Dynamic User Equilibrium Convergence in TRANSIMS

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Abstract: This paper presents a dynamic user equilibrium process in TRANSIMS. While TRANSIMS is characterized by an activity-based travel demand model, it can also be used as a trip-based model by using origin-destination trip data with departure patterns. The main foci of this paper are to investigate TRANSIMS’s dynamic traffic assignment characteristics and to evaluate its convergence property associated with its traffic assignment methods. In a test network, various postulated equilibrium process scenarios are experimented and their convergence properties are compared. Dynamic user equilibrium convergence properties obtained from day-to-day evolutional approach are compared in a highway work zone case. Various dynamic user equilibrium analyses provide intuitions on model convergence and travelers’ behavioral characteristics.

Key Words: TRANSIMS; Dynamic traffic assignment; Dynamic user equilibrium; Day-to-day Evolution

1. INTRODUCTION

While the conventional four-step modeling approach is dominant for travel demand analyses, many inherent weaknesses have been pointed out by many studies (Transportation Research Board, 2007). One of the main criticisms is its inability to represent dynamic conditions for the transportation system. More specifically, the four-step process is unable to produce estimates of time-specific volumes or speeds on routes. There have been two major activities to overcome these weaknesses in the conventional four-step modeling approach.

As an effort to include time dimension in travel demand analysis, there have been ample studies on dynamic traffic assignment (DTA) since Merchant and Nemhauser (1978) formulated the first DTA model as a mathematical optimization problem for a dynamic system optimal route choice model. While analytical dynamic traffic assignment problems (Ran et al., 1996) have been widely studied, more attention has been paid to simulation-based DTA. With Federal Highway Administration (FHWA)’s support, DYNASMART and DynaMIT have been developed as tools for dynamic traffic assignment. These DTA models were aimed to deploy for large-scale real-time traffic state estimation and planning applications (Mahmassani, 2001). DTA has been appealing to researchers and practitioners thanks to its ability to capture traffic dynamics with
time-dependent demand; however, obtaining proper time-dependent demand and its model convergence and solution uniqueness have been questioned in applying DTA to practical applications. As for DTA model convergence, the total amount of shifted trip or the relative gap between consecutive iterations has been used. In simulation-based DTA, the method of average success (MSA) has been applied as a solution technique for user equilibrium state (Peeta and Mahmassani, 1995; Sbayti et al., 2007; Florian and Mahut, 2008). Recently Chiu and Bustillos (2009) have proposed a gap function vehicle-based procedure and showed its outperformance over MSA. Another effort to overcome weaknesses led to the development of TRANSIMS (TRansportation ANalysis and SIMulation System) as a part of FHWA’s Travel Model Improvement Program (TMIP). While TRANSIMS is characterized by activity-based travel analysis, its traffic assignment module incorporates DTA through its routing and microscopic simulation modules. Like other DTA models, TRANSIMS heuristically computes near equilibrium solutions using an iterative routing process. The traffic assignment in TRANSIMS is completely pre-trip-based and iterates until less than certain percentage of traveler can improve their total travel time by changing paths.

The objectives of this paper are to explore properties of dynamic user equilibrium in TRANSIMS and to compare its convergence with the relative gap typically used in other DTA. TRANSIMS employs a network stabilization procedure before implementing the user equilibrium process. This study specifically investigates effectiveness of these procedures and proposes ways of improving model convergence and computational efficiency. This study also compares such DTA models and dynamic user equilibrium (DUE) states from different perspectives. This paper first discusses on dynamic traffic assignment and user equilibrium. It is followed by user equilibrium process in TRANSIMS with an introduction to traffic assignment modules in TRANSIMS. Later part of the paper includes various experiments associated with TRANSIMS’s equilibrium processes, DTA, and day-to-day evolutional approaches.

2. DYNAMIC TRAFFIC ASSIGNMENT AND USER EQUILIBRIUM

2.1. Dynamic User Equilibrium

As opposed to conventional static traffic assignment models, dynamic traffic assignment problem have been evolved from analytical DTA to simulation-based DTA. Unlike analytical DTA models, the simulation-based models treat vehicles as individual vehicles or group of vehicles. Such simulation-based DTA models can be categorized into two: mesoscopic model and microscopic model. While many mesoscopic models have been used to seek dynamic user equilibrium (DUE), microscopic models, which move vehicles according to car-following and lane-changing models, have widely been applied to traffic operation analysis rather than seeking DUE. Recently many microscopic DTA simulation models are capable of seeking DUE through a hybrid approach with a small microsimulation area. (Liu et al., 2005)

The dynamic user equilibrium (DUE) problem is a dynamic extension of Wardrop’s 1st principle (1952), which leads to a static user equilibrium problem. A solution of the problem can also be called a travel-time-based ideal dynamic user-optimal state as defined in Ran and Boyce (1996). The dynamic user equilibrium conditions are defined as follows:

$$\eta_p^r (t) - \pi^r (t) \geq 0 \quad \forall p, r, s_i$$  

(1)
\[
\begin{align*}
  f_p^r(t) \cdot [\eta_p^r(t) - \pi^s(t)] &= 0 \quad \forall p, r, s; \\
  f_p^r(t) &\geq 0 \quad \forall p, r, s;
\end{align*}
\]  

(2) \hspace{5cm} (3)

denoting \( f_p^r(t) \) as path flow from origin \( r \) to destination \( s \) over path \( p \) at time \( t \), \( \eta_p^r(t) \) as the actual travel time, and \( \pi^s(t) \) as the minimal actual travel time.

In DUE, travelers are assumed to make their decisions before departure with their perfect knowledge on network travel time. Decision variables are individual traveler’s paths and the objective is to minimize each traveler’s travel time. Regardless of model types, the solution algorithm for DUE can be described as shown in FIGURE 1.

![FIGURE 1 General Structure of DUE Solution Algorithm](image)

**2.2 Day-to-day Evolution Approach and Equilibrium**

Dynamic user equilibrium also can be analyzed from a travelers’ day-to-day behavioral evolution. Horowitz (1984) first developed a drivers’ information acquisition process through their own experiences from stochastic equilibrium, and Ben-Akiva et al. (1984) and Cascetta (1989) proposed a stochastic process approach for analyzing day-to-day dynamics in a transportation network. The day-to-day dynamics approach in Mahmassani and Chang (1986) considered the choice of departure time and route according to the schedule delay governed by traveler’s daily learning process. Oh et al. (2003) applied this approach to investigate the effect of less-equilibrated data on model estimation by incorporating a nested logit model in the process.

Unlike analytical equilibrium models, the day-to-day dynamic framework enables the analysis of traveler choice changes over a time horizon based on traveler’s updated knowledge. Previous studies (Horowitz, 1984; Nakayama et al., 1999; Polak and Hazelton, 1989) found the drivers’ route choice could be affected by their experiences in early times. Srinivasan (2003) considered disutility of changing path to describe route choice behavior, and Kim et al. (2009) investigated traffic patterns when the user equilibrium assumptions are relaxed employing a day-to-day evolutional approach. It would also be of interest to compare the DUE with the day-to-day evolutional approach in achieving equilibrium.
3. TRANSIMS EQUILIBRIUM PROCESS

3.1 Overview of TRANSIMS

TRANSIMS is an agent-based travel simulation system designed to meet the state departments of transportation and Metropolitan Planning Organizations' (MPOs') need for more accurate and more sensitive travel forecasts for transportation planning and emissions analysis. By employing advanced computational and analytical techniques, it creates an integrated environment for regional transportation system analysis. TRANSIMS is different from the conventional travel demand forecasting models in its underlying concepts and structure. These differences include a consistent and continuous representation of time; a detailed representation of persons and households; time-dependent routing; and a person-based microsimulator.

TRANSIMS includes following four primary modules:

- Population synthesizer,
- Activity generator,
- Route planner, and
- Traffic microsimulator.

Using these components, TRANSIMS estimates activities for individuals and households, plans trips satisfying those activities, assigns trips to routes, and creates a microsimulation of all persons, vehicles and resulting traffic on modeled transport systems in given study area. Despite strengths in TRANSIMS, its applications in practice are relatively less than other software packages mainly because of its comprehensive data needs. However, while the original activity and tour-based method characterizes TRANSIMS strengths, it is also possible to apply a trip-based approach for easiness of practical application as a stepping stone to full implantation of activity-based approach.

Since developed at the Loa Alamos National Laboratory in 1990’s, TRANSIMS has been tested in many places including Texas, Portland, Washington D.C., Chicago, Detroit, etc. Studies have explored TRANSIMS characteristics in its flow model and dynamic user equilibrium, and applied to dynamic value pricing and a metropolitan planning organization’s model (Rilett, 2001; Jeihani et al., 2006; Lee et al., 2003; Lawe et al., 2009; Oh et al., 2011). Recently many case studies are being carried out with supports from the Federal Highway Administration, and latest TRANSIMS source codes along with test data and user manuals are available through a website (http://transims-opensource.net/).

3.2 Traffic Assignment in TRANSIMS

TRANSIMS consists of two main modules for traffic assignment, Router and Microsimulator, and supplementary modules for assisting travel plan updates. The Router module is to generate travel plans for each person in each household. The plans generated by Router include the time and duration of each activity and travel path between activities. The Router module develops time-dependent shortest paths among activity-locations using information on network, travel demand by time of day, time-dependent link travel times. Its capability computing shortest path on a multimodal network is a unique feature among other packages. The Router module can be regarded as the path flow generation routine in FIGURE 3 although it generates individual traveler’s path rather than path flow.
The Microsimulator module simulates persons and vehicles in a multimodal transportation network and generates performance statistics, track individual travelers, and summarize events. TRANSIMS Microsimulator uses cellular automata logic for moving vehicles and all movements within the network are processed as cells per time step. The simulation is carried out in discrete intervals of one second or less over the course of a day. In each time step, a vehicle in the network accelerates, decelerates, changes lanes, or stops based on the behavior of nearby vehicles and of traffic controls. An iterative feedback process is used in updating link travel times and adjusting travel plans. The feedback process is assisted by supporting modules, such as PlanSelect, PlanMerge, and PlanCompare. Their functions are briefly described below.

- **PlanSelect**: selects households for re-routing using four basic criteria: Volume-to-Capacity Ratio, Time of Day, Select Link or Node, Travel Time Difference
- **PlanMerge**: merges the re-routed plans with the full plan set
- **PlanCompare**: compares the travel time for each traveler with the travel time stored in the previous plan file

### 3.3 Network Stabilization

Network stabilization is a unique procedure in TRANSIMS. In this procedure as shown in FIGURE 2, households (travelers) for rerouting are selected through the PlanSelect module based on their travel time differences between before and after the network updated. The Router module reroutes the selected households with the updated network condition, and then their new travel plans are updated for use in Microsimulator.

The criteria for selecting households are typically based on travel time or the percentage of travel time difference. The traveler is chosen to be possibly rerouted if his/her travel time difference is greater than the specified indifference band. Among those, only user-specified percentage of households are randomly chosen for rerouting. The Router in the stabilization process is computationally fast than that in user equilibrium because only selected travelers are rerouted unlike in user equilibrium.

### 3.4 User Equilibrium

In the user equilibrium process in TRANSIMS, all households are rerouted with the updated network condition and their travel times are compared with those in the previous iteration as shown in FIGURE 3. This rerouting process is regarded as a direction finding step in DUE algorithms as the time-dependent shortest paths are recomputed for the rerouting plan. Then, the new travel plans for the next microsimulation iteration are determined using the PlanMerge module. The plan merge is similar to the step-size determination in DUE algorithms.

The equilibrium process in TRANSIMS differs from those in other DTAs. As shown in equation (4) Typical DUE models generate auxiliary path flow \((f_{prs}(t))\) by temporarily fixing each link cost at each time interval \((t)\) and combines auxiliary path flows with the currently path flows \((f_{prs}(t))\) using the method of successive average (MSA) where the step size \(\theta = 1/n\) to determine the path flows for the next iteration \((f_{prs}(t))\). However, TRANSIMS deals with individual travelers rather than path flow between its origin and destination. As such, the TRANSIMS’s equilibrium process redevelops travelers’ travel plans and compares them with the previous plans. Through the comparison, travel plans with bigger difference are selected based on a set of predefined selection criteria, and then a given percentage of the selected are
updated to the new travel plan for the next iteration. Accordingly, these parameters in the
criteria are determinant factors for model convergence.

\[ f_{prs}^{n+1}(t) = (1 - \theta \cdot f_{prs}^n(t) + \theta \cdot f_{prs}^n(t)) \quad \forall p,r,s,t; \] (4)

FIGURE 2 Network Stabilization Procedure in TRANSIMS

FIGURE 3 User Equilibrium Procedure in TRANSIMS

In DUE, the relative gap is typically used as a convergence criterion. The relative gap is
defined as in equation (5):

In DUE, the relative gap is typically used as a convergence criterion.
where $\eta^t_{prs}$ and $\pi^t_{prs}$ are the actual travel time and the minimum travel time from origin r, to destination s through path p departing at time $\tau$. Theoretically, perfect equilibrium is achieved when $RG$ equals to 0 although it is hard to obtain such equilibrium.

The equilibrium process differs from the network stabilization process in that the Router in the equilibrium process reroutes all travelers as opposed to the Router in the network stabilization rerouting only chose travelers. Accordingly, the equilibrium process requires heavier computation than the stabilization process.

4. CASE STUDY

4.1 Test Network

This study uses the Kalamazoo area transportation network that consists of 402 traffic analysis zones covering the whole Kalamazoo County with a population of 200,000. The area includes two major freeways, I-94 and US-131, the Kalamazoo/Battle Creek Airport, and many schools including Western Michigan University. The Kalamazoo travel demand model was developed by the Michigan Department of Transportation using the TransCAD package and is being used for the analysis of long-range transportation planning. FIGURE 4-(a) depicts the Kalamazoo network, and FIGURE 4-(b) is a subarea extracted from the Kalamazoo network for further analysis.

In this study, trip-based TRANSIMS model was developed to investigate the convergence properties in TRANSIMS’s traffic assignment module. The network and the origin-destination demand were converted to input format for TRANSIMS, using the GIS-based conversion tool developed through the TransCAD GISDK. After the data conversion and arrangement, the transportation network, time-dependent OD demands, and traffic control systems were developed using TRANSIMS complementary modules, called TransimsNet, ConvertTrip, and IntControl. The processed TRANSIMS network includes 716 nodes, 1,070 links, 4,634 activity locations and a total of 865,563 trips during a 24-hour period. A travel departure pattern used in this study is depicted in FIGURE 5. The extracted subarea includes 208 links and 703 activity locations, and a total of 344,471 trips.
FIGURE 4 Kalamazoo Transportation Network

(a) Kalamazoo Network

(a) Subarea Network
4.2 Model Convergence Performance

TRANSIMS network stabilization and user equilibrium processes require a plan selection criterion. For this initial investigation, the criterion was set to choose travelers whose travel time difference is more than 10% and 2 minutes. This criterion is applied to both PlanSelect for stabilization as in FIGURE 2 and PlanCompare for user equilibrium as in FIGURE 3.

In order to test convergence properties associated with the percentage of rerouting, four scenarios were prepared as shown in TABLE 1. Two parameters, selection percentage and maximum percentage, were varied in each scenario. The new travel plans are developed as follows:

1. Find those whose travel time difference is greater than the section criteria
2. Select up to the selection percentage among them.
3. If the selected is more than the maximum percentage of the total traveler, then select only up to the maximum percentage.

<table>
<thead>
<tr>
<th>TABLE 1 Analysis Scenarios and Computation Time</th>
</tr>
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<tbody>
<tr>
<td>Plan</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Selection Percentage (%)</td>
</tr>
<tr>
<td>Maximum Percentage (%)</td>
</tr>
<tr>
<td>Computational Time per Iteration</td>
</tr>
<tr>
<td>Microsimulation</td>
</tr>
<tr>
<td>Total</td>
</tr>
<tr>
<td>Note) Selection criterion of 10% and 2 minutes travel time difference applied.</td>
</tr>
</tbody>
</table>

While the first three scenarios were implemented without microsimulation stabilizations, the last scenario included 12 iterations of microsimulation stabilizations. It was to investigate the
effectiveness of the stabilization process. In the third scenario, the method of successive average (MSA) was applied to reduce the travelers to reroute as the iteration goes. However, the MSA was not fully applied until 12th iteration because the percentage selected by the selection criterion was less than the MSA rates. Accordingly, the percentages of traveler selected for rerouting for both UE_2 and UE_3 in FIGURE 8-(a) were the same until 12th iteration. However, after 12th iteration, UE_3 with MSA showed a little better performance than UE_2. It implies that MSA forces to converge at the later iterations by updating less number of travelers than the UE_2 case.

In this comparison, UE with 100% selection percentage (UE_2 and UE_3) outperformed. UE_1’s convergence was slow as less number of travelers was rerouted at the next iteration by randomly selecting 50% among travelers with problem but up to 10% of the total travelers. For the cases of UE_1 and Stabilization+UE1, actual percentage of travelers rerouted was less than 10% due to the maximum percentage. The maximum percentage was set to avoid possible oscillations by rerouting too many travelers, but it made slow convergence in our case.

An interesting finding was that the stabilization process well behaves in its equilibrium process. As explained in FIGURE 2, the stabilization process does not need to reroute all travelers. Rather, it identifies traveler to reroute by updating the network condition and reroutes only those fixing other travelers. Although its convergence speed is a little slower than UE_2 and UE3 and its level of equilibrium moves up a little when checking its equilibrium status later with PlanCompare, it reduces Router computational burden by rerouting only selected travelers and the last convergence was equivalent to those. Therefore, using stabilization at the early iterations seems to effectively impact on the model convergence.

In the computation time comparison in TABLE 1, the case with stabilization was faster than the others. It was mainly because of the Router’s computational time in the stabilization process. Because the stabilization process reroutes only selected travelers, its computational burden is far less than those of other cases.

![Selected Traveler for Rerouting](image-url)
In addition to investigation of the travelers for rerouting, the relative gap typically used in DUE was computed for all travelers based on traffic analysis zones for comparison purpose. As shown in FIGURE 6-(b), changes in the relative gap were similar to the percentage selected for rerouting. However, the relative gap values for the cases of UE_1 and Stabilization+UE1 were lower than the others. The relative gap in the Stabilization+UE1 showed the least among others, but it does not mean that the convergence was the best. It is rather because the selected for actual rerouting was less than other cases. In other words, the smaller updates naturally led to smaller values of relative gap. Although the relative gap and the percentage selected for rerouting are highly correlated as in FIGURE 7, the relative gap may mislead its interpretation.
4.3 Convergence Performance by Departure Time
In our analysis, a 24-hour simulation was implemented. The convergence performance reported was all aggregated measures, so the 24-hour convergence was broken down into hourly departure pattern to see the pattern associated with the level of congestion. FIGURE 8 depicts changes in hourly relative gaps during a 24-hour period. In general, the relative gaps decrease as the iteration goes. The relative gap clearly shows the AM and PM peaks. It implies that the convergence may be hard to reach in congested networks.

![FIGURE 8 Relative Gap Pattern by Departure Time](image)

4.4. Microsimulation Stabilization in the Subarea Network
As discussed in FIGURE 6, the microsimulation stabilization procedure converges fairly well. In order to confirm its performance, both the microsimulation stabilization and user equilibrium procedures were implemented with the same selection criterion (a selection percentage of 100% and a maximum percentage of 100%) in the subarea network as described in FIGURE 4-(b). As shown in FIGURE 9, the results in both the selected for rerouting and the relative gap were consistent with the case in the Kalamazoo network.

![Selected for Rerouting in the Subarea Network](image)
4.5 Day-to-day Evolution and User Equilibrium for a Highway Work Zone

As discussed in previous section, DUE can also be achieved through a day-to-day evolutional approach. To compare DUE with the network equilibrium from day-to-day evolution, a highway work zone case was investigated. Before the highway construction, the network condition is near user equilibrium state; however the highway work zone breaks the equilibrium and drivers begin seeking another user equilibrium based on their day-to-day experience. In this analysis, one of the major arterials was assumed to be closed. Two different methods were applied. The first case is when the day-to-day evolutional approach is applied, and the second case is when DUE is applied.

To implement the day-to-day evolutions, first, the travelers who used the link were selected and rerouted, and then all travelers in the network were assumed to seek their optimal routes. In this case, the day-to-day evolution starts from the equilibrium state and travelers begin switching their routes because of the highway closure. It is assumed that up to 50% among those whose travel time difference is more than 2 minutes and 10% of their travel time are rerouting each day. The second case is seeking new DUE regardless of previous equilibrium achieved without highway link closure. In this case, a new equilibrium is sought after closing the highway link. For a faster convergence, all travelers who needed were assumed to reroute.

FIGURE 10 shows model convergence and network performance between two approaches. The DUE approach model was quickly converged with a better convergence rate while the day-to-day approach needs to reroute more traveler at each iteration. This implies that the travel pattern before the highway work zone led to some kind of inertia in reaching DUE. As shown in FIGURE 10-(b), overall network performance in DUE was better than that in the day-to-day case. The average speed before the work zone was 42.2 mph, and the highway work zone congestion reduced the overall speed to 35 mph at the beginning of day-to-day. The overall speed improved with travelers’ day-to-day adjustment, but the speed was around 5 mph less than that before the work zone and 1 mph less than that of DUE.
5. CONCLUSION

This paper investigated dynamic user equilibrium processes in TRANSIMS using a trip-based approach. This study examined various equilibrium process scenarios in a real network and compared their convergence properties. The convergence properties were compared with two measures: the percentage of travelers for rerouting and the relative gap. This comparison showed characteristics of these measures associated with other selection criteria. This study also identified that the microsimulation stabilization process in TRANSIMS played a good role in reaching an equilibrium state.

The dynamic user equilibrium was also compared with the day-to-day evolution approach. Through a highway work zone case, their performances and convergence properties were investigated. Interestingly, the equilibrium state from the day-to-day approach was not the same as that from the DUE approach because of the equilibrium state before the highway work zone. Comparison of DUE and the day-to-day approach provided intuitions on model convergence and travelers’ behavioral characteristics. Various dynamic user equilibrium analyses in this study shed light on studies of DTA models and DUE. As DTA models are moving toward applications in practice, it is important to understand such model properties.
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