Dynamic Equilibrium for Combined Signal Settings and Dynamic Traffic Assignment

Li-Wen CHEN\textsuperscript{a}, Ta-Yin HU\textsuperscript{b}

\textit{Department of Transportation and Communication Management Science, National Cheng Kung University No.1, University Road, Tainan City 701, Taiwan, R.O.C.}

\textsuperscript{a} E-mail: yoyo.lwchen@gmail.com

\textsuperscript{b} E-mail: tayinhu@gmail.com

Abstract: Intelligent Transportation Systems (ITS) focus on increasing the efficiency of existing surface transportation systems through the use of advanced computers, electronics, and communication technologies. The interaction between signal setting and traffic assignment is an important issue in designing efficient ATMS. This research focuses on finding dynamic user equilibrium between signal setting and route assignment. The problem is solved through a bi-level framework. The upper level solves for signal setting parameters, including cycle length, green splits, and offsets. The lower level solves for user equilibrium Dynamic Traffic Assignment (UEDTA) flows in a traffic network. The signal setting is adjusted through two methods: the Webster formula and Adaptive Signal Control. UEDTA flow patterns are solved through a simulation-based DTA model, DynaTAIWAN. This bi-level framework can be applied in cities with signalized networks. Numerical experiments are conducted based on a sub-network of Kaohsiung City to illustrate the proposed framework.

Keywords: Bi-level framework, Dynamic equilibrium, Time-dependent user equilibrium assignment, Network design problem

1. INTRODUCTION

Traffic congestion has become a common and important problem in many urban areas around the world. Intelligent Transportation Systems (ITS), which focus on increasing the efficiency of existing surface transportation systems through the use of advanced computers, electronics, and communication technologies, have been promoted to address traffic congestion problems. Two major subsystems considered in this paper are Advanced Traveler Information Systems (ATIS) and Advanced Traffic Management Systems (ATMS). Under these subsystems, traffic information is collected, processed, and utilized. The traffic information is used to provide travel information to trip makers and flow distributions are used to improve traffic control strategies. Based on these collected data, the Metropolitan Planning Operators (MPO) and the Traffic Management Center (TMC) can provide efficient management strategies (such as signal settings) through the traffic control systems.

Signal settings are designed based on given flow distributions in a traffic network. However, flow distributions are also influenced by the trip makers’ route choice decisions. Thus, the interaction between signal settings and route choice is an important issue in designing efficient ATMS. Although extensive researches for studying the problem have been done, there are limitations of previous researches which can be summarized as follows:

(1) Fixed cycle length is assumed to simplify the problem;

(2) There is no consideration of time-dependent issue, which might be an important concern in the ITS environment; and

(3) Algorithms are designed and illustrated through small networks.
This research focuses on finding dynamic equilibrium between signal setting and route choice behavior. The User Equilibrium Dynamic Traffic Assignment (UEDTA) rule extends Wardrop's UE conditions to a time-dependent case. Under real-time descriptive information on network conditions, a time-dependent UE would hold when no user can improve his/her individual cost by unilateral route switching.

This research proposes a bi-level framework for solving such problems. The upper level solves for signal setting parameters, including cycle length, green splits, and possibly offsets. The lower level solves for UEDTA flows in a traffic network. The signal setting is adjusted through the Webster formula and Adaptive Signal Control logic. In addition, UEDTA flow patterns are solved through a simulation-based dynamic traffic assignment model, DynaTAIWAN.

Numerical experiments are conducted based on a sub-network of Kaohsiung City to illustrate the proposed framework, and the results are used to explore fundamental properties of UEDTA. The contribution of this paper includes the following: (1) to provide an assessment of the existence of dynamic equilibrium through numerical experiments; (2) to propose a framework for designing network problems; (3) to incorporate signal cycle length which is not fixed; (4) to consider time-dependent flow patterns in the ITS environment; and (5) this framework can be applied in cities with signalized networks.

The next section presents a brief review of related models. The proposed framework and associated modeling issues are described in Section 3. The experimental design is explained in Section 4, followed by a brief summary of the expected contributions and recommendations of this paper.

2. LITERATURE REVIEW

Extensive researches have been conducted on the combined problem of signal setting and network assignment. Research directions include: (1) problem structure (Cantarella et al., 1991a; Cipriani and Gori, 2000; Taale and van Zuylen, 2001), (2) solution algorithms, and (3) efficient algorithms for finding optimal solutions.

According to Ghatee and Hashemi (2007), the approaches can be classified into three methods:

- Algorithm consisting of meta-heuristic algorithms (Simulation-optimization approaches).
- Algorithms composed of analytical methods from non-linear programming.
- Algorithms using integer programming algorithms combined with some heuristics to find the optimal signal times.

In this section, approaches for solving the combined problem are reviewed, and these approaches are classified into mathematical-based models, iterative heuristic approaches, and bi-level programming models.

2.1 Mathematical-based Model

In general, mathematical models include decision variables, such as signal parameters and flow variables, and constraints.

Smith (1979a, 1979b) proposed an integrated model to illustrate the problem in mathematical form. He proved the existence of multiple solutions; however, some solutions might lead to an unstable equilibrium state and worsen possible traffic conditions. The results (Smith, 1980) show that travel time could increase by 30% when introducing flow-responsive
traffic signals. A new signal setting policy, $P_0$, is proposed to maximize intersection capacity, and the policy ensures stability and consistency of user equilibrium.

According to Cascetta et al. (1998), there are two sub-problems, Goss and Lossp, in signal setting and traffic assignment. The first one is the global optimization of signal settings problem (Goss), where the signals of the network are set to minimize the objective function describing the global network performances (Marcotte, 1983); the other one is the local optimization of signal settings problem (Lossp) concerning flow-responsive signals, which are set independently with each other either to minimize a local objective function (Gartner, 1977) or following a given criterion, like equi-saturation or $P_0$ policy (Smith, 1980).

Cantarella et al. (1991b, 1995) extended the problem by including cycle length and offsets in addition to green split settings. They showed a lower normative and an upper descriptive bound of the solution of the equilibrium network signal setting. They also compared global and local approaches which revealed that within the same valley of a possible local solution, the Lossp solution is sub-optimal if compared with the Goss solution.

### 2.2 Iterative Heuristic Algorithm

Pavese (1968) is the first one who proposes the problem about the interaction between signal settings and traffic assignment. He formulated the node functions that relate link performances to traffic flows of every approach at the downstream intersection.

Allsop (1974) and Gartner (1974) developed an iterative procedure for solving the two problems sequentially, by alternately updating green split signal settings for fixed flows and solving the traffic equilibrium problem for fixed signal settings.

### 2.3 Bi-Level Programming Approach

In general, bi-level programming methods are applied to solve the combined problem by treating signal and route choice problems individually. The upper-level solves the traffic control optimization problem, and the lower-level problem solves for traffic flow based on traffic assignment principles, such as user equilibrium.

Lee and Machemehl (1998, 1999) solved the equilibrium network design problem (ENDP) via stochastic equilibrium link flows on signal settings. The GA optimization is applied for individual signalized intersection. The objective function is a function of green split and the UE link flows. When searching the optimal solution, they showed that the greater the network size and/or saturation degree are, solution methods and the choice of starting points likely will lead to different solutions.

Yin (2000) and Yang and Yagar (1995) applied the GA algorithm to small-scale networks. The objective function was to maximize the network reserve capacity. For the signal setting problem, they only considered ‘green split’ for an isolated signalized junction. The offsets and common network common cycle time optimization were ignored. UE assignment traffic flow equilibrium is also considered in these researches. The resultant solutions, however, were about the same as those of the SAB (sensitivity-analysis-based) algorithm in their small-scale example network.

Ceylan and Bell (2004, 2005) used Genetic Algorithm to solve the upper-level problem for a congested signalized road network and applied TRANSYT traffic model to obtain network performance index (PI) and fitness index. Stochastic user equilibrium (SUE) traffic assignment was applied at the lower-level, and solved by Path Flow Estimator (PFE). The different characteristic of their algorithm is that the formulation considered three factors with cycle, offset and green timings. The main discussion of this research is on the operation of the
tool named GATRANSPFE and the GA process applied for finding the optimal solution. The authors compared the result with Mutually Consistent (MC) to determine the efficiency and the performance of the algorithm. Mutually Consistent (MC) is originally proposed by Allsop (1974) and Gartner (1974) and Carried out by Cantarella et al. (1991). Allsop and Charlesworth (1977) also applied it for a medium size road network.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Main contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith (1979a, 1979b)</td>
<td>• Proposed an integrated model to illustrate the problem in mathematical form.</td>
</tr>
<tr>
<td></td>
<td>• The existence of multi-solutions might worsen traffic conditions.</td>
</tr>
<tr>
<td></td>
<td>• Proposed Po policy.</td>
</tr>
<tr>
<td>Cascetta et al. (1998)</td>
<td>• Discussed about Goss and Lossp problems.</td>
</tr>
<tr>
<td>Pavese (1968)</td>
<td>• First who proposed about the interaction between the two problems.</td>
</tr>
<tr>
<td>Allsop (1974) and Gartner (1974)</td>
<td>• Developed an iterative procedure for solving the two problems sequentially.</td>
</tr>
</tbody>
</table>

Table 2. The signal setting variables in the related literature

<table>
<thead>
<tr>
<th>Author</th>
<th>Signal setting variables</th>
<th>Assignment Principles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cantarella et al. (1991b, 1995)</td>
<td>(1) cycle length</td>
<td>SUE link flows</td>
</tr>
<tr>
<td></td>
<td>(2) offsets</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3) green split settings</td>
<td></td>
</tr>
<tr>
<td>Lee and Machemehl (1998, 1999)</td>
<td>green split</td>
<td>SUE link flows</td>
</tr>
<tr>
<td>Ceylan and Bell (2004, 2005)</td>
<td>fixed cycle time</td>
<td>Stochastic user equilibrium (SUE)</td>
</tr>
<tr>
<td>Cipriani and Fusco’s (2004)</td>
<td>green split ratios λ*</td>
<td>User equilibrium</td>
</tr>
</tbody>
</table>

In Cipriani and Fusco’s (2004) bi-level formulation, the Gossp was formulated to minimize an objective function Z, and determine the green split ratios λ* under descriptive user equilibrium conditions subject to admissibility constraints.

Ghatee and Hashemi (2007) discussed the combined problem with Mutually Consistent (MC). A two-layer algorithm was developed. The external layer solves optimal signal setting according to flows on network links, and the internal layer solves a multi-commodity problem for shipping the travellers on the shortest paths from the given O-D. Convergence is achieved when it fulfills the stopping criteria.

3. RESEARCH FRAMEWORK

3.1 Problem Statement

Consider a traffic network G(N,A) consisting of a set of nodes N connected by a set of directed arcs A. In a traffic system with given network characteristics, system performance (in terms of flow distribution) is determined through the interaction between demand and supply factors. Supply factors include traffic control (C) and route information (routes and associated
attributes), and demand factors include time-dependent O-D trips (D) and route choice (R). The system can be described in the following equations:

\[ F = f(C_s, C_r, D) \]  
\[ C_s = f(F) \]  
\[ C_r = f(F) \]

where,

\[ F \]: flow distribution in the traffic system,  
\[ D \]: set of all trip makers,  
\[ C_s \]: signal control policy, and  
\[ C_r \]: route control policy.

As shown in equation (1), flow distribution is the function of signal control and route control. \( D \), the set of trip makers, is assumed to be fixed in this study. The decision variable of signal control and route control are flow distributions. In this representation, signal control \( (C_s) \) and route control \( (C_r) \) are considered independently.

A more advanced system would combine signal and route control as an integrated system. It can be described as:

\[ C_s = f(F, C_r) \]  
\[ C_r = f(F, C_s) \]

However, two basic questions arise in these representations. The first question is on whether or not the dynamic equilibrium exists, and the other question is on how to find such equilibrium. In this study, a solution procedure is constructed to illustrate the evolution of flow distributions under the interaction of signal and route control.

### 3.2 Bi-level Framework

The proposed bi-level framework is illustrated in Figure 1. The upper level deals with signal control strategies under given flow distributions, while the lower level deals with UEDTA route assignment under assumed traffic control policies. The objective of the upper level is to minimize the total delay, while the objective of the lower level is to find the UEDTA state. The UEDTA principle implies that the travel times are the same for trip makers with the same O-D pair departing at the same time. The new flow patterns are again used in designing signal parameters. The process is repeated until the equilibrium state is reached.

Dynamic equilibrium is defined as the condition when flow variations for most links are stable, i.e. when the flow distributions are invariant from iteration \( i-1 \) to iteration \( i \). The threshold of the difference is set as \( \varepsilon \).

\[ \frac{F_i - F_{i-1}}{F_{i-1}} \leq \varepsilon \]  

where,

\[ F_i \]: flow in \( i \)th iteration,
As shown in Figure 1, with a set of initial flow pattern \((F_0)\), the signal settings \((S_i, i = 1)\) are updated according to the signal control policy (such as Webster’s formulation or actuated signal control) for each intersection. DynaTAIWAN is then used to generate the new UEDTA flow patterns.

Figure 1. Process of the combined problem using bi-level method

### 3.3 Dynamic User Equilibrium

In this research, a simulated-based DTA procedure is applied to generate UEDTA flow patterns. The User Equilibrium Dynamic Traffic Assignment (DUE) assignment rule extends Wardrop's UE conditions to the time-dependent case. Under real-time descriptive information on network conditions, a time-dependent UE would hold when no user can improve his/her individual cost by unilateral route switching. However, there is no theoretical or empirical justification to expect convergence to a UE pattern under inherently dynamic conditions.

The process is shown in Figure 2. In the UEDTA algorithm, vehicles are assigned into the simulation network according to their own properties, vehicle types and routes. The simulation calculates the UE path according to the updated travel and delay time from the flow pattern.

A brief summary of the solution approach is described below:

Step 1. Set the iteration counter \(i = 0\). Obtain initial paths for all user classes for each
Step 2. Assign the O-D demands to the given paths and simulate the resulting traffic pattern using Sim-DynaTAIWAN. The path for each vehicle is obtained from the current simulation experience according to its behavioral assumption.

Step 3. Compute the average travel times on links and the number of vehicles on links obtained in post-simulation data (from Step 2).

Step 4. Using the time-dependent shortest path algorithm, compute the shortest travel time paths for each O-D pair and assignment time step.

Step 5. Perform an all-or-nothing assignment on all O-D demands for each UE user class using the shortest travel time paths computed in the previous step.

Step 6. Update paths and the number of users assigned to those paths for the UE user class. Update of paths is done by checking if the path identified in Step 4 already exists (i.e., has carried vehicles in at least one prior iteration) for that O-D pair and including it if it does not. The update of the number of vehicles (assignment of vehicles to the various paths currently defined between the O-D pair after the path update) is performed using the Method of Successive Averages (MSA), which takes a convex combination of the current path and corresponding auxiliary path numbers of vehicles, for each O-D pair and each time step.

Step 7. Check for convergence using an $\varepsilon$-convergence criterion (currently, the
convergence of the path vehicle numbers for UE user class is used as the criterion).

Step 8. If convergence criterion is satisfied, stop the program. Otherwise, update the iteration counter $i = i + 1$ and go to Step 2 with the updated data on paths and the number of vehicles assigned to each of those paths for the UE user class; and with the paths obtained in Step 2 of the current iteration as the initial paths for vehicles of user class 3 for the next iteration.

3.4 Signal Setting

In this research, two types of signal control are considered: pre-timed and actuated signal control.

3.4.1 Pre-timed Signal Control

Pre-timed control is the simplest form of signalization. In pre-timed signal control, the cycle length, phases, green times, and change intervals are all preset. The signal is updated through this defined cycle in repetitive order. Depending on the controller equipment, several preset timing patterns may be used. Multi-dial controllers provide for different timing plans that can be initiated by time-clock at preset times of the day. For example, three-dial controllers usually provide different signal timing plans for AM, off-peak, and PM conditions.

In a coordinated progressive signal system, the cycle length and the initiation of green phases are fixed. This coordination is achieved by offsets, which are defined as the time displacement of one signal relative to neighboring signals.

In this research, Webster formula is applied in pre-timed signal settings. Webster developed the optimum cycle length formula according to minimum total vehicle delay. In an isolated intersection, the cycle length can be calculated according to equation (7).

$$C_0 = \frac{1.5L + 5}{1 - \sum_{i=1}^{n} Y_i}$$

where

$C_0$ : optimum cycle length (sec)
$L$ : total lost time (sec)
$L = nL + R$
$n$ : number of phases
$l$ : average lost time per phase (sec)
$R$ : time duration when all signals display red light (sec)
$Y_i$ : critical lane flow, saturation flow (veh/hr)

3.4.2 Actuated Signal Control

Instead of detecting individual vehicles, this research uses an approximate macroscopic method that determines equivalent green times that are updated to reflect prevailing approach volumes.
The approach is illustrated in Figure 3. In this method, the green time for a given phase is determined based on the number of vehicles that would have reached the intersection at the end of the current simulation intervals. This green is subsequently extended as appropriate at each simulation interval until "max out" is reached, or terminated if no longer needed, thereby emulating "gap out".

Assume the current time is \( t \), and the start time of the green of phase \( i \) is \( t_{i,s} \). The current green time of phase \( i \) is \( G_{i,T-1} \), which is assigned to phase \( i \) at the simulation interval \( T - 1 \). For each simulation interval, the green time is evaluated to determine whether the extension is needed. Vehicles that would have reached the stop line at the end of the simulation time interval \( T \) for phase \( i \) is denoted as \( VR_{i,T} \). If the remaining green time, \( G_{i,T-1} - (t - t_{i,s}) \), is not enough to accommodate \( VR_{i,T} \), the green time is extended appropriately to accommodate the incoming vehicles. If the allocated \( G_{i,T} \) exceeds the maximum green time, maximum green time is specified. Otherwise, the signal enters the changing interval and switches to another phase. Thus, the next phase \( j \) starts at time \( t_{j,s} = G_{i,T} + (t_{i,s} + y_i) \) and the green time \( G_{j,T} = G_{\text{min}} \).

4. NUMERICAL EXPERIMENTS

4.1 Network Description

Numerical experiments are conducted in a sub-network of Kaohsiung City (Figure 4), which includes 27 demand zones, 132 nodes (56 signalized intersections) and 363 links. The
network consists of several arterials and one highway (No. 1 Highway). The O-D demand data is generated from transportation planning projects. The total number of vehicles generated is about 85,300 and the number of motorcycles is about 46,700.

The experiments are summarized as follows:

1. Basic Experiments
   The basic experiments are designed to observe basic network characteristics.

2. Experiments based on pre-time signal assumptions
   The experiments are used to illustrate the framework under pre-timed signal strategy, and observe the equilibrium process and result.

3. Experiments based on actuated signal assumptions
   The experiments are used to illustrate the framework under actuated signal strategy.

### 4.2 Basic experiment

In these experiments, the initial route assignment follows the current best paths for trip makers. The results are summarized in Table 3. There are three demand levels, 0.5, 0.75, and 1.0. The pre-timed signal policy is used in these basic experiments. When the demand factor is equal to 0.5, the average travel time (ATT) is 11.68 min/vehicle, and the average travel distance (ATD) is 5.57 km/vehicle. When the demand factor increases to 0.5 and 1.0, the ATTs increase to 23.1 minutes and 50.84 minutes, respectively. The results show the exponential growth of average travel time with respect to demand levels.

<table>
<thead>
<tr>
<th>Demand Level</th>
<th>Car Veh #</th>
<th>Motor Veh #</th>
<th>System Att (minutes)</th>
<th>Car System Att (minutes)</th>
<th>Motor System Att (minutes)</th>
<th>Car System Ast (m)</th>
<th>Motor System Ast (m)</th>
<th>Car System Atd (m)</th>
<th>Motor System Atd (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>58,857</td>
<td>39,506</td>
<td>19,351</td>
<td>11.68</td>
<td>11.28</td>
<td>11.50</td>
<td>4.12</td>
<td>4.45</td>
<td>3.43</td>
</tr>
<tr>
<td></td>
<td>5,568.52</td>
<td>5,488.34</td>
<td>5,609.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>98,077</td>
<td>60,170</td>
<td>37,907</td>
<td>23.10</td>
<td>24.93</td>
<td>20.19</td>
<td>10.43</td>
<td>13.37</td>
<td>5.75</td>
</tr>
<tr>
<td></td>
<td>5,685.89</td>
<td>5,842.34</td>
<td>5,437.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>132,037</td>
<td>85,294</td>
<td>46,743</td>
<td>50.84</td>
<td>53.66</td>
<td>45.68</td>
<td>27.30</td>
<td>31.92</td>
<td>18.86</td>
</tr>
<tr>
<td></td>
<td>5,980.12</td>
<td>6,175.26</td>
<td>5,624.06</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.3 Results Analysis: Pre-timed Signal Policy

Numerical experiments are conducted to observe flow variation under the interaction of flow distributions and pre-timed signal settings. Some background conditions and parameter settings are as follows:

- **Demand level:** (Car, Motorcycle) = (1.0, 1.0)
- **Total number of vehicles:** about 132,000
- **Maximum iteration:** 10
- **Period of analysis:** 30-90 minutes simulation time

(1) Differences in Flows

The differences in flows between consecutive iterations are used to observe the changes in network flow distribution under $C_s$ and $C_r$. The difference in flow is defined as

$$D_i = \left(\frac{\text{Flow}_i - \text{Flow}_{i-1}}{\text{Flow}_i}\right) \times 100\%.$$  

As shown in Table 4, the links are categorized with respect to level of difference, and the results show the number of links for each category from iteration to iteration. There are a total of 363 links and 5 levels. The number of links in the first level (< 20%) increase from 156 (42.98%) to about 302 (83.20%) with respect to iterations. The number of stable links is about 302 when the equilibrium state is reached ($D_i=83.20\%$). The results show that the differences in flows decrease which indicate that the flow distributions tend to reach stable conditions.

<table>
<thead>
<tr>
<th>Compare case</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>$D_5$</th>
<th>$D_6$</th>
<th>$D_7$</th>
<th>$D_8$</th>
<th>$D_9$</th>
<th>$D_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;20%</td>
<td>156</td>
<td>246</td>
<td>266</td>
<td>251</td>
<td>279</td>
<td>285</td>
<td>260</td>
<td>281</td>
<td>302</td>
<td>N</td>
</tr>
<tr>
<td>% of links (&lt;20%)</td>
<td>42.98%</td>
<td>67.77%</td>
<td>73.28%</td>
<td>69.15%</td>
<td>76.86%</td>
<td>78.51%</td>
<td>71.63%</td>
<td>77.41%</td>
<td>83.20%</td>
<td></td>
</tr>
<tr>
<td>20-40%</td>
<td>94</td>
<td>72</td>
<td>59</td>
<td>68</td>
<td>52</td>
<td>56</td>
<td>76</td>
<td>59</td>
<td>44</td>
<td>N</td>
</tr>
<tr>
<td>40-60%</td>
<td>78</td>
<td>31</td>
<td>27</td>
<td>33</td>
<td>20</td>
<td>16</td>
<td>17</td>
<td>12</td>
<td>10</td>
<td>N</td>
</tr>
<tr>
<td>60-80%</td>
<td>20</td>
<td>10</td>
<td>4</td>
<td>5</td>
<td>7</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>N</td>
</tr>
<tr>
<td>&gt;100%</td>
<td>15</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>3</td>
<td>6</td>
<td>8</td>
<td>4</td>
<td>Y</td>
</tr>
</tbody>
</table>

The network-level differences can be observed through measured value indexes MAPE and RMPSE. The two indexes are calculated using equations (8) and (9).

$$\text{MAPE} = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{Y(I) - Y'(I)}{Y(I)} \right| \times 100\%$$  

$$\text{RMSPE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{Y(I) - Y'(I)}{Y(I)} \right)^2}$$

Where,

- $Y(I)$ : observation value,
- $Y'(I)$ : forecast value, and
- $n$ : number of sample.
MAPE is a useful measure in comparing the accuracy of forecasts between different items since it measures relative performance. The interpretation of MAPE values is shown in Table 5 (Lewis, 1982). The results of MAPE and RMSPE are summarized in Table 6. The results show that a value of MAPE lower than 20% indicates good forecasting. The values of MAPE and RMSPE decrease with respect to iterations and both measures indicate that the network is reaching an equilibrium state.

<table>
<thead>
<tr>
<th>MAPE (%)</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;10</td>
<td>Highly accurate forecasting</td>
</tr>
<tr>
<td>10-20</td>
<td>Good forecasting</td>
</tr>
<tr>
<td>20-50</td>
<td>Reasonable forecasting</td>
</tr>
<tr>
<td>&gt;50</td>
<td>Inaccurate forecasting</td>
</tr>
</tbody>
</table>

(Reference: Lewis, 1982)

(2) Variation of Average Travel Time

Another alternative performance index is average travel time, based on the simulation results during 30-90 minutes. The results of average travel time for these iterations are summarized in Table 7. In the base case as described in Section 4.1, the average travel time during the 30-90 minute simulation time is about 91.17 minutes. However, under the bi-level framework applied with pre-timed signal control strategy, average travel times range from 74 to 96 minutes.

The convergence index α for UEDTA reaches about 0.87, and this shows that the UEDTA equilibrium state is obtained in each iteration. But in each iteration, when the UEDTA flow pattern is provided to the upper level to calculate signal settings, the new flow patterns which are affected by the new signal settings varies until iteration 10 (where η=83.20% > 80%). In dynamic equilibrium status, the average travel time of vehicles during the 30-90 minute simulation time is 84.95 minutes.

<table>
<thead>
<tr>
<th>Compare case</th>
<th>$D_1$</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>$D_5$</th>
<th>$D_6$</th>
<th>$D_7$</th>
<th>$D_8$</th>
<th>$D_9$</th>
<th>$D_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSPE (%)</td>
<td>37.29</td>
<td>25.03</td>
<td>24.32</td>
<td>25.27</td>
<td>22.58</td>
<td>20.29</td>
<td>22.44</td>
<td>22.10</td>
<td>19.43</td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Average travel time of vehicles during the 30-90min simulation time
(pre-timed signal policy)

**Table 5. Interpretation of MAPE values**

<table>
<thead>
<tr>
<th>MAPE (%)</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;10</td>
<td>Highly accurate forecasting</td>
</tr>
<tr>
<td>10-20</td>
<td>Good forecasting</td>
</tr>
<tr>
<td>20-50</td>
<td>Reasonable forecasting</td>
</tr>
<tr>
<td>&gt;50</td>
<td>Inaccurate forecasting</td>
</tr>
</tbody>
</table>

* UEDTA equilibrium threshold = 0.8
4.4 Results Analysis: Actuated Signal Policy

Numerical experiments are conducted to observe flow variations under the interaction of flow distributions and actuated signal settings. The experimental parameters are summarized as follows:

- Actuated Signal Policy: (minimum green time, maximum green time) = (25, 40)
- Demand level: (Car, Motorcycle) = (1.0, 1.0)
- Total number of vehicles: about 132,000
- Maximum iteration: 10
- Period of analysis: 30-90 minutes simulation time

(1) Differences in Flows

The results are shown in Table 8. The number of links in the first level (< 20%) increase from 127 (34.99%) to 331 (91.18%) as the number of iterations increase. The number of links is about 302 ($D_y = 83.20\%$) when the equilibrium state is reached.

Table 8. $D_y$ frequency table (actuated signal policy)

<table>
<thead>
<tr>
<th>Case</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>$D_5$</th>
<th>$D_6$</th>
<th>$D_7$</th>
<th>$D_8$</th>
<th>$D_9$</th>
<th>$D_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;20%</td>
<td>127</td>
<td>256</td>
<td>302</td>
<td>302</td>
<td>328</td>
<td>332</td>
<td>330</td>
<td>327</td>
<td>331</td>
</tr>
<tr>
<td>/363 (%)</td>
<td>34.99%</td>
<td>70.52%</td>
<td>83.20%</td>
<td>83.20%</td>
<td>90.36%</td>
<td>91.46%</td>
<td>90.08%</td>
<td>91.18%</td>
<td></td>
</tr>
<tr>
<td>20-40%</td>
<td>101</td>
<td>83</td>
<td>43</td>
<td>37</td>
<td>24</td>
<td>22</td>
<td>18</td>
<td>24</td>
<td>20</td>
</tr>
<tr>
<td>40-60%</td>
<td>97</td>
<td>11</td>
<td>16</td>
<td>13</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>60-80%</td>
<td>23</td>
<td>8</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>8</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>&gt;100%</td>
<td>15</td>
<td>5</td>
<td>0</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Dynamic equilibrium?</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>($\eta&gt;80%, \epsilon&lt;0.2$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results of MAPE and RMSPE are summarized in Table 9. The results show that the value of MAPE is lower than 20% which indicates good forecasting. After the fifth iteration, the values of MAPE are lower than 10% which indicates ‘highly accurate forecasting’. Similar results are observed in the RMSPE index. The values of MAPE and RMPSE decrease with respect to iterations, which indicate that the network is reaching an equilibrium state.

Table 9. Measured values of $D_y$ (actuated signal policy)

<table>
<thead>
<tr>
<th>Compare case</th>
<th>$D_2$</th>
<th>$D_3$</th>
<th>$D_4$</th>
<th>$D_5$</th>
<th>$D_6$</th>
<th>$D_7$</th>
<th>$D_8$</th>
<th>$D_9$</th>
<th>$D_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAPE(%)</td>
<td>32.31</td>
<td>15.95</td>
<td>10.36</td>
<td>10.99</td>
<td>8.03</td>
<td>6.71</td>
<td>8.04</td>
<td>7.81</td>
<td>8.40</td>
</tr>
<tr>
<td>RMSPE(%)</td>
<td>39.86</td>
<td>23.51</td>
<td>16.34</td>
<td>20.35</td>
<td>15.11</td>
<td>13.04</td>
<td>16.09</td>
<td>15.23</td>
<td>16.93</td>
</tr>
</tbody>
</table>

(2) Variations of Average Travel Time

The results of average travel time for the various iterations are summarized in Table 10. The ATTs range from 88 to 97 minutes. In some of the iterations, the ATT is longer than the base case (91.17 minutes). Although the flows under actuated control policy reach convergence conditions, the benefit in terms of travel time is not observed. The convergence index $\alpha$ of UEDTA increases from 0.58 to 0.86, and this shows that the UEDTA equilibrium state is obtained after iteration 3. Dynamic equilibrium is attained in iteration 4 and the average travel time of vehicles during the 30-90 minute simulation time is 83.79 minutes.
Table 10. Average travel time of vehicles for the 30-90min simulation time (actuated signal policy)

<table>
<thead>
<tr>
<th>ATT-iteration</th>
<th>Based</th>
<th>ATT-1</th>
<th>ATT-2</th>
<th>ATT-3</th>
<th>ATT-4</th>
<th>ATT-5</th>
<th>ATT-6</th>
<th>ATT-7</th>
<th>ATT-8</th>
<th>ATT-9</th>
<th>ATT-10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Travel time (min)</td>
<td>91.17</td>
<td>97.02</td>
<td>88.12</td>
<td>95.03</td>
<td>83.79</td>
<td>92.08</td>
<td>90.02</td>
<td>93.61</td>
<td>86.50</td>
<td>89.18</td>
<td>97.43</td>
</tr>
<tr>
<td>α of DUE</td>
<td>-</td>
<td>0.5823</td>
<td>0.7729</td>
<td>0.8221</td>
<td>0.8449</td>
<td>0.8502</td>
<td>0.8545</td>
<td>0.8591</td>
<td>0.8605</td>
<td>0.8619</td>
<td></td>
</tr>
<tr>
<td>DUE equilibrium</td>
<td>-</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Dynamic equilibrium (as Table 8)</td>
<td>-</td>
<td>N</td>
<td>N</td>
<td>N</td>
<td>Y</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* UEDTA equilibrium threshold = 0.8

From Table 7 and Table 10, the difference between the two signal control policies can be observed. The variation in actuated signal control is less compared with the pre-timed control policy, and the average travel times under the pre-timed signal policy are generally better than the actuated signal policy.

5. CONCLUSION AND SUGGESTION

This research proposes a bi-level framework for solving the network equilibrium problem in combination with the signal control problem. The UEDTA principle was applied to the trip makers’ route decision and two signal control policies were used for signal settings optimization. Unique features of this research include the consideration for a changeable cycle length and time-dependent UE assignment.

From the results of the experiments, variations in link flows become stable with respect to the number of executed iterations. The results illustrate equilibrium flow conditions under signal control and route assignment. However, the average travel times under pre-timed signal policy are generally better than the actuated signal policy.

Although the existence of dynamic equilibrium was illustrated from the numerical experiment, a more rigorous formulation is needed to be developed to prove equilibrium from a theoretical point of view. In addition to delays at intersections, other performance measurements for arterial or network optimization can be considered in future researches.

REFERENCES


Smith, M.J. (1980) A local traffic control policy which automatically maximises the overall travel capacity of an urban road network. *Traffic Engineering and Control*, 21 (6),
298-302.