A Stochastic User Equilibrium Assignment Model under Stochastic Demand and Supply Following Lognormal Distributions

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Abstract: To construct a traffic network that is resilient to natural and man-made disasters, it is necessary to assess the reliability of the network. This article proposes a stochastic user equilibrium assignment model under stochastic origin–destination (OD) demand and link capacity following lognormal distributions. The nonnegative property of the lognormal distribution makes it possible to assume any degraded state of links, including disruption or reduction of lane(s), by varying the stochastic link capacities. This model assigns the OD demand to a road network based on the stochastic user equilibrium principle. Thus, the model can compare the link (or path) flows and link (or path) travel costs in the normal state with those in the degraded state, and is formulated as a logit-based fixed-point problem. We conducted numerical experiments to compare the assigned flow and travel cost in the normal state with those in the degraded state.

Keywords: Stochastic User Equilibrium, Degraded Network, Natural and Man-made Disasters.

1. INTRODUCTION

Our urbanized society is constantly exposed to many risks, such as natural and man-made disasters. To maintain socioeconomic activities, we must prepare traffic measures against these disasters in advance. The resilience of a transportation network that enables rapid evacuation, restoration, and reconstruction immediately after a disaster is particularly important. In many Asian countries, the road networks are subject to the harsh natural environment, with heavy rainfalls, land slips, and earthquakes. Therefore, it is necessary to evaluate the connectivity and travel time reliability of a degraded road network following a natural disaster.

A number of studies have evaluated the resistance of road networks to disasters. Bell and Iida (1997) proposed the concept of connectivity reliability and showed a method for its calculation. In general, connectivity reliability is the probability of a path or an origin–destination (OD) pair being passable.

Research that addresses the vulnerability of transportation networks has also been carried out. The concept of vulnerability can be explained by using a situation where a node or link causes a catastrophic impact on the whole network in spite of its low disruption occurrence probability. Taylor et al. (2006) evaluated vulnerability to network degradation based on the accessibility measure considering the socioeconomic impacts in the degraded

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Chen et al. (2006) presented network-based accessibility measures by considering travel time or generalized cost derived from logit-based stochastic user equilibrium (SUE) traffic assignment models.

From a supply-side perspective, the concept of capacity reliability was proposed by Chen et al. (1999). Capacity reliability is the probability that a network can accommodate a certain demand flow. Sumalee and Kurauchi (2006) extended the concept of capacity reliability by including the rerouting behavior derived from a probity-based SUE model with Monte Carlo simulation. They evaluated traffic regulation considering OD demand after a disaster. In the case of connectivity reliability, the state of links is described by two discrete states, operating or failed (e.g., Asakura, 1999). However, the state of links can change successively, so the successive states of a degraded network can be represented by the concept of capacity reliability or stochastic capacity.

For the evaluation of network reliability, studies have also considered travel time reliability and demand uncertainty. Asakura (1999) evaluated the reliability of a degraded network by combining the concepts of connectivity reliability and travel time reliability, and used a multiclass SUE model that considers the effect of providing path choice information to drivers. Nakayama and Takayama (2003) assumed binomial distribution as an OD demand. Clark and Watling (2005) proposed a SUE model with stochastic OD demand following a Poisson distribution. Lo et al. (2006) set up path choice criteria by using mean and variance of path travel time, that is, the travel time budget, under the assumption of a stochastic link capacity that follows a uniform distribution. Lam et al. (2008) formulated a SUE model with uncertain OD demand and link capacity. The link travel time was affected by weather, that is, rainfall intensity. In stochastic road networks, which are assumed in the above works, several empirical studies show that many drivers exhibit risk-averse travel behaviors (e.g., Liu et al., 2004; Brownstone et al., 2003; Lam and Small, 2001). In contrast, some different approaches have been adopted to deal with uncertainty of travel cost. Lou et al. (2010) proposed the boundedly rational user equilibrium model, in which drivers are assumed to accept almost maximum utility. Miralinaghi et al. (2016) proposed the multiclass fuzzy user equilibrium model, and introduced the driver’s imprecise path choice behavior based on fuzzy travel times.

In modeling the stochastic demand flow, many studies have adopted the normal distribution for representing OD demand (e.g., Uchida, 2015; Lam et al., 2008; Shao et al., 2006; Chen et al., 2002). However, the normal distribution can take a negative value theoretically, which never happens in a real traffic situation. The normal distribution has an advantage over the other theoretical distributions in terms of easy calculation of the network flows due to its reproductive property of the linear combination of normal distributions. In contrast, Sumalee and Xu (2011), Zhou and Chen (2008) and Zhao and Kockleman (2002) adopted lognormal distributions for OD demands. The nonnegativity and asymmetricity of lognormal distributions are suitable for actual traffic phenomena. Indeed, Uno et al. (2009) showed that lognormal distributions can represent observed travel times obtained from bus probe data. From this, it may be reasonable to assume that the OD demand follows a lognormal distribution, if travel time is represented by a lognormal distribution. In previous studies, the approximation method proposed by Fenton (1960) was applied for representing the link flow as a lognormal distribution. Fenton’s method approximated the sum of lognormal distributions with a lognormal distribution. Since the approximation method is valid only under independent lognormal distributions, the correlation of link flows is not correctly considered in calculating the link flow distribution, that is, the variance of the link flow may be underestimated. Thus, such an effect can propagate to the calculation of link or path travel time.
This article proposes a SUE traffic assignment model by considering stochastic OD demand and link capacity that follows lognormal distributions. As noted, studies that have adopted a lognormal distribution for OD demand cannot properly address the correlation of link flows in calculating travel time. In contrast, here we take the correlation of link flows into account in calculating travel time distribution by adopting the approximation method proposed in Abu-Dayya and Beaulieu (1994), which extended the method proposed in Fenton (1960). Therefore, the model we propose expresses the link flow and link travel time that follow multivariate lognormal distributions with the correlation of the link flows. In addition, the application of a lognormal distribution to the link capacity makes it possible to express various states of network degradation, for example, partial damage or disruption of a link by decreases in its mean capacity. Note that the disruption of a link is represented by the lognormal distribution that is obtained by substituting a negligible small value for the mean value of the lognormal capacity in this article. Due to this formulation, this model can assign OD demand to a degraded network. Heavy congestion can occur in a degraded network as a result of man-made and natural disasters, and it interferes with the traffic related to the restoration or reconstruction work around the degraded road network. Therefore, it is important to consider congestion in the degraded network when evaluating network reliability.

The remainder of this article is organized as follows. Section 2 describes the stochastic OD demand that follows the lognormal distribution and the derived flows in the network, and shows the approximation method applied in this study. Section 3 formulates the stochastic travel time and the SUE model. Section 4 shows the results of numerical experiments for two test networks. Section 5 provides concluding remarks and suggestions for future tasks.

2. FLOWS IN A NETWORK

2.1 Notation

The following notations are used in this article. Note that the random variables and the deterministic variables are written in uppercase letters and lowercase letters, respectively.

- **W**: Set of origin–destination (OD) pairs in the network
- **K_w**: Set of paths serving OD pair \( w \)
- **N**: Set of nodes in the network
- **A**: Set of links in the network
- **Q_w**: Demand flow for OD pair \( w \)
- **q_w**: Mean demand flow for OD pair \( w \)
- **cv_w**: Coefficient of variation in traffic demand for OD pair \( w \)
- **F_w,k**: Flow of path \( k \) serving OD pair \( w \)
- **f_w,k**: Mean flow of path \( k \) serving OD pair \( w \)
- **V_a**: Flow of link \( a \)
- **C_a**: Capacity of link \( a \)
- **T_a**: Travel time for link \( a \)
- **T_w,k**: Travel time for path \( k \) serving OD pair \( w \)
- **p_w,k**: Proportion of drivers choosing path \( k \) serving OD pair \( w \)
- **\eta_{w,k}**: Travel cost of path \( k \) serving OD pair \( w \)
\[ \delta_{w,k,a} : \text{Variable that equals 1 if link } a \text{ is a part of path } k, \text{ and 0 otherwise} \]

2.2 Assumptions

Assumptions employed in this article are given below.

A1. The OD demand, \( Q_w \), follows an independent lognormal distribution as assumed in the literature (e.g., Sumalee and Xu, 2011; Zhou and Chen, 2008). Its standard deviation is calculated as the product of the mean value and the given coefficient of variation (e.g., Uchida, 2014; Shao et al., 2006).

A2. The path flow, \( F_{w,k} \), is calculated as the product of path choice probability, \( p_{w,k} \), and the OD demand, \( Q_w \). Thus, the path flow also follows a lognormal distribution (e.g., Lam et al., 2008). From A1 and A2, the coefficient of variation of path flow, \( F_{w,k} \), is equal to that of the OD demand, \( Q_w \) (e.g., Uchida, 2014; Shao et al., 2006).

A3. The link flow, which is the sum of correlated path flows, can be approximated by a lognormal distribution. This assumption is supported by the formulas shown in Abu-Dayya and Beaulieu (1994).

A4. The link capacities follow independent lognormal distributions. This assumption is not a limitation of this study, since the assumption can be easily relaxed without technical difficulties.

A5. There is no correlation between the link flow and the link capacity (e.g., Uchida, 2014). To our knowledge, there is no evidence for a statistical correlation between link flow and link capacity.

2.3 Stochastic Demand Flow

In this article, the OD demands are assumed to follow lognormal distributions (e.g., Sumalee and Xu, 2011; Zhou and Chen, 2008; Zhao and Kockleman, 2002). Thus, the demand flow for OD pair \( w \) is:

\[ Q_w \sim LN(\mu_{Q_w}, \sigma_{Q_w}^2) \]  

By taking the logarithm of this demand distribution, we obtain the corresponding normal distribution with the mean of \( \mu_{Q_w} \) and the variance of \( \sigma_{Q_w}^2 \) as (1). The variance of the stochastic OD demand is defined as the square of the product of its mean and the coefficient of variation (e.g., Uchida, 2014; Shao et al., 2006). The mean and variance of the OD demand are respectively defined as:

\[ E[Q_w] = q_w \]  
\[ \text{var}[Q_w] = (cv_w \cdot q_w)^2 \]

Note that the variance or standard deviation of the OD demand is defined by the increasing function related to its mean. This study defines the standard deviation as the product of its mean and coefficient of variance (e.g., Lam et al., 2008; Shao et al., 2006), while Sumalee and Xu (2011) and Zhou and Chen (2008) defined the variance as the product of its mean and
the variance-to-mean ratio (VMR). The path flow that follows a lognormal distribution is calculated as the product of its path choice probability and the demand flow:

\[ F_{w,k} = p_{w,k} \cdot Q_w \]  

(4)

Its mean and standard deviation are respectively given by:

\[ f_{w,k} = E[F_{w,k}] = p_{w,k} \cdot E[Q_w] = p_{w,k} \cdot q_w \]  

(5)

\[ \sqrt{\text{var}[F_{w,k}]} = p_{w,k} \sqrt{\text{var}[Q_w]} = p_{w,k} \cdot q_w \cdot cv_w \]  

(6)

Then, by considering the variance and covariance matrix of path flows, the OD demand variance is equal to the sum of the path flow covariance related to the demand:

\[ \text{var}[Q_w] = \sum_{k \in K_w} \sum_{j \in K_w} \text{cov}[F_{w,k}, F_{w,j}] \]  

(7)

\[ = \sum_{k \in K_w} \sum_{j \in K_w} p_{w,k} \cdot p_{w,j} (cv_w \cdot q_w)^2 \]

\[ = (q_w \cdot cv_w)^2 \left( \sum_{k \in K_w} p_{w,k} \right)^2 = (q_w \cdot cv_w)^2 \]

Note that the OD demand variance is preserved only within the same OD pair. The link flow is represented by the sum of the path flows that pass through the link given by:

\[ V_a = \sum_{w \in W} \sum_{k \in K_w} \delta_{w,k,a} \cdot F_{w,k} \]  

(8)

This study assumes the additivity of lognormal distributions so that both path flows and link flows can follow lognormal distributions. Detailed discussion for this assumption is provided in Section 2.4. The mean and covariance of the link flow are respectively given by:

\[ E[V_a] = \sum_{w \in W} \sum_{k \in K_w} \delta_{w,k,a} \cdot E[F_{w,k}] \]  

(9)

\[ \text{cov}[V_a, V_b] = \text{var} \left[ \sum_{w \in W} \sum_{k \in K_w} \delta_{w,k,a} \cdot \delta_{w,k,b} \cdot F_{w,k} \right] \]

\[ = \sum_{w \in W} \sum_{k \in K_w} \sum_{j \in K_w} \delta_{w,k,a} \cdot \delta_{w,j,a} \cdot \text{cov}[F_{w,k}, F_{w,j}] \]  

(10)

\[ (\text{if } a = b, \ \text{var}[V_a] = \text{cov}[V_a, V_b]) \]

In (10), the covariance is equivalent to the variance if \( a = b \).

### 2.4 Lognormal Distribution

If a random variable, \( Z_a \), follows a lognormal distribution, then \( Z_a \) is identified by using two parameters, \( \mu_{Z_a} \) and \( \sigma_{Z_a}^2 \), as shown by:
where $\mu_{Z_a}$ and $\sigma^2_{Z_a}$ are the mean and variance of the normal distribution obtained by taking the logarithm of $Z_a$.

The mean of the lognormal distribution is expressed by using the mean and variance of the corresponding normal distribution as shown by (12). In a similar manner, the covariance of the lognormal distribution is expressed as shown by (13):

$$\mu_{Z_a} = \ln(E[Z_a]) - \frac{1}{2} \ln \left(1 + \frac{\text{var}[Z_a]}{(E[Z_a])^2}\right)$$

$$\sigma_{Z_a, Z_b} = \ln \left[1 + \frac{\text{cov}[Z_a, Z_b]}{E[Z_a] \cdot E[Z_b]}\right]$$

(12) and (13) can be rewritten as:

$$E[Z_a] = \exp \left[\mu_{Z_a} + \frac{1}{2} \sigma^2_{Z_a}\right]$$

$$\text{cov}[Z_a, Z_b] = E[Z_a] \cdot E[Z_b] \cdot \left(\exp(\sigma_{Z_a, Z_b}) - 1\right)$$

The sum of lognormal distributions does not follow a lognormal distribution theoretically, but a method proposed by Fenton (1960) makes it possible to approximate the sum of lognormal distributions with a lognormal distribution. However, in this approximation method, independent lognormal distributions are assumed. Abu-Dayya and Beaulieu (1994) proposed a method that approximates the sum of the correlated lognormal distributions with a lognormal distribution (Appendix A). By applying this approximation method, the sum of the correlated lognormal distributions can be represented by a lognormal distribution whether or not there are correlations. This method is employed in this article in calculating link flows, that is, the link flow shown by (8) is assumed to follow the lognormal distribution in this article. However, if a total demand flow follows a lognormal distribution, the corresponding link flows follow the lognormal distributions without loss of generality (Appendix B).

3 TRAVEL TIME AND PATH CHOICE

3.1 Travel Time

We use the following BPR function (Bureau of Public Roads, 1964) to represent the link travel time, following many other studies:

$$t_a(v_a, c_a) = t^0_a \left[1 + \beta_a \left(\frac{v_a}{c_a}\right)^{n_a}\right]$$

In (16), $t^0_a$ is the free-flow travel time of link $a$. $\beta_a$ and $n_a$ are link-specific calibration parameters. In general, by substituting the deterministic link flow, $v_a$, and the link capacity, $c_a$, into the BPR function, the deterministic link travel time is obtained. However, here, to represent a stochastic network, the stochastic link travel time is calculated by substituting the
stochastic link flow, $V_a$, and the stochastic link capacity, $C_a$, into the BPR function. This assumption was applied in Uchida (2014). The resultant stochastic link travel time is:

$$t_a(V_a, C_a) = t_a^0 \left( 1 + \beta_a \cdot \left( \frac{V_a}{C_a} \right)^{\alpha_a} \right)$$

(17)

Here, both the link flow and the link capacity are assumed to follow lognormal distributions; thus, they can be integrated as the random variable, $D_a$, which also follows a lognormal distribution. The corresponding stochastic link travel time can be rewritten easily as:

$$t_a(V_a, C_a) = T_a(D_a) = t_a^0 \cdot \left( 1 + \beta_a \cdot D_a^{\alpha_a} \right)$$

(18)

where

$$D_a = \frac{V_a}{C_a}$$

(19)

This integration of the two variables, $V_a$ and $C_a$, into one variable, $D_a$, shown in (19) was adopted in Uchida (2015). The mean and the covariance of the link travel time can be respectively given by:

$$E[T_a] = t_a^0 (1 + \beta_a \cdot E[D_a^{\alpha_a}])$$

(20)

$$\text{cov}[T_a, T_b] = E[T_a \cdot T_b] - E[T_a] \cdot E[T_b]$$

$$\text{if } a = b, \text{ var}[T_a] = \text{cov}[T_a, T_b]$$

(21)

where

$$E[T_a \cdot T_b] = E\left[ t_a^0 \left( 1 + \beta_a \cdot D_a^{\alpha_a} \right) \cdot t_b^0 \left( 1 + \beta_b \cdot D_b^{\alpha_b} \right) \right]$$

$$= E[t_a^0 \cdot t_b^0 \left( 1 + \beta_a \cdot D_a^{\alpha_a} \right) \cdot \left( 1 + \beta_b \cdot D_b^{\alpha_b} \right)]$$

$$= t_a^0 \cdot t_b^0 \left( 1 + \beta_a \cdot E[D_a^{\alpha_a}] + \beta_b \cdot E[D_b^{\alpha_b}] + \beta_a \cdot \beta_b \cdot E[D_a^{\alpha_a}] \cdot E[D_b^{\alpha_b}] \right)$$

(22)

The mean and variance of the normal distribution corresponding to $D_a^{\alpha_a}$ shown by (18) are respectively given by:

$$\mu_{D_a^{\alpha_a}} = E[n_a \cdot X_a] = n_a \cdot E[X_a]$$

$$= n_a \cdot \mu_{D_a}$$

(23)

$$\sigma_{D_a^{\alpha_a}}^2 = \text{var}[n_a \cdot X_a] = n_a^2 \cdot \text{var}[X_a]$$

$$= n_a^2 \cdot \sigma_{D_a}^2$$

(24)
Note that $D_{a}^{n}$ in (18) follows a lognormal distribution since $D_{a}$ follows a lognormal distribution. Therefore, $D_{ab}^{n} = D_{a}^{n} \cdot D_{b}^{n}$ in (22) also follows a lognormal distribution. The mean and variance of the normal distribution corresponding to $D_{ab}^{n}$ are respectively given by:

$$
\begin{align*}
\mu_{D_{ab}^{n}} &= E[n_{a} \cdot X_{a} + n_{b} \cdot X_{b}] \\
&= n_{a} \cdot \mu_{D_{a}} + n_{b} \cdot \mu_{D_{b}} \\
\sigma_{D_{ab}^{n}}^{2} &= \text{var}[n_{a} \cdot X_{a} + n_{b} \cdot X_{b}] \\
&= n_{a}^{2} \cdot \text{var}[X_{a}] + n_{b}^{2} \cdot \text{var}[X_{b}] + 2 n_{a} \cdot n_{b} \cdot \text{cov}[X_{a}, X_{b}] \\
&= n_{a}^{2} \cdot \sigma_{D_{a}}^{2} + n_{b}^{2} \cdot \sigma_{D_{b}}^{2} + 2 n_{a} \cdot n_{b} \cdot \sigma_{D_{a}, D_{b}}
\end{align*}
$$

Due to the correlated link flows, the random variables, $X_{a}$, are also correlated. The covariance of the two random variables, $X_{a}$ and $X_{b}$, is:

$$
\begin{align*}
\text{cov}[X_{a}, X_{b}] &= \sigma_{D_{a}, D_{b}} \\
&= \text{cov}[\ln D_{a}, \ln D_{b}] \\
&= \text{cov}[\ln V_{a} - \ln C_{a}, \ln V_{b} - \ln C_{b}] \\
&= \sigma_{V_{a}, V_{b}}
\end{align*}
$$

In (30), it is assumed that the link flow and the link capacity are statistically independent.

In many studies that consider the stochastic link flow to follow a normal distribution, the link travel cost is expressed by a polynomial function of mean link flow or is approximated by an $m$th-order Taylor expansion around a mean link flow (e.g., Uchida, 2015; Clark and Watling, 2005). However, note that the $n_{a}$th-order moment of $D_{a}$, $D_{b}^{n}$, is directly calculated here. Nevertheless, in a case where $D_{a}$ follows a normal distribution, its moment needs to be calculated by applying analytical methods, e.g., the method proposed by Isserlis (1918).

According to (18), the link travel time is composed of a deterministic term and a random term that follows a lognormal distribution. Therefore, it is represented by a shifted lognormal distribution given by:

$$
\tilde{T}_{a} = T_{a} - t_{a}^{0} ~ LN(\beta_{a} + \mu_{D_{a}^{n}}, \sigma_{D_{a}^{n}}^{2})
$$

The path travel time, which is the sum of the link travel times, is:

$$
T_{w,k,a} = \sum_{a \in A} \delta_{w,k,a} \cdot T_{a}
$$

The path travel time is also composed of a deterministic term and a random term. It can be approximated by a shifted lognormal distribution by applying the method proposed in
Abu-Dayya and Beaulieu (1994) to the random term. The path travel time that follows the shifted lognormal distribution is given by:

$$\hat{T}_{w,k} = T_{w,k} - \sum_{a \in A} \delta_{w,k,a} \cdot t^0_a \sim LN(\mu_{\hat{T}_{w,k}}, \sigma^2_{\hat{T}_{w,k}})$$

(33)

Two parameters, $\mu_{\hat{T}_{w,k}}$ and $\sigma^2_{\hat{T}_{w,k}}$, in (33) can be obtained by applying the method proposed in Abu-Dayya and Beaulieu (1994). The mean and variance of the path travel time are respectively given by:

$$E[T_{w,k}] = \sum_{a \in A} \delta_{w,k,a} \cdot E[T_a]$$

(34)

$$\text{var}[T_{w,k}] = \text{cov}\left[ \sum_{a \in A} \delta_{w,k,a} \cdot T_a, \sum_{b \in A} \delta_{w,k,b} \cdot T_b \right]$$

$$= \sum_{a \in A} \sum_{b \in A} \delta_{w,k,a} \cdot \delta_{w,k,b} \cdot \text{cov}[T_a, T_b]$$

(35)

The formulation shown by (35) is employed in many studies (e.g., Lam et al., 2008). Note that total travel time of link $a \in A$, $T_a \cdot V_a$, in which $T_a$ follows a shifted lognormal distribution and $V_a$ follows a lognormal distribution, can be approximated by a lognormal distribution. It is calculated as $T_a \cdot V_a = \hat{T}_a \cdot V_a + t^0_a \cdot V_a$ in which both terms of $\hat{T}_a \cdot V_a$ and $t^0_a \cdot V_a$ follow lognormal distributions. By applying the method proposed in Abu-Dayya and Beaulieu (1994), $\hat{T}_a \cdot V_a + t^0_a \cdot V_a$, which is the sum of two lognormal distributions, can be approximated by a lognormal distribution. Total travel time in the network can be approximated by a lognormal distribution since it is the sum of total travel times of all links in the network. Similar to the flow calculation in a deterministic network model, total travel time in the network can be also obtained by summing the total travel time of each path, $T_{w,k} \cdot F_{w,k}$, in the network. Therefore, the total travel time in the network that follows lognormal distribution is calculated as either $\sum_{a \in A} T_a \cdot V_a$ or $\sum_{w \in W} \sum_{k \in K_w} T_{w,k} \cdot F_{w,k}$.

### 3.2 Driver’s Path Choice Behavior

By using the formulas provided above, the path travel cost in this article is defined as a linear function of the mean and variance of travel time for a path. Thus, the driver in the network is assumed to make a risk-averse path choice. In the deterministic network model proposed by Uchida (2014), a driver traveling on OD pair $w \in W$ is assumed to choose path $k \in K_w$, which minimizes the following path travel cost:

$$\eta_{w,k} = E[T_{w,k}] + \gamma \cdot \text{var}[T_{w,k}]$$

(36)

$\gamma \geq 0$ in (36) represents the degree of risk aversion. Here, we adopt the variance of path travel time as a measure of path travel variability. The proposed path choice criteria can be derived by the scheduling preference proposed in Engelson and Fosgerau (2011). In our model, path travel cost is applied to random utility theory to express the driver’s perception error of the travel cost. As an error term, an identically and independently distributed Gumbel distribution is assumed. Then, a logit-based SUE traffic assignment model with a dispersion parameter, $\theta$,
can be formulated as the following fixed-point problem:

\[ f_w = q_w \cdot p_w(\eta_w(f)) \quad (37) \]

where

\[ p_{w,k} = \frac{\exp(-\theta \cdot \eta_{w,k})}{\sum_{k \in K_w} \exp(-\theta \cdot \eta_{w,k})} \quad (38) \]

\[ f_w = (f_{w,1}, \ldots, f_{w,|K_w|})^T \quad (39) \]

\[ f = (f_1, \ldots, f_{|W|})^T \quad (40) \]

\[ p_w = (p_{w,1}, \ldots, p_{w,|K_w|})^T \quad (41) \]

\[ \eta_w = (\eta_{w,1}, \ldots, \eta_{w,|K_w|})^T \quad (42) \]

The superscript \( T \) denotes the transposition operator of a vector.

Note that the driver assumed in this article is exposed to two kinds of uncertainties in choosing a path in the network: the uncertainties of travel time and perception of the travel cost.

4. NUMERICAL EXPERIMENTS

This section demonstrates the proposed model and shows the results obtained for two test networks: a small network and a relatively large network. Two parameters, \( \beta_w \) and \( n_w \), used for the BPR function shown by (17) are set as 2 and 6, respectively. Both parameters, \( \gamma \) and \( \theta \), which respectively represent the risk-adverse measure in (34) and the dispersion of the driver’s perception error in (37), are equal to 1. Each link in the network has a length of 3 [km] and a free traveling speed of 60 [km/h]. Thus, the free travel time of each link is 0.05 [h]. The mean and the variance of the stochastic capacity of each link in the network are 1,000 [PCU/h] (PCU, passenger car unit) and 100^2 [(PCU/h)^2] in the normal state, respectively. Figure 1 shows an example of link capacity distributions in normal and degraded states. The mean and the variance of the degraded link capacity shown in Figure 1 are 10 [PCU/h] and 100^2 [(PCU/h)^2], respectively. In all calculations, the method of successive average (MSA) is adopted as a solution algorithm (Sheffi, 1985). Note that the number of iterations required to obtain the equilibrium flows for test network 1 was around 100 and that for test network 2 was around 300. All results of numerical calculations here are calculated by Matlab 2007b installed on Dell Optiplex 3020 (Windows 10, CPU: Intel Core i3-4160 CPU @ 3.60GHz, RAM: 4.00GB). For example, the computational time for the calculation result in Section 4.2, for 300 iteration times, is around 2.55 CPU s.
4.1 An OD Pair

This section shows the results of a numerical experiment on test network 1 shown in Figure 2. This network has three paths, five links, four nodes, and an OD pair. Table 1 shows the stochastic OD demand. Table 2 shows the set of paths, which are represented by the sequences of links for this network. In the proposed model, since the travel cost of a path is not the sum of the travel costs of the links that comprise the path, we prepared a set of paths for solving a problem by MSA. In this numerical experiment, link 5 is assumed to be a degraded link.

Table 1. Stochastic OD demand (Test network 1)

<table>
<thead>
<tr>
<th>OD pair</th>
<th>Mean [PCU/h]</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 4)</td>
<td>1000</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 2. Set of paths (Test network 1)

<table>
<thead>
<tr>
<th>OD pair</th>
<th>Path number</th>
<th>Link sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1, 4)</td>
<td>1</td>
<td>1-4</td>
</tr>
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<td></td>
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<td>1-2-5</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3-5</td>
</tr>
</tbody>
</table>

As noted in Section 1, a link capacity is always nonnegative. Then, various combinations of its mean and variance can be assumed in the proposed model. Figure 3 shows the equilibrium mean flow on link 5 for each combination of mean and variance of stochastic...
capacity of link 5. The mean and variance of the stochastic capacity are gradually changed from 1 to 1,000 [PCU/h] and from 10 to 1,000,000 [(PCU/h)^2], respectively. From Figure 3, we can see that the mean flow of link 5 increases with an increase in the mean capacity of link 5 and increases with a decrease in the capacity variance of link 5. The flows on links 2 and 3, which share paths 2 and 3 with link 5, decrease with a decrease in the mean of capacity on link 5 and decrease with an increase in the capacity variance of link 5. Figure 3 demonstrates that the proposed model is able to address various kinds of stochastic link capacities.

![Figure 4. Link flow (Normal, Test network 1)](image)

![Figure 5. Link flow (Degraded, Test network 1)](image)

Figures 4 and 5 show the link flow distributions in the normal state and those in the degraded state, respectively. In the degraded state, the mean capacity of link 5 is set as 10 [PCU/h], while capacity variance does not change.

Due to the symmetric topology of the network, the flow distributions of links 3 and 4 and those of links 1 and 5 are overlapped in Figure 4. Figure 5 shows that the demand flow concentrates on links 1 and 4 due to the disruption of link 5. This is because links 2 and 3 share paths with link 5. The path flows that pass through link 5 in the normal state shift toward links 1 and 4 in the degraded state.

![Figure 6. Path travel time (Normal, Test network 1)](image)

![Figure 7. Path travel time (Degraded, Test network 1)](image)

Figures 6 and 7 show path travel time distributions in the normal state and those in the degraded state, respectively. The travel time distributions are represented by the shifted lognormal distribution as discussed in Section 3.

Figure 7 shows how the disruptions of paths 2 and 3, which include the degraded link 5, affect the travel time of path 1. Only the travel time of path 1 is for practical use. The OD
demand concentrates only on path 1. The travel time mean and variance of paths 2 and 3 can be interpreted as positive infinities (cf Table 4). Thus, the travel time distributions of paths 2 and 3 shown in Figure 7 represent the state of disruption. The disrupted paths, that is, paths 2 and 3, carry almost zero flow. As a result, the travel time mean and variance of path 1 are higher than those in the normal state. The graphs in Figures 4–7 are based on the detailed results shown in Tables 3 and 4, which respectively show the link flows and path flows in both states.

Tables 3 and 4 show that the coefficient of variation of the link flow and that of the path flow are equal to the coefficient of variation of the OD demand shown in Table 1. This is because all link flows and path flows are generated by the same OD demand. In this case, (8) makes the coefficient of variation of OD demand equal to those of the link flows and the path flows in the network. As shown in the next experiment, this relationship does not hold in the case of multiple OD pairs.

Table 3. Stochastic link flows and link travel costs in normal and degraded networks (Test network 1)

<table>
<thead>
<tr>
<th>Link</th>
<th>Normal Mean</th>
<th>Normal Standard deviation</th>
<th>Disaster Mean</th>
<th>Disaster Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>658.8</td>
<td>131.8</td>
<td>1000.0</td>
<td>200.0</td>
</tr>
<tr>
<td>2</td>
<td>317.6</td>
<td>63.5</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>341.2</td>
<td>68.2</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>341.2</td>
<td>68.2</td>
<td>1000.0</td>
<td>200.0</td>
</tr>
<tr>
<td>5</td>
<td>658.8</td>
<td>131.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4. Stochastic path flows and path travel times in normal and degraded networks (Test network 1)

<table>
<thead>
<tr>
<th>Path</th>
<th>Normal Mean</th>
<th>Normal Standard deviation</th>
<th>Disaster Mean</th>
<th>Disaster Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
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<td>341.2</td>
<td>68.2</td>
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</tr>
<tr>
<td>2</td>
<td>317.6</td>
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<td>0.0</td>
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<td>341.2</td>
<td>68.2</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

4.2 Multiple OD Pair

This section shows the results of a numerical experiment for the network shown in Figure 8. This network, taken from Nguyen and Dupuis (1984), has 25 paths, 19 links, and 13 nodes with four OD pairs shown in Table 5. We prepared two sets of OD demand shown in Table 6. The coefficients of variation differ between the two sets of OD demand. In Table 5, the coefficients of variation of demand flows for OD pairs (1, 2) and (4, 2) are 0.2 and those for OD pairs (1, 3) and (4, 3) are 0.25. In Table 6, all of the coefficients of variation are set as 0.2. In contrast, all the mean OD demands are the same for the two tables. Paths in the network are shown in Table 5 as link sequences. In the degraded network, the mean of the capacity of link 2 decreases from 1000 [PCU/h] to 10 [PCU/h] in the degraded state, whereas the variance of the capacity remains unchanged.
Figure 8. Test network 2

Table 5. The combination of paths and links (Test network 2)

<table>
<thead>
<tr>
<th>OD pair</th>
<th>Path number</th>
<th>Link sequence</th>
<th>OD pair</th>
<th>Path number</th>
<th>Link sequence</th>
</tr>
</thead>
<tbody>
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<td>(1, 3)</td>
<td>14</td>
<td>1-6-13-19</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1-5-7-9-11</td>
<td></td>
<td>15</td>
<td>1-5-7-10-16</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1-5-7-10-15</td>
<td></td>
<td>16</td>
<td>1-5-8-14-16</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1-5-8-14-15</td>
<td></td>
<td>17</td>
<td>1-6-12-14-16</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1-6-12-14-15</td>
<td></td>
<td>18</td>
<td>2-17-7-10-16</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>2-17-7-9-11</td>
<td></td>
<td>19</td>
<td>2-17-8-14-16</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>2-17-7-10-15</td>
<td>(4, 3)</td>
<td>20</td>
<td>4-13-19</td>
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<td>2-17-8-14-15</td>
<td></td>
<td>21</td>
<td>4-12-14-16</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>4-12-14-15</td>
<td></td>
<td>22</td>
<td>3-6-13-19</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>3-5-7-9-11</td>
<td></td>
<td>23</td>
<td>3-5-7-10-16</td>
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<td>3-5-7-10-15</td>
<td></td>
<td>24</td>
<td>3-5-8-14-16</td>
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<td></td>
<td>12</td>
<td>3-5-8-14-15</td>
<td></td>
<td>25</td>
<td>3-6-12-14-16</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>3-6-12-14-15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6. Stochastic OD demands

<table>
<thead>
<tr>
<th>OD pair</th>
<th>Stochastic OD demands 1</th>
<th>Stochastic OD demands 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean [PCU/h]</td>
<td>Coefficient of variation</td>
</tr>
<tr>
<td>(1, 2)</td>
<td>1000</td>
<td>0.2</td>
</tr>
<tr>
<td>(4, 2)</td>
<td>1500</td>
<td>0.2</td>
</tr>
<tr>
<td>(1, 3)</td>
<td>800</td>
<td>0.25</td>
</tr>
<tr>
<td>(4, 3)</td>
<td>1000</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 7 shows the link flows and the link travel times calculated by using the OD demands shown in left half of Table 6. Table 8 shows the link flows and the link travel times calculated by using the OD demands shown in the right half of Table 6.

Figures 9 and 10 show the flows on links 1, 2, and 5 in the normal state and those in the degraded state, respectively. These link variables are calculated by using the OD demands shown in the left half of Table 6. By comparing the results shown in Figure 9 with Figure 10, it is clear that the flows on links 1 and 5 increased due to the disruption of link 2. Since the mean flow on link 2 is almost zero in the degraded state, the link flow distribution seems to disappear in Figure 10.
Table 7. Stochastic link flows and link travel costs in normal and degraded networks
(Test network 2, OD demands 1)

<table>
<thead>
<tr>
<th>Link</th>
<th>Normal Mean</th>
<th>Normal Standard deviation</th>
<th>Disaster Mean</th>
<th>Disaster Standard deviation</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>695.7</td>
<td>140.1</td>
<td>1800.0</td>
<td>282.8</td>
</tr>
<tr>
<td>2</td>
<td>1104.3</td>
<td>181.7</td>
<td>0.0</td>
<td>0.38</td>
</tr>
<tr>
<td>3</td>
<td>1269.4</td>
<td>200.4</td>
<td>1152.3</td>
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<tr>
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<td>160.6</td>
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</tr>
<tr>
<td>6</td>
<td>832.2</td>
<td>124.4</td>
<td>1385.0</td>
<td>188.8</td>
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<tr>
<td>7</td>
<td>1283.3</td>
<td>164.4</td>
<td>1466.1</td>
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<td>43.0</td>
<td>101.1</td>
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<td>9</td>
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<td>102.6</td>
<td>1163.7</td>
<td>165.2</td>
</tr>
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<td>10</td>
<td>663.9</td>
<td>85.0</td>
<td>302.5</td>
<td>35.9</td>
</tr>
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<td>1280.7</td>
<td>185.5</td>
<td>1163.7</td>
<td>165.2</td>
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</tr>
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<td>87.5</td>
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<td>661.3</td>
<td>132.3</td>
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<td>19</td>
<td>1042.0</td>
<td>203.6</td>
<td>1310.2</td>
<td>236.5</td>
</tr>
</tbody>
</table>

Figures 9 and 10 show the link flows in the normal state (Test network 2, OD demands 1) and those in the degraded state, respectively. These link variables are calculated by using the OD demands shown in the left half of Table 6. Since both the mean and variance of link travel times shown in Figure 11 are relatively small, the travel time distributions depicted in the figure are leptokurtic, having a higher peak. Figure 12 indicates that the travel time variance for links 1 and 5 increased due to the disrupted link. Note that the travel time distribution of link 2 shown in Figure 12 does not imply that the link travel time is near to zero. In fact, it implies numerous increments of both mean and variance of the link travel time.

The coefficients of variation of the path flows calculated from Table 8 are all equal to those shown in the right half of Table 6, that is, the coefficient of variation is 0.2. However,
the coefficients of variation of the link flows shown in Table 8 are less than 0.2. In the proposed model, OD demands are assumed to be independent, and thus the covariance of path flows is defined only between path flows that are generated from the same OD demand. Therefore, even though the coefficients of variation for all OD demands are the same, those of the link flows in the network are not equal to those values (Appendix C).

![Figure 11. Link travel times in the normal state (Test network 2, OD demands 1)](image1)

![Figure 12. Link travel times in the degraded state (Test network 2, OD demands 1)](image2)

**Table 8. Stochastic link flows and link travel costs in normal and degraded networks (Test network 2, OD demands 2)**

<table>
<thead>
<tr>
<th>Link</th>
<th>Normal Mean</th>
<th>Normal Standard deviation</th>
<th>Disaster Mean</th>
<th>Disaster Standard deviation</th>
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<table>
<thead>
<tr>
<th>Link travel time [h]</th>
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<th>Disaster Mean</th>
<th>Disaster Standard deviation</th>
</tr>
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<td>0.45</td>
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</table>
5. CONCLUDING REMARKS AND FUTURE TASKS

This study proposed a stochastic user equilibrium assignment model. This model aims to evaluate the reliability of a road network that had degraded as a result of a natural or man-made disaster. Thus, our proposed model contributes to improving the resilience of road networks in many Asian countries, which are exposed to risks of natural disasters. This model assumes the stochastic demand flow and link capacity that follow lognormal distributions. Because of the lognormally distributed link flow and link capacity, the link travel time can be calculated without specific approximation, which was required in previous studies. In the model, the correlated link flows that are consistent with the correlated path flows are correctly calculated. In addition, by the assumptions, the proposed model addresses the various kinds of link damage and assigns an OD demand to the degraded network.

Numerical experiments under normal and degraded states for two test networks were carried out to demonstrate the applicability of the proposed model to a stochastic road network including degraded or disrupted links. The results show that the link capacity that approximates the state of disruption can be applied to the model by utilizing the nonnegative property of a lognormal distribution. The results in the simple test network indicate that the assumption of lognormally distributed travel times enables the model to address the heavily degraded or disrupted states of links, which cannot be addressed by the assumption of normally distributed travel times. The results in the larger test network show that our model is applicable to a network with multiple OD pairs. Thus, this model can theoretically address both the damaged stochastic road network and any mean or variance of random variables.

As future tasks, the elastic OD demands corresponding to a disaster situation, namely the cancellation of trips due to the increment of travel costs in the degraded network, need to be considered. After a disaster, specific traffic demand for emergency aid, restoration and reconstruction, and traffic congestion due to damage to the road network or the reduction of link capacity arises. Thus, traffic demand management corresponding to the changed network capacity is required to ensure the passage of priority vehicles. Our proposed model is expected to be applied for examining the demand management system corresponding to the restoration process for the supply side.

In addition, the cancellation of trips should also be explained by differences in drivers. This study assumes single-user class OD demands. However, in the normal or degraded road network, there should be drivers whose price sensitivities differ from each other, which results in differences in travel behaviors or cancellation of trips after a natural disaster.

A strength of this model is that the road network in a normal or degraded network can be evaluated by using the same framework as the network models considering travel time reliability. Thus, the normal and degraded network can be compared, and it is also expected to be extended to a model that analyzes the vulnerability of a road network.

APPENDICES

Appendix A

Based on the method proposed in Abu-Dayya and Beaulieu (1994), the first and second moments of a random variable, $Z$, which is the sum of lognormal distributions, are respectively given by:
The path flow is represented by the product of the proportion of the flow on path $i$ to $k$, that is, $p_{w,k}$, and $q_w$, as given by:

$$F_{w,k} = p_{w,k} \cdot q_w = p_{w,k} \cdot p_w \cdot Q \sim LN(\mu_Q + \ln(p_{w,k}), \sigma_Q^2)$$  \hspace{1cm} (B8)

The mean and covariance of random variable $Z$ can be calculated by using (A1) and (A2). Note that the parameter $\sigma_{Z_i,Z_j}$, which is the covariance between two normal distributions corresponding to $Z_i$ and $Z_j$, is directly used for the calculation.

**Appendix B**

We consider a total demand flow that follows the following lognormal distribution given by:

$$Q \sim LN(\mu_Q, \sigma_Q^2)$$  \hspace{1cm} (B1)

The demand flow for OD pair $w$ is represented by the production of the given proportion of $q_w = E[q_w]$ to $q = E[Q]$, that is, $p_w$, and $Q$, as shown by:

$$Q_w = p_w \cdot Q$$

$$Q_w \sim LN(\mu_Q + \ln(p_w), \sigma_Q^2)$$  \hspace{1cm} (B3)

Its mean and covariance are respectively given by:

$$E[Q_w] = \exp(\mu_Q + \ln(p_w) + \frac{\sigma_Q^2}{2})$$  \hspace{1cm} (B4)

$$\text{cov}(Q_w, Q_{w'}) = E[Q_w \cdot Q_{w'}] - E[Q_w] \cdot E[Q_{w'}]$$  \hspace{1cm} (B5)

where

$$Q_w \cdot Q_{w'} = p_w \cdot p_{w'} \cdot Q^2$$

$$\sim LN(2\mu_Q + \ln(p_w \cdot p_{w'}), 2\sigma_Q^2)$$  \hspace{1cm} (B6)

$$E[Q_w \cdot Q_{w'}] = \exp(2\mu_Q + \ln(p_w \cdot p_{w'}) + 2\sigma_Q^2)$$  \hspace{1cm} (B7)

The path flow is represented by the product of the proportion of the flow on path $k$ to $Q_w$, that is, $p_{w,k}$ and $Q_w$, as given by:

$$F_{w,k} = p_{w,k} \cdot Q_w = p_{w,k} \cdot p_w \cdot Q \sim LN(\mu_Q + \ln(p_{w,k} \cdot p_w), \sigma_Q^2)$$  \hspace{1cm} (B8)
Its mean and covariance are respectively given by:

\[
E[F_{w,k}] = \exp\left(\mu_Q + \ln(p_{w,k} \cdot p_a) + \frac{\sigma_Q^2}{2}\right)
\]

\[
\text{cov}[F_{w,k}, F_{w,k}] = E[F_{w,k} \cdot F_{w,k}] - E[F_{w,k}] \cdot E[F_{w,k}]
\]

(B9)

(B10)

where

\[
E[F_{w,k} \cdot F_{w,k}] = \exp(2\mu_Q + \ln(p_{w,k}^2 \cdot p_a) + 2\sigma_Q^2)
\]

(B11)

The link flow is the sum of the path flows that pass through the link as given by:

\[
V_a = \sum_{w \in W} \sum_{k \in W_k} \delta_{w,k,a} \cdot F_{w,k} = \sum_{w \in W} \sum_{k \in W_k} \delta_{w,k,a} \cdot p_w \cdot p_{w,k} \cdot Q
\]

\[
= \left(\sum_{w \in W} \sum_{k \in W_k} \delta_{w,k,a} \cdot p_w \cdot p_{w,k}\right) \cdot Q
\]

\[
\sim LN\left(\mu_Q + \ln\left(\sum_{w \in W} \sum_{k \in W_k} \delta_{w,k,a} \cdot p_w \cdot p_{w,k}\right), \sigma_Q^2\right)
\]

(B12)

Its mean and covariance are respectively given by:

\[
E[V_a] = \exp\left(\mu_Q + \ln\left(\sum_{w \in W} \sum_{k \in W_k} \delta_{w,k,a} \cdot p_w \cdot p_{w,k}\right) + \frac{\sigma_Q^2}{2}\right)
\]

\[
\text{cov}[V_a, V_a] = E[V_a \cdot V_a] - E[V_a] \cdot E[V_a]
\]

(B13)

(B14)

where

\[
E[V_a \cdot V_a] = \exp\left(2\mu_Q + 2\sigma_Q^2 + \ln\left(\sum_{w \in W} \sum_{k \in W_k} \delta_{w,k,a} \cdot p_w \cdot p_{w,k}\right) \cdot \left(\sum_{w \in W} \sum_{k \in W_k} \delta_{w,k,b} \cdot p_w \cdot p_{w,k}\right)\right)
\]

(B15)

The variance of the total demand is equal to that of the sum of OD flows as shown by:

\[
\text{var}\left[\sum_{w \in W} Q_w\right] = \text{var}\left[\sum_{w \in W} p_w \cdot Q\right] = \sum_{w \in W} \text{var}[Q_w] + 2 \sum_{w_1 \in W} \sum_{w_2 \in W} \text{cov}[Q_{w_1}, Q_{w_2}]
\]

\[
= \sum_{w \in W} p_w^2 \cdot \text{var}[Q] + 2 \sum_{w_1 \in W} \sum_{w_2 \in W} p_{w_1} \cdot p_{w_2} \cdot \text{var}[Q]
\]

(B16)

Thus, in this set of formulations, there is no need to apply a specific approximation such as that of Abu-Dayya and Beaulieu (1994) to sum the correlated random variables following lognormal distributions.
Appendix C

As noted in Section 2, the OD demands are assumed to be independent in this article. The correlation of path flows is then defined only between the path flows that are generated from the same OD demand. In this case, the coefficient of variation of the link flows that are composed of several OD demands is not equal to those of the OD demands, even though all the coefficients of variation of the OD demands are the same. In the following, this fact will be shown.

The mean and variance of the link flow are respectively given by:

\[
E[V_a] = \frac{1}{(2T - 1)^2} \sum_{i=1}^{2T} \sum_{k=1}^{2T} \sum_{a} \delta_{w,k,a} \cdot F_{w,k}
\]

\[
= \sum_{w} \sum_{k} \delta_{w,k,a} \cdot E[F_{w,k}]
\]

\[
= \sum_{w} \sum_{k} \delta_{w,k,a} \cdot P_{w,k} \quad q_w
\]

\[
\text{var}[V_a] = \text{var}\left(\sum_{w} \sum_{k} \delta_{w,k,a} \cdot F_{w,k}\right)
\]

\[
= \sum_{w} \sum_{k} \sum_{a} \sum_{a} \delta_{w,k,a} \cdot \delta_{w,k,a} \cdot \text{cov}[F_{w,k}, F_{w,k}]
\]

\[
= \sum_{w} \sum_{k} \left(\sum_{a} \delta_{w,k,a} \cdot P_{w,k}\right)^2 \cdot (c_{w} \cdot q_w)^2
\]

\[
= \sum_{w} \left(\sum_{k} \left(\sum_{a} \delta_{w,k,a} \cdot P_{w,k}\right)^2 \cdot q_w^2\right) \cdot c_{w}^2
\]

\[
= \sum_{w} \left(\sum_{k} \left(\sum_{a} \delta_{w,k,a} \cdot P_{w,k}\right)^2 \cdot q_w^2\right) \cdot c_{w}^2
\]

\[
= \left(\sum_{w} \left(\sum_{k} \left(\sum_{a} \delta_{w,k,a} \cdot P_{w,k}\right)^2 \cdot q_w\right) \cdot \left(\sum_{k} \left(\sum_{a} \delta_{w,k,a} \cdot P_{w,k}\right) \cdot q_w\right) \cdot c_{w}^2
\]

\[
= \left(\sum_{w} \left(\sum_{k} \left(\sum_{a} \delta_{w,k,a} \cdot P_{w,k}\right)^2 \cdot q_w^2\right) \cdot c_{w}^2
\]

\[
= E^2[V_a] \cdot c_{w}^2
\]

where

\[
c_{w} = c_{w}^2 \quad \forall w \in W
\]

\[
\text{cov}[Q_{w1}, Q_{w2}] = q_{w1} \cdot q_{w2} \cdot c_{w}^2
\]

\[
= p_{w1} \cdot p_{w2} \cdot (c_{w} \cdot q)^2
\]
Then, if the correlation between two OD demands is assumed, the covariance between two path flows that are derived from different OD demands can be calculated. In this case, the variance of the link flow is the summation of covariance of path flows that go through the link. The variance of link flow is shown by:

\[
\text{var}[P_a] = \text{var}\left[\sum_{w \in W} \sum_{k \in W_k} \delta_{w,k,a} \cdot F_{w,k}\right]
\]

\[
= \sum_{w_1 \in W} \sum_{w_2 \in W} \sum_{k \in W_k} \sum_{l \in W_l} \delta_{w_1,k,a} \cdot \delta_{w_2,l,a} \cdot \text{cov}[F_{w_1,k}, F_{w_2,l}]
\]

\[
= \sum_{w_1 \in W} \sum_{w_2 \in W} \sum_{k \in W_k} \sum_{l \in W_l} \delta_{w_1,k,a} \cdot \delta_{w_2,l,a} \cdot p_{w_1,k} \cdot p_{w_2,l} \cdot \text{cov}[Q_{w_1}, Q_{w_2}]
\]

\[
= \sum_{w_1 \in W} \sum_{w_2 \in W} \sum_{k \in W_k} \sum_{l \in W_l} \delta_{w_1,k,a} \cdot p_{w_1,k} \cdot p_{w_2,l} \cdot q_{w_1} \cdot q_{w_2} \cdot \text{cv}_Q^2
\]

(C8)

Note that the mean of a link flow is equal to that in (C4) in this network.

The coefficient of variation of the link flow derived from (C4) and (C5) is given by:

\[
\text{cv}_Q = \frac{\left(\sum_{w \in W} \left(\sum_{k \in W_k} \delta_{w,k,a} \cdot p_{w,k}\right) \cdot q_w^2 \right)^{1/2}}{E[P_a] \cdot \text{cv}_Q}
\]

(C9)

(C9) shows that the coefficient of variation of the link flow derived from (C4) and (C5) is always smaller than those of the OD demands, if the independent OD demands are assumed. Note that the coefficient of variance of a link flow is equal to those of the OD demands, if the correlated OD demands are assumed.

REFERENCES


