Investigating Travel Time Reliability Measures in Toll Design Problem

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Abstract: Congestion pricing has been regarded as an effective method of reducing network-wide travel cost. Previous work on the toll-design problem focused on the deterministic network, assuming that the network operates deterministically without uncertainty. This essentially precludes investigating the recently proposed travel time reliability indices as a system performance measure to evaluate and prioritize the selection of future investment strategies. In this study, we investigate various recently proposed travel time reliability measures in determining the optimal toll strategies to improve travel time reliability in the network. A simulation-based genetic algorithm (SGA) procedure is developed to solve the stochastic bi-level programming problem. Our experimental results using a simple test network show that optimal toll determined by the misery index can improve both travel time reliability for network users and total travel time for the network planner.

Key Words: Travel time reliability, Network design problem, Toll pricing, Stochastic program

1. INTRODUCTION

Travel time, delay, and reliability are important performance indicators that characterize transportation system mobility and reliability. These indicators can serve as the basis for both monitoring transportation system performance in a manner that is directly relevant to the system users, and at the same time being linked closely to evaluation and prioritization criteria that influence the selection of future investment strategies. Travel time, delay, and reliability are user-oriented measures since they focus on the total trip experience of the traveler or goods transporter and can be appreciated and understood by the system users. Therefore individuals can use these measurements to make better informed decisions to reduce the uncertainty and loss of productivity due to congestion. Such measures will be also useful in transportation planning and decision making for capital and operational investments and for quality-of-service monitoring and evaluation.

In addition, many recent empirical studies (Abdel-Aty et al., 1995; Small et al., 1999; Lam, 2000; Brownstone et al., 2003; Cambridge Systematics et al., 2003; van Lint et al., 2008; Franklin and Karlström, 2009) have revealed that travel time reliability plays an important role in travelers’ route choice decisions. Abdel-Aty et al. (1995) found that travel time reliability was either the most or second most important factor for most commuters. In the study by Small et al. (1999), they found that both individual travelers and freight carriers were strongly averse to scheduling mismatches. For this reason, they were willing to pay a premium to avoid congestion and to achieve greater reliability in travel times. From the two
value-pricing projects in Southern California (California Department of Transportation, Sacramento, 1998), Lam (2000) and Brownstone et al. (2003) also consistently found that travelers were willing to pay a substantial amount to reduce variability in travel time (or to improve travel time reliability). Based on the empirical data collected on the Netherlands freeways, travel time distributions are not only very wide but also heavily skewed with a long tail (van Lint et al., 2008). The implication of these positively skewed travel time distributions has a significant impact on travelers’ route choice decisions. For example, it has been shown that about 5% of the “unlucky drivers” incur almost five times as much delay as the 50% of the “fortunate drivers” on the densely used freeway corridors in the Netherlands. In the United States, the cost of unexpected delay for truck freight is estimated to be 50 to 250 percent higher than the expected delay cost (FHWA, 2004). Recently, Franklin and Karlström (2009) estimated the “mean lateness” factor in the departure time choice model, and used it to conduct a benefit-cost analysis for the roadway projects in Stockholm, Sweden. These empirical studies show that travel time distribution is not only heavily skewed with a long fat tail, but also the consequences of these late trips may be much more serious than those of modest delays, and can have a significant impact on both travelers and freight shippers and carriers.

However, most toll design problems in the literature focused on minimizing total travel time, maximizing social welfare, or maximizing revenue or profit without considering network uncertainty (e.g., see Yang and Bell, 1998). Only a few studies examined tolling strategies under uncertainty. Gardner et al. (2008) determined the first-best robust pricing scheme using a mean-variance model under demand uncertainty, and Li et al. (2008) applied the mean-standard deviation model to the toll design under both demand and supply uncertainty. However, these two models only consider variance (or standard deviation) of travel time as a travel time reliability measure in determining the optimal toll. In this paper, we investigate various recently proposed travel time reliability measures to determine the optimal toll strategies to improve travel time reliability in the network. The remainder of the paper is organized as follows. In section 2, several recently proposed travel time reliability measures are reviewed. Section 3 presents the toll design problem as a stochastic bi-level programming (SBLP) formulation. Section 4 describes a simulation-based genetic algorithm procedure for solving the SBLP formulation. Section 5 presents the numerical results using selected travel time reliability measures in the toll design problem. Section 6 provides some concluding remarks.

2. TRAVEL TIME RELIABILITY MEASURES

Reliability measures are statistical inferences in relation to the expected condition during any given time and could vary depending travelers’ expectation. The reliability measures inferred from historical travel time data need definitional agreement and a guideline in order to be used as objective system performance measures. There are also many different sources of travel time variability, such as incident, weather, special events, road maintenance and construction work, traffic control systems, and fluctuations in demand. Reliability measures themselves are useful measures for transportation planners and operators to identify problems and improve transportation systems.

Turner et al. (1996) defined travel time reliability as the range of travel times experienced during a large number of daily trips. The range of travel time is based on the 85th percentile of
travel time distribution. However, the range of travel time is not very meaningful unless it is used to make comparisons of conditions along the same facility. Lomax et al. (2001) defined reliability as the difference in delay experienced on incident days versus non-incident days. This measure does not consider recurrent delay. Ikhrate and Michell (1997) defined reliability as the probability of arriving within the expected travel time. This measure does not reflect the variation in travel time. In the Urban Mobility Report, Schrank and Lomax (2002) proposed the buffer index as the difference between the average travel time and the 95th percentile travel time. It implicitly assumes that 95th percentile travel time is an acceptable threshold, which is higher than the 85th percentile used in the travel time range method.

On the other hand, state DOTs are using different measures for reliability. California Transportation Plan (Booz-Allen & Hamilton, Inc., 1998) defined reliability as the variability between the expected travel time and the actual travel time. Florida DOT adopted the method by Shaw and McLeod (1998) by defining acceptable travel time as the average travel time plus the acceptable additional time (marginal time). The reliability is the percentage of travel that is acceptable (or the travel time less than the acceptable travel time). It is obvious that the reliability measure should be able to reflect both variance of travel time and the acceptance of users. Also different aggregate levels (e.g., road segment, corridor, and network) need different reliability measures. van Lint et al. (2008), on the other hand, focused on travel time unreliability on the Netherlands freeways. Since most of the travel time distributions are not only very wide but also heavily skewed with a long fat tail, they proposed some skew and width measures for assessing travel time (un)reliability.

In short, the various travel time reliability measures can be summarized into five categories as shown in Table 1 and Figure 1, and are briefly discussed below.

- **Statistical range** represents the most often theorized or conceptualized measures. Typically standard deviation statistics are used to present an estimate of the range of transportation conditions that might be experienced by travelers. These usually appear as “variability” measures.
- **Buffer time** measures indicate the effect of irregular conditions in the form of the amount of extra time that must be allowed for a traveler to achieve their destination in a high percentage of the trips. In practice this might be thought of as “I need to allow enough time so that I arrive on-time for (some percentage) of my trips”. These measures usually illustrate “reliability”.
- **Tardy trip indicators** answer the question “how often will a traveler be unacceptably late?” The time can be either a percentage of the trip time, an increased time in minutes above the average or some absolute value in minutes. These measures usually illustrate “reliability”.
- **Probabilistic measures** represent the probability that a trip between a given origin-destination pair can be made successfully within a given time interval and at a specified level-of-service (Asakura and Kashiwadani, 1991; Bell et al., 1999).
- **Skew and width measures** attempt to measure the skew and width of the travel time distribution using percentiles. A large skew value indicates that the probability of encountering extreme travel times (relative to the median) is high, while a large width value indicates the range (or width) or the travel time distribution is large relative to its median value.
Table 1. Summary of travel time reliability measures (modified from Lomax et al. 2003 and van Lint et al. 2008)

<table>
<thead>
<tr>
<th>Category</th>
<th>Name</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical range</td>
<td>Travel time window</td>
<td>Average travel time ± standard deviation</td>
</tr>
<tr>
<td></td>
<td>Percent variation</td>
<td>Standard deviation / average travel time × 100</td>
</tr>
<tr>
<td></td>
<td>Variability index</td>
<td>Difference in peak-period confidence intervals / difference in off-peak-period confidence intervals</td>
</tr>
<tr>
<td>Buffer time measures</td>
<td>Buffer time</td>
<td>95th percent travel time – average travel time</td>
</tr>
<tr>
<td></td>
<td>Buffer index</td>
<td>(95th percent travel time – average travel time) / average travel time×100%</td>
</tr>
<tr>
<td></td>
<td>Planning time index</td>
<td>95th percentile travel time index</td>
</tr>
<tr>
<td>Tardy trip indicators</td>
<td>Florida reliability index</td>
<td>100% - (percent of trips with travel times greater than expected)</td>
</tr>
<tr>
<td></td>
<td>On-time arrival</td>
<td>100% - (percent of travel rates greater than 110% of the average travel rate)</td>
</tr>
<tr>
<td></td>
<td>Misery index</td>
<td>(Average of the travel rates for the longest 20% of the trips – average travel rates for all trips) / average travel rate</td>
</tr>
<tr>
<td>Probabilistic</td>
<td>Probabilistic</td>
<td>Pr (travel time &gt; α · travel time threshold)</td>
</tr>
<tr>
<td>Skew and width measures</td>
<td>( \lambda^{var} )</td>
<td>(90th percentile travel time – 10th percentile travel time) / (50th percentile travel time)</td>
</tr>
<tr>
<td></td>
<td>( \lambda^{skew} )</td>
<td>(90th percentile travel time – 50th percentile travel time) / (50th percentile travel time – 10th percentile travel time)</td>
</tr>
<tr>
<td></td>
<td>( UI_r )</td>
<td>( \lambda^{var} \ln(\lambda^{skew}) / ) travel time per unit length</td>
</tr>
</tbody>
</table>

3. MODEL FORMULATION

For convenience, notation is provided first, followed by demand uncertainty generation, travel time reliability measures, and the toll design problem formulated as a stochastic bi-level programming problem.

3.1 Notation

- \( A \): set of links in the network
- \( \overline{A} \): set of toll links in the network
- \( W \): set of origin-destination (O-D) pairs
- \( R_w \): set of routes between O-D pairs \( w \in W \)
- \( f_{r,w} \): flow on route \( r \in R_w \)
- \( v_a \): flow on link \( a \in A \)
- \( \mathbf{v} \): vector of link flows
3.2 Demand Uncertainty

To generate random demands, we adopt the method described by Asakura and Kashiwandani (1991). The method is simple and has the ability to generate correlated demands. Let \( Q = (\ldots, Q_w, \ldots)^T \) be the random demand vector defined on the probability space \((\Omega, \Theta, Pr)\), where \( \Omega \) is a set of all outcomes of a random experiment (a non-empty set), \( \Theta \) is called a \( \sigma \)-algebra, and \( Pr \) is referred to as a probability measure. For each \( \omega \in \Omega \), \( q = Q(\omega) \) is a realization of the random demand vector \( Q \) generated according to the following equations:

\[
\begin{align*}
    u_a &: \text{ toll on link } a \in A \\
    u_a^{\text{max}} &: \text{ upper bound of } u_a \\
    u &: \text{ vector of link tolls} \\
    t_a(v_a, u_a) &: \text{ travel time on link } a \in A \\
    Q_w &: \text{ random demand between O-D pair } w \in W \\
    Q &= \text{ vector of random O-D demands } Q_w \\
    q_w &: \text{ realization of } Q_w \\
    q &: \text{ vector of realization } q_w \\
    \delta^{\omega}_{rw} &: \text{ 1 if route } r \text{ of O-D pair } w \text{ uses link } a, \text{ and 0 otherwise}
\end{align*}
\]
\[ q^w_n = \bar{q}^w (1 + \varepsilon^w_n) \]  
\[ \varepsilon^w_n = \sigma (\lambda \delta^w_n + (1 - \lambda) \eta^w_n) \]  

where \( \bar{q}^w \) is the mean demand of OD pair \( w \); \( q^w_n \) is the \( n \)th realization of random demand of OD pair \( w \); \( \sigma \) is scale parameter; \( \lambda \) is the correlation parameter, \( 0.0 \leq \lambda \leq 1.0 \); \( \delta^w_n \) and \( \eta^w_n \) are independent random variables from the standard normal distribution \( N(0,1) \). The random part of OD demand is described by using two random variables, \( \delta \) and \( \eta \). The value of the scale parameter \( \sigma \) corresponds to the value of the coefficient of variation (COV) of OD demand (assumed to be common for every OD pair). The parameter \( \lambda \) represents the correlation of random demand among OD pairs. The correlation coefficient between each OD pairs is given by

\[ R = \frac{(1 - \lambda)^2}{(1 - \lambda)^2 + \lambda^2} \]  

where \( R = 1.0 \) if \( \lambda = 0.0 \); \( R = 0.0 \) if \( \lambda = 1.0 \).

### 3.3 Travel Time Reliability Measures

In this study, we investigate selected travel time reliability measures in Table 1 to be used as performance measure in the toll design problem. Specifically, we select one measure in each category (standard deviation, buffer time index, misery index, probabilistic, and skew measures) as the travel time reliability measure to determine the optimal toll.

#### Standard deviation (SD):

\[ F_1(v(u, Q), u) = \frac{\sum_{a \in A} v^a_d(u, Q) \cdot \sigma_a(u, Q)}{\sum_{a \in A} v^a_d(u, Q)} \]  

where \( v^a_d(u, Q) \) is the average flow on link \( a \in A \) and \( \sigma_a(u, Q) \) is the standard deviation of travel time on link \( a \in A \).

#### Buffer time index (BI):

\[ F_2(v(u, Q), u) = \frac{\sum_{a \in A} VMT_a^d(u, Q) \cdot \left( \frac{95^{th} TT_a(u, Q) - \mu_a(u, Q)}{\mu_a(u, Q)} \right)}{\sum_{a \in A} VMT_a^d(u, Q)} \]  

where \( VMT_a^d(u, Q) \) is the vehicle miles traveled on link \( a \in A \), \( 95^{th} TT_a(u, Q) \) is the 95th percentile of travel time on link \( a \in A \), and \( \mu_a(u, Q) \) is the average travel time on link \( a \in A \).

#### Misery index (MI):

\[ F_3(v(u, Q), u) = \frac{\sum_{a \in A} VMT_a^d(u, Q) \cdot \left( \frac{20^{th} TT_a(u, Q) - \mu_a(u, Q)}{\mu_a(u, Q)} \right)}{\sum_{a \in A} VMT_a^d(u, Q)} \]  

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where $20^\text{th} TT_a(u, Q)$ is the 20th worst percentile of travel time on link $a \in A$.

**Probabilistic measure:**

$$F_1(v(u, Q), u) = \frac{\sum_{a \in A} v_a^f(u, Q) \cdot \Pr(t_a(u, Q) \geq \alpha \cdot \mu_a(u, Q))}{\sum_{a \in A} v_a^f(u, Q)}$$

(5)

where $\Pr(.)$ is a probability operator, $\alpha$ is a threshold value (e.g., $\alpha=1.2$ or 20% higher than the average travel time), and $t_a(u, Q)$ is the random travel time on link $a \in A$.

**Skew measure ($\lambda_{skew}$):**

$$F_3(v(u, Q), u) = \frac{\sum_{a \in A} v_a^f(u, Q) \cdot \left(\frac{90^\text{th} TT_a(u, Q) - 50^\text{th} TT_a(u, Q)}{90^\text{th} TT_a(u, Q) - 10^\text{th} TT_a(u, Q)}\right)}{\sum_{a \in A} v_a^f(u, Q)}$$

(6)

where $10^\text{th} TT_a(u, Q)$, $50^\text{th} TT_a(u, Q)$, and $90^\text{th} TT_a(u, Q)$ are the 10th, 50th, and 90th percentiles of travel time on link $a \in A$, respectively.

### 3.4 Toll Design Problem

The toll design problem under demand uncertainty can be formulated as a stochastic bi-level programming problem. The upper-level subprogram is to determine the optimal link tolls by optimizing one of the travel time reliability measures given in Section 3.3 subject to some constraints.

$$\min_{u} F_1((v(u, Q), u))$$

(7a)

s.t. $\sum_{a \in A} u_a \cdot v_a \geq G,$

(7b)

$0 \leq u_a \leq u_a^{max}, \quad \forall a \in \bar{A},$

(7c)

where $F_1((v(u, Q), u))$ is a travel time reliability measure; $\bar{A}$ is the set of toll links in the network; $u_a$ is the toll on link $a$; $v_a$ is the flow on link $a$; $G$ is the minimum revenue; and $u_a^{max}$ is the upper bound of toll on link $a$. For a realization $q$ of the random demand $Q$, $v_a(u, q)$ is the equilibrium flow on link $a$, which can be obtained by solving the lower-level subprogram as shown below:

$$\min_{v} \sum_{a \in A} \int_{\Omega} t_a(\omega, u_a) d\omega - \sum_{w \in W} \int_{\Omega} D_w^{-1}(\omega) d\omega$$

(8a)

s.t. $\sum_{r \in R_w} f_r^w = q_w, \quad \forall w \in W,$

(8b)

$v_a = \sum_{w \in W} \sum_{r \in R_w} f_r^w \delta_{ar}^w, \quad \forall a \in A,$

(8c)

$f_r^w \geq 0, \quad \forall r \in R, \ w \in W,$

(8d)

$q_w \geq 0, \quad \forall w \in W$

(8e)
where \( A \) is the set of toll links in the network; \( W \) is the set of O-D pairs; \( R \) is the set of routes between O-D pairs \( w \in W \); \( t_r(\omega, u_a) \) is the travel time on link \( a \); \( D_w^i() \) is the inverse demand function of O-D pair \( w \); \( f_r^w \) is the flow on route \( r \) between O-D pair \( w \); \( q_w \) is the demand of O-D pair \( w \); \( \delta_r \) is the path-link incidence indication: 1 if route \( r \) of O-D pair \( w \) uses link \( a \), and 0 otherwise. Equation (7a) is one of the travel time reliability measures used to determine the optimum toll; equation (7b) imposed a minimum revenue requirement to ensure the viability of the tolling system; and equation (7c) sets the lower and upper bounds of the possible link toll. Equation (8a) is the objective function for the user-equilibrium problem with elastic demand; equation (8b) is the flow conservation constraint; equation (8c) represents the link-path flow relationship; and equations (8d) and (8e) ensures the non-negativity of path flows and O-D flows.

4. SOLUTION PROCEDURE

Stochastic bi-level programs are generally difficult to solve by traditional calculus-based optimization methods. To solve the toll design problem with demand uncertainty, we develop a simulation-based genetic algorithm (SGA) solution procedure to handle the different complexities involved in solving the stochastic bi-level programming formulation. The SGA solution procedure consists of a traffic assignment algorithm, a genetic algorithm, and a Monte-Carlo simulation. The demand uncertainty is addressed by the stochastic (Monte Carlo) simulation. The nonlinear and nonconvex nature of the bi-level program is handled by the genetic algorithm. Bi-level mathematical programs are generally difficult to solve because evaluation of the upper-level objective function requires solving the lower-level subprogram. Here a standard traffic assignment algorithm (known as the convex combination method) is used to solve the lower-level subprogram (Sheffi, 1985). GA implementation generally involves minimizing or maximizing the design variables in the upper-level sub-program as chromosomes, evaluating the fitness of the chromosomes, and performing the basic GA operators (i.e., reproduction, crossover, and mutation) to evolve the chromosomes to obtain better solutions. In the toll pricing problem under demand uncertainty, the chromosomes are represented as a string of real numbers with a length equal to the number of design variables. The major steps of the SGA solution procedure are shown in Figure 2 and are briefly described as follows:

Step 0. Define input parameters: population size, crossover and mutation rates, maximum number of generations, and maximum number of simulations.

Step 1. Generate an initial population that satisfies the minimum revenue constraint and lower and upper bounds of the possible link toll.

Step 2. Evaluate the fitness of all chromosomes in the population pool using the traffic assignment and stochastic simulation procedures.

Step 3. Check whether the predefined maximum generation number is reached or not. If yes, go to step 6; otherwise, go to step 4.

Step 4. Rank the chromosomes based on their fitness values and select new parent chromosomes using the reproduction operator.

Step 5. Update the chromosomes using the crossover and mutation operators, increment the generation index, and go to step 2.

Step 6. Report the best chromosome as the optimal design.
For a detailed description of the SGA solution procedure, the reader may refer to Chen and Yang (2004), Chootinan et al. (2005), and Chen et al. (2006, 2010).

5. NUMERICAL EXAMPLE

5.1 Test network description and GA parameter setting

For the numerical experiments, we illustrate the toll design problem under demand uncertainty using a simple network depicted in Figure 3. The network consists of six nodes, seven links, two origins, two destinations, and four O-D pairs. The link travel time function is based on the standard Bureau of Public Road (BPR) function:

$$t_a = t_a^f \left(1 + 0.15 \left(\frac{v_a}{C_a}\right)^4\right)$$

where $v_a$, $t_a^f$, and $C_a$ are the flow, free-flow travel time, and capacity on link $a$, respectively.

For the O-D demand function, we adopt the following exponential functional form:

$$d_w(c) = \tilde{d}_w \exp(-\gamma c_w), \ w \in W,$$
where \( \hat{d}_w \) is the random potential demand of O-D pair \( w \) generated according to Eq. (2a) in Section 3.2; \( \gamma \) is a scaling parameter which reflects the sensitivity of demand to full trip price (0.05 is used in this study); and \( c_w \) is the travel time of O-D pair \( w \).

The link characteristics are provided in Table 2. The mean demand for O-D pairs (1-3), (1-4), (2-3), and (2-4) are assumed to be 100, 60, 60, and 100, respectively, and \( \lambda = 0.8 \) in Eq. (2b) for generating the random potential demand. For the toll strategy, $1 is used per 10 minutes for links 1, 3, and 5, and the minimum revenue is $790 (i.e., 90% of the maximum revenue).

The following parameters are used in the SGA solution procedure:

- Population size: 128
- Maximum number of generations: 50
- Maximum number of samples: 500
- Probability of crossover: 0.50
- Probability of mutation: 0.80

Without loss of generality, we examine the performance of the SGA solution procedure using the misery index and skewness as the travel time reliability measures in the toll design problem under demand uncertainty. The convergence results of the SGA solution procedure are shown.
in Figure 4. The two reliability measures steadily decrease in the early generations and stabilize to a near-optimal solution after the 25th generation.

![Figure 4. Convergence characteristics](image)

5.2 Numerical results and discussions

The numerical results of the toll design problem using different travel time reliability measures are summarized in Table 3. It consists of the optimal tolls on links 1, 3, and 5 for five travel time reliability measures presented in Section 3.3 as well as the base case (i.e., do-nothing case). For example, when the standard deviation (SD) of travel time is used as the objective function in the toll design problem, the optimal tolls are 4.61, 4.62 and 4.15 for links 1, 3 and 5, respectively. With these recommended tolls, we compute the other travel time reliability measures: Buffer Index (BI), Misery Index (MI), Probability (Prob) and Skewness (Skew). One interesting observation in Table 3 is that when we apply the optimal tolls recommended by the MI measure (4.68, 4.73 and 4.30 for links 1, 3 and 5), the travel time reliability values are better than those determined by the other travel time reliability measures except for the Skew measure. For example, the BI value obtained based on optimizing the BI measure in the toll design problem is 7.42, but this is worse than the BI value calculated from the optimal tolls obtained from optimizing the MI measure (6.83). We attribute this finding to the following reasons. First, the SGA solution procedure does not guarantee the global solution. In the GA process, the population encompasses a range of possible outcomes. Solutions are identified purely on a fitness level, and therefore local optima are not distinguished from other equally fit individuals. Those solutions closer to the global optimum will thus have higher fitness values. Successive generations improve the fitness of individuals in the population until the convergence criterion is met. Due to the probabilistic nature, GA has a higher tendency to find the global optimum. However, GA cannot guarantee finding the optimal solution for the same reasons (Mardle and Pascoe, 1999). Second, the travel time reliability measures in equations (2) to (6) are highly correlated. In other words, we can obtain the minimum value of a particular function through minimizing other objective functions.
Table 3. Optimal tolls and the corresponding travel time reliability values

<table>
<thead>
<tr>
<th>Optimal Toll ($)</th>
<th>Link SD (min)</th>
<th>BI (%)</th>
<th>MI (%)</th>
<th>Prob (%)</th>
<th>Skew (min)</th>
<th>Base case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.61</td>
<td>4.55</td>
<td>4.68</td>
<td>4.61</td>
<td>2.12</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>4.62</td>
<td>4.60</td>
<td>4.73</td>
<td>4.68</td>
<td>2.71</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>4.15</td>
<td>4.05</td>
<td>4.45</td>
<td>4.30</td>
<td>2.41</td>
<td>0.00</td>
</tr>
</tbody>
</table>

All travel time reliability measures from optimization look promising compared to the base case except the skew measure. We attribute this problem to the minimum revenue constraint (Eq. (8b) in the upper-level subprogram). To investigate this problem, we run another experiment to see if we can improve the value of skew measure by varying the amount of minimum revenue requirement from 90% to 10% of the maximum revenue. Table 4 presents the revenue obtained from tolls and its corresponding skew value at different minimum revenue requirement levels.

Table 4. Revenue and skew value at different minimum revenue requirement levels

<table>
<thead>
<tr>
<th>% of maximum revenue</th>
<th>Revenue obtained from tolls</th>
<th>Optimized skew value</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>790</td>
<td>1.3075</td>
</tr>
<tr>
<td>80</td>
<td>702</td>
<td>1.2507</td>
</tr>
<tr>
<td>70</td>
<td>614</td>
<td>1.2180</td>
</tr>
<tr>
<td>60</td>
<td>527</td>
<td>1.1746</td>
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<tr>
<td>40</td>
<td>351</td>
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<td>10</td>
<td>88</td>
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<tr>
<td>Base case</td>
<td>0</td>
<td>1.1729</td>
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</tbody>
</table>

Table 4 shows the tradeoff between the revenue obtained from tolls and the skew value at different minimum revenue requirement levels. As the requirement is relaxed (from 90% to 10%), the revenue collected from tolls drops significantly while the skew value is improved (smaller value is better). Note that when the minimum revenue requirement is 60% or greater, the skew value is worse than the base case (i.e., no tolls). This result implies that collecting revenue greater than 50% may worsen the system’s travel time reliability in terms of the skew measure.

5.3 Travel time distributions at different spatial levels

This section presents the travel time distributions at three spatial levels: (a) link level with toll, (b) O-D level, and (c) system level (see Figures 5 to 7). In each level, we only present the link travel time distributions of the base case and those by minimizing the MI and skew measures. Note that we do not show the link travel time distributions of SD, BI and Prob because they have similar distribution pattern as the MI measure. Specifically, Figure 5 presents the travel
time distributions at the link level for two links with toll; Figure 6 presents the travel time distributions at the O-D level for all four O-D pairs; and Figure 7 presents the total travel time distribution at the system level for the whole network. In each figure, the X-axis is the travel time, while the Y-axis represents the probability density of interest computed using 500 simulations (or trials) of demand uncertainty. As can be seen from Figures 5 to 7, the travel time distributions of the base case (do-nothing without tolls) for all three spatial levels are quite wide and skew toward the right (i.e., high travel times and large variability). Applying tolls to selected links in the network can improve the travel time distributions compared to those of the base case. However, the effect on the travel time distribution is different for different travel time reliability measures adopted in the toll design problem. For example, using the MI measure to determine the link tolls can significantly improve the travel time distributions at all three spatial levels compared to those of the skew measure.

As can be noted from Figures 5(a) and 5(b), the travel time distributions for link 1 and link 5 are improved with tolls determined by the skew measure and are significantly improved with tolls determined by the MI measure (i.e., the travel time distribution shifts toward the left with a smaller variability). Similar improvements are also observed at the O-D level (see Figures 6(a) to 6(d) for the four O-D pairs in the network). Particularly, the travel time distributions for O-D pairs (1-3) and (2-4) have similar characteristics as the travel time distributions for link 1 and link 5 as the tolls on these two links directly affect the main paths serving these two O-D pairs. As for the other two O-D pairs (1-4) and (2-3), the improvements are also significant, but the travel time distributions are not as tight as those of O-D pairs (1-3) and (2-4). For the network level, we show the total travel time distributions in Figure 7. Similar improvements are also observed for the two toll settings determined by the MI and skew measures. It should be noted that these improvements at different spatial levels are possible because some demands are being suppressed by the toll charges (recall the lower-level subprogram is a user-equilibrium with elastic demand). The travel costs under toll settings are considered too high compared to those of the base case without tolls; therefore, some travel demands are either postponed to the off-peak periods or cancelled altogether.

Figure 5. Link travel time distributions
Figure 6. O-D travel time distributions

Figure 7. Total travel time distributions
6. CONCLUSIONS

This paper has presented a toll design problem with the consideration of the recently proposed travel time reliability measures as objective function under demand uncertainty. A stochastic bi-level programming (SBLP) formulation was provided to model the toll design problem under demand uncertainty. The upper-level subprogram minimizes one of the five travel time reliability measures subject to the minimum revenue requirement and lower and upper of tolls, and the lower-level subprogram is a user-equilibrium problem using elastic demand subject to demand uncertainty. Based on the numerical results, using the SD, BI, MI and Prob measures as the objective function in the toll design problem have better results than using those from the skew measure. Particularly, the MI measure appears to be a good candidate as a travel time reliability measure. However, the findings are based on a simple network. We will test our approach on more networks with different topologies and travel demand characteristics to obtain further insights of the recently proposed travel time reliability measures, particularly on the realized demand and suppressed demand with toll pricing.

REFERENCES


