Optimizing Frequency of Bus Services in Mixed-Traffic Urban Streets

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Abstract: In this paper, a new analytical model is proposed to determine optimal frequency for urban bus transit under variable demand by explicitly considering the interaction between different vehicles in urban mixed traffic. The effect of road congestion on in-vehicle travel time associated with each mode is explicitly considered in the formulation of link performance function. The equilibrium flow of vehicle in mixed traffic and the passenger flow on the transit network are computed using multi-modal equilibrium assignment with respect to the frequency selected by Hooke-Jeeves algorithm; finally, the optimal frequency is found by repeating the process. A numerical example is given to illustrate the application of the model and algorithm. From the convergence test and sensitivity analysis, some important conclusions are drawn. The computational result proves that the proposed model is applicable in real aspects of bus network design in mixed traffic.

Keywords: Frequency optimization, Mixed traffic, Combined modal-split assignment model

1. INTRODUCTION

The problems associated with mixed traffic conditions on urban streets of developing countries are unique. Pedestrians, bicycles, buses, cars, motorcycles, auto-rickshaws, cycle-rickshaws, and various other kinds of travel modes share the same street space without lane discipline creating inefficient mobility conditions that reduce the economic potential of cities in developing countries. For various reasons, these problems are yet to be resolved and become more and more complex.

The magnitude and predominance of road transport in urban areas with a large variety of road-based public transport modes underline another difficult issue for transport planners in developing countries. In addition to congestion caused by mixed-traffic, conventional buses, which are the most common form of public transport, have to struggle against various types of vehicle being used for paratransit services. Some of these paratransits operate on fixed routes, such as the Bemo in Indonesia and the Tempo in Bangladesh. Others operate on non-fixed-routes, especially motorcycle-based ones, such as the Motodop in Cambodia and the Ojek in Indonesia, and these are becoming increasingly common as a form of individual public transport in many countries, particularly in South-East Asia. Furthermore, a higher proportion of the population dependent on public transport in these countries causes the problem to be much more critical. Standards of safety, comfort, punctuality, reliability, and user’s expectation are often far lower than those in developed countries. For example, people in some countries do not expect the buses to run on schedule, and are prepared to endure long waiting times, and to travel in conditions which would be unacceptable elsewhere (Iles, 2005). Thus, research on the issues of public transport specifically in developing countries should be
of greater significance to improve the existing conditions; however, no significant effort is made.

The setting of route and frequency are vital to improve bus level-of-service, enhance competitiveness in the market, and ensure profitability. With some meaningful models and solution algorithms, past research on urban transit routing and scheduling commonly consider either cost minimization or service maximization under fixed demand, or profit maximization or passenger trip maximization under variable demand. Transit route setting has been extensively researched, for example Ceder and Wilson (1986), Bajaj and Mahmassani (1991), Pattnaik et al. (1998), and Fan and Machemehl (2006). Transit frequency optimization, which is the focus of this study, has also gained a lot of interest from researchers; however, many of them could not consider the interaction between supply side and demand side and the behavior of demand side appropriately due to the adoption of a single-level model (Furth and Wilson, 1981; Ceder, 1994). Gao et al. (2004), and Dell’Olio et al. (2006) explicitly consider the interaction between supply side and demand side by using a bi-level programming approach. While the former seeks to optimize only service frequency, the latter estimates the optimum of both frequency and distance between bus stops. However, they are restricted to the case where transit demand is constant, and the in-vehicle travel time of public transit is not affected by the congestion on road. In the real conditions of a road network, a link is practically used by more than one mode and the cost of each mode on a link depends on the flow of both the corresponding mode and the other modes (Cascetta, 2001). The problem seems to be more overwhelming when talking about the developing countries where transportation networks exhibit a big combination of different types of vehicle. An earlier study by LeBlanc (1988) introduces a frequency optimization model considering elastic demand for bus service. Where bus and car are considered to represent the interaction between different modes, the model assumes that bus transit vehicle flow has significant impact on car travel time while car traffic has only a minor effect on travel time of bus on the link (Abdulaal and LeBlanc, 1979). In this manner, the application of this model in the context of developing countries will be misleading. When mixed-traffic urban street becomes congested, buses cannot move faster than the other modes. Instead, it is the smaller size vehicles (e.g. car, motorcycle) that can move faster due to their flexibility and ability to maneuver using narrow spaces on the street. A recent study by Uchida et al. (2004) presents a more plausible modeling framework to determine optimal transit frequency under variable demand considering a variety of modes on the network. One main difference from the other studies mentioned earlier in this paper is the concept of adopting probit-based user equilibrium assignment to model user’s response to the frequency change of transit service on a multimodal network. Nevertheless, to what extent the model can accommodate mixed traffic is not mentioned. Additionally, this model is only applicable in the context where only private vehicles and fixed-route transit systems are available for users. It should be noted that in many developing countries in Asia, various kinds of paratransit operating on a non-fixed-route based are also the alternate modes for public transport users. These paratransit modes are generally small in size, motorcycle-based, and strong competitors for bus as they are available at the door step and provide door-to-door service. As they become more common in many urban cities of South-East Asian countries, it is worthwhile to consider these paratransit modes in modeling the setting of bus schedule. Thus, this paper proposes a new analytical model for estimating optimum frequency of urban bus transit under variable demand by explicitly considering the interaction between different vehicles in mixed traffic, targeting bus, non-fixed-route paratransit, car, and motorbike.

The remainder of this paper is organized as follows: In Section 2, network characteristics are described along with notations, assumptions, and definition of cost function to support the
development of the model to be formulated in Section 3. Then in Section 4, the solution algorithm is presented, and it is followed by a numerical example in Section 5. In Section 6, conclusions and future research directions are presented.

2. NETWORK REPRESENTATION

2.1 Notations and Assumptions

In order to represent the multimodal network such that flow of vehicles, passengers, and road elements are connected, we consider a transportation network as a directed graph \( G = (N, A) \) where \( N \) is a set of nodes and \( A \) a set of links. On the same graph, we consider \( \bar{A} \) a sub-set of links on which bus operates (\( \bar{A} \subseteq A \)) and \( \bar{N} \) is the sub-set of nodes representing centroids and bus stops reached by bus services (\( \bar{N} \subseteq N \)). Moreover, in the formulation of mixed traffic, we consider that the network consists of \( m \) different modes, in which \( \bar{m} \) is used to refer to private modes and non-fixed-route paratransit modes, and \( \bar{m} \) is used to refer to bus. To formulate the model we will define:

- \( q_{ij} \): total trip of an OD pair \( ij \) considered as constant over the studied period, say an hour
- \( q_{ij}^m \): total trip of an OD pair \( ij \) by mode \( m \) measured in person trips per hour
- \( f_{ij,r}^m \): flow of mode \( m \) on path \( r \) linking OD pair \( ij \), in person trips per hour
- \( h_{g,r}^m \): flow of mode \( m \) on path \( r \) linking OD pair \( ij \), in vehicle flows per hour
- \( V_{a,m} \): flow of mode \( m \) on link \( a \), in person trips per hour
- \( v_{a,\bar{m}} \): flow of mode \( \bar{m} \) on link \( a \), in vehicle flow per hour (\( v_{a,\bar{m}} = V_{a,\bar{m}} / \lambda_{\bar{m}} \))
- \( \lambda_{\bar{m}} \): average occupancy rate for vehicle mode \( \bar{m} \)
- \( \gamma_m \): passenger car equivalent coefficient for vehicle mode \( m \)
- \( t_{a,m} \): travel time on link \( a \) of mode \( m \)
- \( t_{a,m}^f \): free flow travel time on link \( a \) of mode \( m \)
- \( C_{a,m} \): capacity of link \( a \) for vehicle mode \( m \) measured in passenger car units per hour
- \( \delta_{ij,a} \): link-path incident matrix equal to 1 if link \( a \) is part of path \( r \) and 0 otherwise

\( \alpha, \beta, \eta, \phi \): non-negative parameter associated with link flow

In order to make the model tractable, the following basic assumptions are made:

- Bus line frequency is a continuous variable.
- Though users have access to all modes, all trips made in the urban area are classified into four groups according to modes availability in many cities of developing countries in Asia. The first group is private car users, and the second group is private motorcycle users. The third group is paratransit users, specifically the one that is motorcycle-based and operating without fixed routes. The fourth group is bus users.
- The level of service of each mode, also known as travel time or cost, is not fixed. Therefore, the trip demand for each mode varies accordingly. For example, as the number of people using private car increases on one path of an OD, congestion on that path will increase travel time of car users; as a result, some of them will shift to other modes available. Similarly for bus users, when more and more people use the bus on the same
link, the chance of getting a seat will be lower and the bus will be crowded prompting an uncomfortable atmosphere for users; some users will choose other modes instead. The modal link flow variation of these modes is considered as the main contributor to the level of congestion in the network, and this is the essence of the proposed model that aims to tackle the interaction between various modes when setting the schedule for bus service. This example also illustrates the assumption that users seek to minimize their travel cost, thus the flow of vehicles on road network and the flow of bus passenger on transit network follow the Deterministic User Equilibrium conditions. Additionally, users of car, motorcycle, and paratransit can always choose the shortest path for their travel on the road network, but bus users cannot change their path arbitrarily to get the shortest path on the road network as bus operates on the fixed-route basis. Therefore, transit assignment is also employed in this paper to get the equilibrium flow of passengers on bus network.

### 2.2 Formulation of Cost Functions

In this study, path travel cost, $c_{ij}^{m}$, which is the cost of a trip perceived by the network user when traveling from $i$ to $j$ on path $r$ by mode $m$, is assumed to have three components:

$$c_{ij}^{m} = T_{ij}^{m} + P_{ij}^{m} + W_{ij}^{m}$$

In equation (1), the first component, $T_{ij}^{m}$, denotes in-vehicle travel time experienced by a trip maker. It varies depending on road congestion and differs from one mode to another as shown in Section 3.1. The second component, $P_{ij}^{m}$, refers to fare for one trip, for which paratransit users and bus users have to pay. The third component, $W_{ij}^{m}$, denotes waiting time which is applied only to bus users based on the assumption that paratransit is available everywhere at anytime, i.e. waiting time for paratransit is assumed to be zero.

Another important point to note in equation (1) is that the waiting time is part of the path travel cost. This differentiates this study from the previous studies that include waiting time into link travel time as they adopt the concept of “combined link” for transit equilibrium assignment (Gao et al., 2004; Dell’Olio et al., 2006; Uchida et al., 2004). The computation of waiting time at bus stop in this paper based on an idea originally known as “optimal strategy” (Spiess and Florian; 1989). By definition, a “strategy” is a set of rules that, when applied, allows the traveler to reach his or her destination. Travelers may choose different strategies depending on the information available to them during the trip. However, if no additional information becomes available during the trip, a strategy simply defines a path. Based on this, we assume that there is no additional information available during the trip on the transit vehicle so that the “strategy” can be simplified as the “path” without loss of generality. Therefore, the average waiting time of a bus passenger at origin node $i$ to travel to the destination node $j$ can be computed by:

$$W_{ij}^{m} = \sum_{a \in A \cap A'} f_a$$

where: $w$ : waiting time factor; normally used value is 0.5 corresponding to the
uniform distribution of users arrival at bus stop and the constant headway of buses,

\[ A^*_i : \text{set of outbound links from an origin node } i, \ A^*_i \subseteq A \]

\[ A^*_j : \text{set of links belonging to feasible transit paths connecting } ij, \ A^*_j \subseteq A \]

\[ f_a : \text{total frequency of bus service on link } a, \]

The total frequency of bus service on link \( a \) is determined by the sum of frequency of all lines running on link \( a \). It can be computed by:

\[ f_a = \sum_{l \in \psi(a)} f_l \]  

(3)

where:

\[ \psi(a) : \text{being the set of bus line } l \text{ on link } a \]

\[ f_l : \text{frequency of bus line } l \]

Figure 1. Transit network

Figure 1, which illustrates the meaning of equation (2), shows three bus lines pass by node 1. Waiting time of passenger waiting at node 1 to travel to node 2 is affected by the frequency of line 2 only. For the waiting time of other passengers waiting at the same node 1 to travel to node 3 is affected by the frequency of line 1 and line 3. Then, waiting time for passengers waiting at the same node 1 to travel to node 4 or 5 is influenced by the frequency of all lines 1, 2, and 3.

Under mixed traffic, as mentioned by Cascetta (2001), each traffic mode in a transportation network has an individual cost function which contributes to the overall congestion in an individual way, i.e. the link performance function of any traffic mode should be expressed in a function of the entire traffic flow. Based on this, the in-vehicle travel time for a user of private mode and paratransit operating on a non-fixed-route basis on link \( a \) is dependent on the summation of all vehicle flows on the link including the fixed bus flow:

\[ t_{a,\text{v}} = t_{a,\text{v}}(\sum_{\text{m}} v_{a,\text{m}}, f_a) \]

(4)

By adopting a BPR function, equation (4) can be specified as:
Unlike for private modes and paratransit, the formulation of the in-vehicle link travel time for bus passengers is more complex. Passenger waiting at bus stops in a bus network can be described as a complex queuing process in which people arrive randomly at a bus stop at an average rate and are served by transit vehicles belonging to different lines passing through the stop. This might lead to the idea that different queuing systems, each of them corresponding to one of the lines starting at bus stop, must be considered. The service rate for each queue depends on the capacity available on the line belonging to the set of feasible paths on the transit network. Because a line on one link can be used by other links, the service rate for the link also depends on the volume of the other links. When the arrival rate of transit users on one link exceeds the capacity of this link, the waiting time at a stop will increase heavily, for the arriving transit vehicles are full or without sufficient capacity to carry all users waiting at the stop. At this point, it is clear that if the queuing approach is used to solve such problems, it will lead to a complex formulation that is very hard to solve in practical use. Gao et al. (2004) stated that it is not feasible to use simulation to evaluate passenger waiting time at every bus stop as there might be thousands of stops over the transit network; moreover, it is not what we want to know in transit planning. The important information to be considered should be the equilibrium flow on each link of transit network where transit lines are loaded, which could be computed using an equilibrium method similar to that for the conventional urban transportation network. Different approaches have been developed in order that the travel time of the transit user is well represented. The idea of “common line” by Chriqui and Robillard (1975) and “optimal strategy” by Spiess and Florian (1989) have been highly recognized due to their ability to represent the travel time of passengers. Jaquin and Enrique (1993) extended these two concepts to include the in-vehicle congestion of transit and they formulated the link performance function to be the sum of the in-vehicle travel time, waiting time at bus stop, and delay due to the insufficient capacity of the link:

\[
t_{a,m} = t_{a,m}^o + \left[1 + \alpha \left( \frac{\sum_{\bar{m}} V_{a,\bar{m}} \gamma_{\bar{m}} + f_{a,\bar{m}}}{C_{a,\bar{m}}} \right) \right] \quad \forall \bar{m}, a \in A
\]

where:
- \( t_{a,m}^o \): transit in-vehicle travel time
- \( V_{a,\bar{m}} \): total number of bus passengers boarding at the same link at node \( I \)
- \( \bar{V}_{a,\bar{m}} \): bus passenger flow that compete with the \( V_{a,\bar{m}} \) for the same capacity
- \( K_{a,\bar{m}} \): total link capacity of transit on link \( a \), in number of passengers/hour

Since this paper does not follow the concept of “common line”, “optimal strategy”, or “combined link”, any type of flow (vehicle and bus passenger) is dealt with on the links of the primitive road network. It is, therefore, impossible to apply equation (6) to represent the link travel time of bus users in this study.

A tailored formulation for this paper is developed based on the original concept introduced by Joaquin and Enrique (1993). As waiting time at stop must be considered separately from the components of link travel time, only in-vehicle travel time, and delay due to the capacity insufficiency is taken into account, and equation (6) is reduced to:
\[ t_{a,m} = \overline{t}_{a,m} + \eta \left( \frac{V_{i,m} + \overline{V}_{a,m}}{K_{a,m}} \right)^\phi \]  

As the bus transit vehicle operates in the same road with the other vehicles in mixed traffic, it must experience the same congestion as other modes do. For this reason, the *in-vehicle travel time* of bus, \( \overline{t}_{a,m} \), is formulated in a similar way as the private mode in the same network as in equation (5), such that:

\[ \overline{t}_{a,m} = t_{a,m}^o \left[ 1 + \alpha \left( \frac{\sum V_{i,m} + f_a \gamma_m}{C_{a,m}} \right)^\gamma \right] \]  

Moreover, without adopting the idea of “common line”, the passenger link flow on transit network using link \( a \) can be used to replace the original version proposed by Joaquin and Enrique above, and written as:

\[ V_{a,m} = V_{i,m} + \overline{V}_{a,m} \]  

By replacing equations (8) and (9) in equation (7), the travel time on link \( a \) for bus users can be rewritten as below:

\[ t_{a,m} = t_{a,m}^o \left[ 1 + \alpha \left( \frac{\sum V_{i,m} + f_a \gamma_m}{C_{a,m}} \right)^\gamma \right] + \eta \left( \frac{V_{a,m}}{K_{a,m}} \right)^\phi, \quad a \in \overline{A} \]  

Note that the second term of equation (10) can be interpreted as discomfort inside the bus due to crowded conditions when more and more passengers use buses on the same link. In case the number of users exceed the link capacity, \( K_{a,m} \), the frequency of bus lines running through that link will increase, and this is the core of scheduling bus service in mixed traffic that is considered in this paper.

### 3. MODEL FORMULATION

As an important part of urban transportation planning, transit system planning directly affects the conditions of urban transportation. Modification of the transit service usually affects users’ travel behavior in terms of mode choice and route choice. It is, therefore, reasonable to consider two groups of people. The first group is the planners who determine the characteristics of the transport system independently from users within the limit of feasible constraints. The second group is the users who try to minimize their individual travel cost under the strategy set by the planners, and they produce a flow pattern on the urban network. In this paper, the planner decides the setting of the bus schedule, and the users are allowed to respond to the service frequency of bus lines by changing their intermodal routes. This
response is assumed to follow a Deterministic User Equilibrium condition and can be solved by a combined modal-split assignment model. In the following formulation of the model, the terms “waiting time” and “fare” are included in the minimization problem of both lower level and higher level since we did not include in-link travel cost.

### 3.1 Combined Modal-Split Assignment Model Under Given Bus Line Frequency

To enable the combined modal-split assignment, any feasible set of flow on the network must satisfy the conservation constraints:

\[ \sum_{m} f_{ij}^{m} = q_{ij}, \forall ij \]  \hfill (11)

\[ \sum_{r} v_{a,m}^{ij} = q_{ij}^{m}, \forall m, \forall ij \]  \hfill (12)

\[ v_{a,m}^{ij} = \sum_{r} h_{ij}^{m} D_{r,a}^{ij} = \sum_{r} \sum_{q} \left( q_{ij}^{m} / \lambda_{a}^{m} \right) D_{r,a}^{ij}, \forall a \in A \]  \hfill (13)

\[ V_{a,m} = \sum_{r} f_{ij}^{m} D_{r,a}^{ij}, \forall a \in \bar{A} \]  \hfill (14)

As modal link travel times are known (see equations (5) and (10)), the in-vehicle travel time on path \( r \) connecting OD pair \( ij \) is given as follows:

\[ T_{ij}^{m} = \sum_{a \in A} t_{ij}^{a} D_{r,a}^{ij}, \forall \bar{m}, \forall r, \forall ij \]  \hfill (15)

\[ T_{ij}^{m} = \sum_{a \in \bar{A}} t_{ij}^{a} D_{r,a}^{ij}, \forall \bar{m}, \forall r, \forall ij \]  \hfill (16)

Using equation (1), the path travel cost, \( c_{ij}^{m} \), is obtained. Let \( \mu_{ij}^{m} \) denote the minimum path travel cost for traveling between OD pair \( ij \) by mode \( m \) and let user route choice be governed by Wardrop’s principle; equilibrium path flow should satisfy the following complementarity conditions:

\[ \begin{cases} \left( c_{ij}^{m} - \mu_{ij}^{m} \right) h_{ij}^{m} = 0, \forall \bar{m}, \forall r, \forall ij \\ \left( c_{ij}^{m} - \mu_{ij}^{m} \right) \geq 0 \end{cases} \]  \hfill (17)

\[ \begin{cases} \left( c_{ij}^{m} - \mu_{ij}^{m} \right) f_{ij}^{m} = 0, \forall \bar{m}, \forall r, \forall ij \\ \left( c_{ij}^{m} - \mu_{ij}^{m} \right) \geq 0 \end{cases} \]  \hfill (18)

It is widely known that mode choice is influenced by various factors many of which are difficult to quantify. For simplicity, this study considers only travel cost, \( \mu_{ij}^{m} \), and mode attraction measure that captures the effects of all factors other than travel cost, \( \phi^{m} \), for mode choice which is given by a Multinomial Logit formula with the logit parameter \( \tau \), that is,

\[ q_{ij}^{m} = q_{ij} e^{-\tau (\mu_{ij}^{m} - \phi^{m})} \sum_{n} e^{-\tau (\mu_{ij}^{n} - \phi^{n})}, \forall m, \forall ij \]  \hfill (19)
The model to represent the network in this study is a large scale of equation group in which the Jacobian matrix is asymmetric. This study adopts “diagonalization algorithm” (Sheffi, 1985) that iteratively solves a series of standard UE programs to find the equilibrium flow of the mixed traffic network and the bus network. Suppose that there are four modes namely, car (mode 1), motorcycle (mode 2), paratransit (mode 3), and bus (mode 4). Assume that at \( n^{th} \) iteration, link flows of all modes are known \( (a_1, a_2, a_3, a_4) \), the following mathematical programming can be formulated:

\[
\begin{align*}
\min z^{(n)} &= \sum_{a,c,d} \int_{0}^{\infty} t_{a,c} \left( \phi, v_{a,2}, v_{a,3}, f_{a} \right) d\phi + \sum_{a,c,d} \int_{0}^{\infty} t_{a,2} \left( \phi, v_{a,1}, v_{a,3}, f_{a} \right) d\phi + \\
&\sum_{a,c,d} \int_{0}^{\infty} t_{a,4} \left( \phi, v_{a,1}, v_{a,2}, v_{a,3}, f_{a} \right) d\phi + \int_{0}^{\infty} \left[ \sum_{a,c,d} \sum_{m=1}^{4} \sum_{ij} q_{ij}^{m} \left( \ln q_{ij}^{m} - 1 \right) - \sum_{ij} \varphi_{ij}^{m} \right] d\phi
\end{align*}
\]

subject to:
\[
\begin{align*}
\sum_{m} q_{ij}^{m} &= q_{ij} & \forall ij \\
\sum_{m} f_{ij}^{m} &= q_{ij} & \forall ij, m \\
f_{ij}^{m} &\geq 0 & \forall ij, r, m
\end{align*}
\]

In equation (20), number of bus vehicle flows (or total bus frequency on link \( a \)), \( f_{a} \), is constant regardless of the iteration number as this equilibrium assignment problem allows users to respond under a service frequency of buses being fixed. Therefore, waiting time, \( W_{ij}^{4} \), is also constant. Naturally, fare, \( P_{ij}^{m} \), is always constant as urban public transport service is paid for one trip from \( i \) to \( j \).

3.2 Bus Frequency Optimization Model

To determine an optimal frequency of bus service, there are numerous possible objectives to be considered. Similar to Uchida et al. (2004), the objective function dealt with in this paper is to minimize the total cost on the transport network including users of all modes as well as the transit operating cost. Thus, the model is specified as a summation of travel time of all passengers on the network, total waiting time of bus passengers, total fare for using bus and paratransit, and the operating cost of the bus company. It is formulated as:

\[
\begin{align*}
\min C(f) &= \sum_{a} \sum_{m} t_{a,m} \left( V_{a,m}(f), f_{a} \right) V_{a,m}(f) + \sum_{m} \sum_{ij} q_{ij}^{m} W_{ij}^{m} + \sum_{m} \sum_{ij} q_{ij}^{m} P_{ij}^{m} + \vartheta \sum_{l} f_{l} F_{l}
\end{align*}
\]

subject to:
\[
\begin{align*}
f_{l} &\geq 1 & \forall l
\end{align*}
\]

where
- \( f \) : vector of line frequency
- \( F_{l} \) : operating cost per frequency increase on transit line \( l \)
- \( \vartheta \) : converts operating cost due to frequency setting to travel time
Since the frequency of bus line is a function of number of bus passengers, the constraint \((f_i \geq 1)\) ensures that there is at least one bus operating on every bus line even though the number of passengers is extremely low.

4. SOLUTION ALGORITHM

In this section, a description of how optimal frequency of the bus transit can be computed is given. First of all, it is important to consider the changes of equilibrium link flows by all modes resulting from the changes of the line frequencies in transit network in the way that the constraint, equation (25), is not violated. In this sense, the multi-modal equilibrium assignment must be solved for each fixed frequency assigned to each line. The equilibrium flow is then inserted in equation (24) for evaluation, and then another new frequency is assigned. So the whole process belongs to two different levels: (1) operator or planner decides on the frequency, (2) the users of all modes react following the Deterministic User Equilibrium condition. A solution algorithm is proposed and is described below:

\(\text{Step 1: Initialize a feasible line frequency vector } f \text{ that satisfies the constraint in the frequency design model.}\)

\(\text{Step 2: given } f, \text{ combined modal-split assignment model solves for the equilibrium flow on road network and on bus network producing equilibrium link flow vector } V^*.\)

\(\text{Step 3: the equilibrium link flow } V^* \text{ and the feasible frequency } f \text{ are inserted into the objective function. The objective function is then evaluated.}\)

\(\text{Step 4: Hooke and Jeeves algorithm (Hooke and Jeeves, 1961) is used to find a new frequency of each bus line one at a time, and then return to step 2.}\)

This procedure is repeated until a better frequency that reduces the cost of the system in comparison to the previous iteration is found.

Hooke-Jeeves algorithm is a pattern search procedure widely used to optimize non-linear functions that are not necessarily continuous or differentiable (Alkhamis and Ahmed, 2005). It is also known to be a simple and fast with high accuracy when the number of decision variables is not too large (LeBlanc, 1988; Dell’Olio et al., 2006). From the initial bus line frequency, the algorithm takes a step \(\Delta\) in various directions and conducts a new model run. If the new line frequency scores better than the previous one, it uses this new point as its best guess. If it is worse, then the algorithm retains the old point. It proceeds in a series of these steps, each step slightly smaller than the previous one \((\Delta = 0.5\Delta)\). When the algorithm finds a point which it cannot improve on with a small step in any direction, then it accepts this point as being the solution.

It should be noted that the proposed algorithm has the form of a heuristic approach, so there is no guarantee of global optimum solution. In order to maximize the chance of reaching a global solution, an appropriate initial frequency value with an appropriate step size \(\Delta\) of the Hooke-Jeeves algorithm must be selected prior to the simulation using the following steps:

- Consider a number of different values of initial frequency, and propose a variety of different step size \(\Delta\).
- Use an initial frequency value one at a time, with the each step size (one by one) to compute the objective functions.
- Compare the value of objective functions. Usually, most of them have equal value which is the minimum value; that is believed to be the global solution. However, a few of them...
have a higher value than the minimum (the local optimum solution); the use of these step size values should be avoided.

5. NUMERICAL EXAMPLE

To illustrate the operation of the model and the algorithm proposed in this paper, a numerical example using Nguyen and Dupuis (1984) network is conducted. This example considers four traffic modes having different sizes and characteristics in order to capture the impact of mixed-traffic congestion and the existence of non-fixed-route paratransit on the optimal setting of bus frequency. These four modes are car (mode 1), private motorcycle (mode 2), motorcycle-taxi (mode 3), and bus (mode 4). A motorcycle-taxi may have different local names, but it is commonly a motorcycle serving as a paratransit, operating without fixed routes, on which passengers are supposed to sit just behind its driver. On the test network, three bus lines operate as shown in figure 2. There are four OD pairs: (1-2), (1-3), (4-2), and (4-3) with the corresponding demand of 1000, 800, 1000, and 800 trips per hour respectively.

The link performance functions for each mode are assumed to be:

\[ t_{a,\bar{a}} = t_{a,\bar{a}}^o \left[ 1 + 0.15 \left( \frac{v_{a,1} \gamma_1 + v_{a,2} \gamma_2 + v_{a,3} \gamma_3 + f_{a} \gamma_4}{C_{a,\bar{a}}} \right)^4 \right] \]

for modes 1, 2, and 3; and

\[ t_{a,\bar{a}} = t_{a,\bar{a}}^o \left[ 1 + 0.15 \left( \frac{v_{a,1} \gamma_1 + v_{a,2} \gamma_2 + v_{a,3} \gamma_3 + f_{a} \gamma_4}{C_{a,\bar{a}}} \right)^4 \right] + \left( \frac{V_{a,\bar{a}}}{K_{a,\bar{a}}} \right)^2 \]

for mode 4.

As for link performance characteristics (Table 1.a), it is assumed that two-wheelers (mode 2 and 3) have longer free-flow travel time than mode 1 and 4 on every link. Also, this example considers that all links of this network have the same capacity, but the capacity of each link varies according to the traffic mode being referred to. In Table 1.a, link capacities for mode 2 and 3 are higher than that for mode 1 and 4. One possible reason is that when traffic flow reaches capacity, the flow of smaller vehicles is supposed to have higher density than the flow of larger vehicles on the same urban road link, but their speeds may not be much different. Other necessary input values are also shown in Table 1.b.

![Figure 2. Test network](image-url)
The three bus lines remain: different initial points, the algorithm converges at similar value with the optimal solution for the proposed model are used. The values in the bracket in Table 2 show the initial values of line 1, line 2, and line 3 respectively. The result shows that despite the contrast, share of mode 2, 3, and 4 increase, changing from 9%, 7%, and 3% at initial frequency (1, 1, 1) to about 17%, 16%, and 16% respectively at the optimum. In road network. It is important to note that at the initial solution (iteration 0), where bus frequency is low (1, 1, 1), 80% of trips are made by mode 1 which generate excessive congestion on road network (figure 3-(a)). This congestion strongly affects mode 4 by making.

First of all, in order to test the convergence of the algorithm, different initial values of bus line frequency to solve the proposed model are used. The values in the bracket in Table 2 show the initial values of line 1, line 2, and line 3 respectively. The result shows that despite different initial points, the algorithm converges at similar value with the optimal solution for the three bus lines remains: \((f_1^*, f_2^*, f_3^*) = (4.66, 3.00, 3.00)\). Therefore, it can be concluded that the solution may be close to the global solution of the problem.

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First of all, in order to test the convergence of the algorithm, different initial values of bus line frequency to solve the proposed model are used. The values in the bracket in Table 2 show the initial values of line 1, line 2, and line 3 respectively. The result shows that despite different initial points, the algorithm converges at similar value with the optimal solution for the three bus lines remains: \((f_1^*, f_2^*, f_3^*) = (4.66, 3.00, 3.00)\). Therefore, it can be concluded that the solution may be close to the global solution of the problem.
its travel cost as high as 241 minutes (figure 3-(b)). At the same time, under such congestion, the travel cost of mode 1, 2, and 3 are almost similar. However, when the traffic becomes less congested, travel cost of mode 1 turns out to be smaller than mode 2 and 3. This verifies that vehicles with smaller size are less sensitive to congestion.

![Modal share and corresponding travel cost of OD pair 1-2 by iteration](image)

Figure 3. Modal share and corresponding travel cost of OD pair 1-2 by iteration:
(a) modal share of OD pair 1-2 and (b) travel cost of OD pair 1-2

We will observe how the changes in demand of some OD pairs will affect the schedule setting of every bus lines in the network. In this case, demand shares of OD pairs (1-2) and (1-3) gradually increase from 10% to 60%, while the demand shares of O-D pairs (4-2) and (4-3) are held fixed. From Figure 4, the optimal frequencies of all lines generally increase with respect to the demand increase of OD pairs 1-2 and 1-3. An important increase can be found on the frequency of line 1 because any feasible transit paths connecting OD pairs 1-2 and 1-3 must go through link 1 on which only bus line 1 is in service. When the demand shares of O-D pairs 1-2 and 1-3 increase 10%, 20%, 30%, 40%, 50%, and 60%, the frequency of line 1 increases by 2%, 13.7%, 32.2%, 44.4%, 56.2%, 66.9%; and frequency of line 3 also increases by 19%, 22%, 27%, 30.7%, 34.7%, 34.9% respectively. For line 2, except the case of increase of 10% and 20%, the rest makes frequency of line 2 to increase by 10.9%, 17.8%, 23.4%, and 30.9% respectively.

![Optimal frequency under the increase of demand for OD pair (1-2) and (1-3)](image)

Figure 4. Optimal frequency under the increase of demand for OD pair (1-2) and (1-3)

In practical situations, there are many factors that combine together to give a definition of the operating cost. Any variation of those factors will surely affect cost, for example, increase of drivers’ salary, gas price increase, etc. Since this paper considers the operating cost to be defined by variation of service frequency, it can be observed how the optimal frequency would change when the operating cost increases. Figure 5 shows that when the cost per unit frequency is increased by 20%, the optimal frequency of each line decreases by 18% for line 1, while line 2 and line 3 remain the same. The line frequency of line 2 and line 3 remain...
unchanged until the cost per unit frequency reached 40%. It is observed that when the cost per frequency reaches 100% (i.e. double), the reductions that resulted in each line frequency are 24.9% for line 1, 34.7% for line 2, and 6.3% for line 3.

![Figure 5. Optimal frequency under variation of operating cost](image)

The selection of larger bus capacity for operation has significant impact on the optimal frequency simply because the ability to carry more may reduce the line frequency (Figure 6.(a)), thus, increasing the waiting time. However, some interesting property is found in this analysis. Figure 6.(b) shows that within the range of bus capacity increase in this numerical example, the number of bus users of all OD pairs increases, or at least stays the same with the

![Figure 6. Impact of bus vehicle capacity on: (a). setting of bus schedule, (b). bus ridership, (c). in-vehicle travel time of OD (1-2), (d). in-vehicle travel time of OD (1-3), (e). in-vehicle travel time of OD (4-2), (f). in-vehicle travel time of OD (4-3)](image)
base situation. At this point, it is clearly seen that when bus vehicle capacity is large, implying that the level of congestion inside the bus vehicle is low, more passengers are willing to ride buses. In this manner, the term $(V_{a,m} / K_{a,m})$ proposed in equation (11) makes sense. When the bus capacity is larger, $K_{a,m} = f_a, k_{bus}$ (where $k_{bus}$ is the vehicle capacity of bus) is also large.

Figure 6.(c), 6.(d), 6.(e), and 6.(f) show that with larger bus capacity, the in-vehicle travel times for all modes reduce for all OD pairs due to less congestion occurring on urban roads. This result gives an interesting implication on how increasing the bus capacity positively affects both the travel time of the whole network and the bus ridership. This measure is good both from the viewpoint of transit operator and transport planners provided that many car users shift to transit.

6. CONCLUSIONS

In this paper, a new analytical framework to determine the setting of bus frequency in a mixed traffic network is proposed by considering the fact that road congestion will increase in-vehicle travel time for all modes in the same network. Moreover, the in-vehicle congestion occurring within the bus transit is also included in the link travel time of bus transit. Hence, the transit link travel time is the function of both vehicle flows on the mixed traffic as well as the bus passenger flow on the transit network. The essence of this proposed model is not only to cover the mixed traffic, but also to replicate the existence of paratransit operating on a non-fixed route, which is one of the most common forms of public transport in Asian developing countries.

Based on the proposed model, a numerical example is also given to illustrate the application of the model and the algorithm. Sensitivity analysis is conducted in various directions to check the performance and the accuracy of the proposed model. From these analyses, some conclusions can be drawn. The optimal frequencies would increase according to the increase rate of the total OD demand; therefore, it is necessary for planners to make an optimal modification periodically in order to alleviate the strain on transit conditions resulting from the increase of OD demand, and thus optimize the performance of the multi-modal network. Moreover, the analysis also pointed out that the change of factors such as transit vehicle, driver income, gasoline price, and others will result in the change of operating cost per unit frequency on some lines, thus the change of system performance. So it is necessary to modify the frequency for some lines to optimize system performance. It is also demonstrated that the frequency will decrease accordingly with the vehicle capacity of transit bus; and at the same time the OD travel time will also decrease with respect to bus capacity while bus ridership increases.

As there are some important points that are not handled in this paper, some potential avenues for future improvement of the proposed model are as follows:

- The access time and transfer time should be included in the generalized cost of travel since these two terms as well as waiting time are even more important factors in determining which route is chosen (Sheffi, 1985).
- The quality of the multinomial logit model used in this study should be improved so that it can include more aspects affecting the passenger’s mode choice behavior.
- The contribution of each mode to the congestion in mixed traffic, represented by PCE factors in this paper, should be treated more comprehensively so that it will be more insightful of vehicle interaction under non-lane-discipline mixed traffic flow conditions in developing countries.
• On a practical level, a real-size network should be used for the case study so that it can closely represent the real aspects of urban transport, especially those of Asian countries.

REFERENCES


