle hours),

<table>
<thead>
<tr>
<th>Link no.</th>
<th>Separated design approach</th>
<th>Integrated design approach</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SDN-SUE$_1$</td>
<td>SDN-SUE$_2$</td>
</tr>
<tr>
<td>1</td>
<td>318</td>
<td>319</td>
</tr>
<tr>
<td>2</td>
<td>611</td>
<td>613</td>
</tr>
<tr>
<td>3</td>
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<tr>
<td>16</td>
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<td>252</td>
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<tr>
<td>17</td>
<td>256</td>
<td>257</td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Z ($\times 10^3$)</th>
<th>19,051</th>
<th>22,171</th>
<th>23,971</th>
<th>31,767</th>
<th>34,013</th>
<th>53,678</th>
<th>59,864</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expansion cost ($\times 10^3$)</td>
<td>1,861</td>
<td>1,872</td>
<td>6,594</td>
<td>12,237</td>
<td>15,540</td>
<td>34,428</td>
<td>42,438</td>
</tr>
</tbody>
</table>
the rest of the SDN-SO conditions violate the desired values. The results of the other SDN-SO design problems are in a similar manner. For the cases of SDN-SUE\textsubscript{1} and SDN-SUE\textsubscript{2}, the ETT values under all SDN-SO conditions violate the desired values because the desired ETT constraints are not considered in the design problems for these cases. In contrast, all ETT values obtained from the integrated design are less than or equal to the desired values, i.e. 10,135 vehicle hours (SDN-SO\textsubscript{1}), 11,983 vehicle hours (SDN-SO\textsubscript{2}), 12,094 vehicle hours (SDN-SO\textsubscript{3}), and 15,000 vehicle hours (SDN-SO\textsubscript{4}). With respect to the minimum value of ETT under each network condition (last column), the ETT values of the integrated design under all conditions are not the minimum. However, the ETT value under the SDN-SO\textsubscript{2} (11,988 vehicle hours) almost reaches the minimum (11,983 vehicle hours). The results can be implied that although the integrated design approach cannot provide the minimum ETT as compared to the separated design approach, the integrated design approach achieves all desired ETT constraints for emergency situations (whereas the separated design cannot).

Table 7 Cross-comparison of expected total travel times under the six network conditions evaluated from the optimal solutions to the seven designs

<table>
<thead>
<tr>
<th>Design Condition</th>
<th>Separated design approach</th>
<th>Integrated design approach</th>
<th>Min. ETT under each network condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDN-SUE\textsubscript{1}</td>
<td>10,882</td>
<td>10,872</td>
<td>10,005</td>
</tr>
<tr>
<td>SDN-SUE\textsubscript{2}</td>
<td>12,426</td>
<td>12,425</td>
<td>11,983</td>
</tr>
<tr>
<td>SDN-SO\textsubscript{1}</td>
<td>12,054</td>
<td>12,022</td>
<td>10,569</td>
</tr>
<tr>
<td>SDN-SO\textsubscript{2}</td>
<td>13,845</td>
<td>13,843</td>
<td>11,983</td>
</tr>
<tr>
<td>SDN-SO\textsubscript{3}</td>
<td>24,546</td>
<td>24,530</td>
<td>13,118</td>
</tr>
<tr>
<td>SDN-SO\textsubscript{4}</td>
<td>48,238</td>
<td>48,212</td>
<td>39,989</td>
</tr>
</tbody>
</table>

Note: The values in brackets are the desired values of the total network travel time for evacuation purpose. The total travel time that exceeds the desired value is expressed by the bold letters.

The final test examines the sensitivity of different probabilities of demand growth (Pr\textsubscript{1} and Pr\textsubscript{2}) to the link capacity expansion patterns. As shown in Table 8, the patterns of the expanded links from different cases are similar. However, different cases provide different amounts of optimal link capacity improvements. The improvement costs are from $41,954x10^3 to $44,531 x10^3. The trend of Z continuously increases when the value of Pr\textsubscript{1} increases, i.e. from $58,846 x10^3 (Pr\textsubscript{1}=1.0) to $62,986x10^3 (Pr\textsubscript{1}=0.0). Note that the ETT values under all SDN-SO conditions satisfy the desired values.
6. CONCLUSION AND DISCUSSION

This paper proposed the integrated scenario tree model for SDNDP, which simultaneously considered both recurrent network congestions (due to different levels of uncertain future demand growth) and sporadic network disruptions (caused by sporadic disasters). Following the framework of stochastic degradable road network (SDN), the SDN-SUE was used to evaluate stochastic traffic pattern in a normal network under recurrent congestion, whereas the SDN-SO was used to assess the traffic pattern in a degraded network under a sporadic disaster. The scenario tree was used to represent a set of different network conditions under uncertain demand growth and sporadic disaster. The integrated scenario tree model for SDNDP was then proposed. The solution algorithm based on the SQP gradient-based method and the CCA was also developed for solving the proposed model.

The proposed model and solution algorithm were tested with a hypothetical network. The first test evaluated the values of ETT of the existing network (without any network improvement) under different predefined conditions. The second test determined the optimal link capacity expansion under the separated and integrated design approaches. The results showed the pitfalls of the separated design approach in that, as the design focuses on the network condition considered only, the values of ETT under other conditions cannot satisfy the desired constraints. Although the integrated design approach requires a larger amount of improvement cost as compared to the separated design approach, the ETT values under all network disruption conditions achieve the desired constraints.
The proposed model can be considered as a single-stage NDP. Future studies should address the shortcomings of this NDP by developing a multi-stage NDP capable of making flexible decisions (e.g., delay, change, abandonment) over multiple time periods. Facility and demand-supply balance constraints could also be considered in the case of sporadic disasters. Real-world case studies and the model validation will be considered in our further work.

REFERENCES


