THE USE OF A SOCIAL WELFARE FUNCTION FOR PLANNING PURPOSES IN A FREE-ENTERPRISE ECONOMY*

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Let me start by explaining the very limited purpose of this paper. We are confining our attention to the decisions of a government in a basically free-enterprise developed and industrialised economy such as the United Kingdom or Japan. We start from the familiar general assumption that competitive economic efficiency is promoted by the market process whereby each buyer—unable himself to affect any market price—turns from the more expensive to less expensive source of supply, while each seller—also himself unable to affect any market price—turns from the less to the more valuable outlet for what he has to sell. Thus we start with the thought that if the government refrains from interfering with the market all will be well, unless—as it rather quickly occurs to us—there be certain blemishes and imperfections in this competitive process which might be wholly or partially corrected by some judicious interventions by the government. That is to say, we are dealing not with a fully planned command economy, but with a free enterprise market economy with a number of ad hoc governmental interventions.

But when we come to consider the matter we realise that there are rather a large number of possible imperfections which may call for governmental intervention. These I will group under six headings.

(1) It may be considered desirable to influence the demographic developments in the community, that is to say, to raise or to lower the rate of growth of the population.

(2) It may be considered desirable to affect the distribution of income and wealth between the present and future generations, that is to say, to raise or to lower the proportion of the national income which is not consumed at once but which is saved and invested for the advantage of the citizens in the future.

(3) It may be considered desirable to take steps to redistribute income and wealth between the various classes of the citizens of any one generation, that is to say, to increase the welfare of the poor and needy at the expense of the rich and prosperous.

(4) For reasons which are now familiar it will probably be necessary for the government to intervene in order to stabilise total effective demand so as to preserve a high and stable level of employment and to prevent general economic depression, on the

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one hand, and inflationary developments, on the other hand.

(5) The free-enterprise competitive system will not work in the efficient manner of the perfectly competitive system if there are important indivisibilities in the economy which are bound to lead to the existence of monopolistic or monopsonistic influences in the economy. The government may wish to intervene in order to correct distortions to the efficient pattern of the use of resources caused by monopolistic elements.

(6) Finally, there is the huge category of external economies and diseconomies. This includes, at the one extreme, the need for the government itself to provide certain services, such as police, the benefits from which are entirely social or 'external' as contrasted, with the 'individual' or 'internal' benefits gained from some activity which affects directly only one or two particular citizens. At the other extreme, this category covers those cases where the individual activity has only some incidental external effects (such as the proverbial smoke nuisance of a factory) which the government can hope to control by some partial regulation or tax without taking over the whole activity from the private sector of the economy. Since, as we are now told, the external diseconomies of fumes from the petrol engine may so pollute the atmosphere as to put an end to all human life, it would be somewhat unrealistic not to allow for such factors in the catalogue of imperfections of the laissez-faire system.

The purpose of my paper is simply to enquire into the general manner in which the government of such a basically free-enterprise economy might consider these various diverse objectives in evaluating its policies. Since my purpose is limited in this way, I shall, solely in order to simplify the exposition, assume that the economy is a closed economy (unlike the United Kingdom or Japan!) and that one does not have to fuss about the balance of international payments, treaties about tariffs or customs unions, and so on. Even so it is clear that there is a vast range of considerations which the government must somehow or other have simultaneously in mind in devising its policies for intervention in the economy.

I shall not in this paper discuss the relative merits or defects of the different instruments of control which the government may use for these interventions. I shall, in fact, have at the back of my mind the assumption that the main instruments of control will be (i) monetary policy to make it more or less difficult for people to get hold of new funds of money to spend on capital developments or other purposes, (ii) rates of taxation and subsidisation on various economic flows or activities in order to influence the distribution of income or to induce people to spend more or less on this or that product or to indulge in a greater or smaller scale in this or that activity, and (iii) the direct purchase by the government of goods and services for social purposes such as defence, police, education, health, etc. My analysis does not rule out other kinds of regulation (such as the insistence on smoke-free zones) or the socialisation or nationalisation of certain industries; but it does assume that the basic pattern is one of private free-enterprise with a diversity of different governmental interventions, mainly of the indirect monetary, fiscal, and budgetary kind.

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The use of these controls by the government in order to achieve social objectives under the six main headings which I gave at the outset of this paper must be considered in a dynamic setting. To take but one very simple and obvious example, today's level of taxation on motor vehicles will affect not only today's use of the roads but by encouraging or discouraging the purchase of new cars today will affect the number of cars crowding on to the roads tomorrow and the next day, this future number of cars being also affected by future rates of motor vehicle taxation. Thus today's rates of tax on motor vehicles must be planned with an eye on the factors affecting congestion on the roads tomorrow, and these factors will include tomorrow's rates of motor vehicle taxation and also today's road-building program. Thus in order to consider the optimum level of today's rates of motor vehicle taxation and of today's expenditure on road building the government must have some plan for the optimum future course of motor-vehicle taxation and of expenditure on road-building. This is, of course, merely one very simple example of the problem of devising a dynamic optimal control plan.

Moreover, all this must be done in conditions of uncertainty. Thus, to illustrate the problem with a simple example, let us suppose that there is a technological advance on which research is going forward which, if successful, would greatly reduce the cost of railway transport. No one knows for certain whether it will be successful tomorrow or not. If it is, the demand for road transport and the crowding on the roads will be much less than if it is not successful. The government's present road-building and motor-vehicle taxation and its plans for the future road policies must take into account both possibilities.

In order to simplify my exposition I shall make the following very simple assumptions about the dynamic setting of our general control problem. First the government's plan for its controls covers only three periods which I will call Today, Tomorrow, and the Next Day or days 0, 1, and 2 respectively. Second, the only basic uncertainty about the future environmental state of the world is whether the weather will be Wet or Fine each day. We are faced then with the following schema:
We start at the beginning of Today knowing already whether it is Wet or Fine. We are then faced with four possible environmental paths over the remaining two days of the plan, namely sequences of (1) Wet–Wet, (2) Wet–Fine, (3) Fine–Wet and (4) Fine–Fine. I will call these environmental paths 1, 2, 3, and 4 respectively. This means that there are 11 relevant states of the economy to be considered. There are the four terminal points which I will call \( T_1 \), \( T_2 \), \( T_3 \), and \( T_4 \) respectively which refer to the states of the economy which will exist at the end of day 2 (i.e. the beginning of day 3) if the weather has moved on paths 1, 2, 3, or 4 respectively. There are also 7 other points. We start this morning in a known state of the world knowing that we are at the start of paths 1, 2, 3, and 4; and accordingly I call this starting point \( 1234 \). If it is Wet tomorrow, we shall know tomorrow that we are on paths 1 or 2; and I call this point \( 12 \). Similarly, for points \( 34 \), \( 1 \), \( 2 \), \( 3 \), and \( 4 \) on the diagram. The points on my diagram thus refer to points in time and in environmental states.

In order to make a rational choice of policies the governmental authorities must form some estimate of the probabilities of moving along the different environmental paths. This involves estimating (1) \( E_{12} \), i.e. the probability of moving from point \( 1234 \) to point \( 12 \) which also implies \( E_{34} \) the probability of moving to point \( 34 \), since \( E_{12} + E_{34} = 1 \), (2) \( E_1 \) i.e. the probability of moving from point \( 12 \) to point \( 1 \) which implies \( E_2 = 1 - E_1 \), and (3) \( E_3 \) i.e. the probability of moving from point \( 34 \) to point \( 3 \), which implies also \( E_4 = 1 - E_3 \).

If we make the assumption that in some sense or another the community clocks up units of Social Welfare or Utility as it passes through the various points along any one environmental path and that the objective of the government is so to set the levels of its controls as to maximise the total Social Welfare or Utility which it (the government) expects to be clocked up over the period of the plan, we can express our problem formally as the formulation today of a plan for the use of the governmental controls along each environmental path which will maximise

\[
E(U) = U_{1234} + E_1 U_{11} + E_{34} U_{34} + E_{12} E_{12} (U_1 + U_{T_1}) + E_2 E_{12} (U_2 + U_{T_2}) + E_3 E_{34} (U_3 + U_{T_3}) + E_4 E_{34} (U_4 + U_{T_4})
\]

where \( U_q \) stands for the amount of Social Welfare which would be clocked up if and when the economy passed through point \( q \) and \( U_{T_q} \) stands for the social valuation to be placed on the state in which the economy would be left at the end of path \( q \).

The purpose of my paper is to consider what might be the forms of the social welfare function (i.e. of \( U_q \) and \( U_{T_q} \)) if one was to try to formulate it and, more realistically, how far the sort of deliberations and actions of actual governments in the choice of policies could in reality be interpreted and even perhaps assisted by working out some of the implications of any such social welfare function. This is the sole purpose of my paper which is thus concerned with the ‘normative’ side of planning; and it means that I am completely neglecting the whole of the ‘positive’ side of the problem, namely all the questions connected with the estimation of the actual effect.
which any given time pattern of the governmental controls will actually have on the course of the economy—i.e., on the total national income, on its distribution between rich and poor, on the level of employment, on the output and consumption of various particular goods and services, and so on, as the economy moves along any one environmental path. I must simply assume that by means of clever work in its statistical, economic, and planning offices, whether through the construction of econometric models or through consultation with the private sectors of the economy in the formulation of an indicative plan, the authorities make the best possible estimates of the effect which any given time pattern of their controls along each given environmental time path will have upon the relevant variables in the economy.

I turn now to my main problem, namely the way in which the government might handle its policy objectives in such a setting.

Suppose that there are only two products X and Y—the generalisation to n products would present no difficulty of principle, I think. Then if one tried in our liberal free-enterprise economy to formulate a function $U_q$—namely, the amount of Social Welfare clocked up by the community as one passed through point $<q>$—one might express it as

$$U_q = a\{eN\bar{U}(\bar{C}_x, \hat{C}_y, \hat{L}_a)\}_q + b\{eN\bar{U}(\bar{C}_x, \hat{C}_y, \hat{L}_b)\}_q + \{(1-e)N\bar{U}(\bar{C}_x, \hat{C}_y)\}q + \{U_2(C_x, C_y, \bar{X}, \bar{Y})\}q + \{U_0(G_x, G_y)\}q$$

(2)

where $\bar{X}$ and $\bar{Y}$ are the total gross outputs of X and Y, $C_x$ and $C_y$ are the total amounts consumed by individuals, $G_x$ and $G_y$ are the amounts bought by the government for public use, $N$ is the total population, $e$ is the proportion of the population in work so that $1-e$ is the proportion unemployed, $a$ and $b$ (where $a+b=1$) are two constant fractions which divide the employed population into two groups $aeN$ and $beN$ such that the least rich person in group $aeN$ is better off than (or perhaps rather, at least as well off as) the least poor person in group $beN$, and the $\hat{C}$'s and the $\hat{L}$'s represent the amounts of consumption per head and of work done per head in the various groups in society.

The philosophy behind this is, I hope, clear. Social Welfare is represented primarily as the sum of the welfare of the individual citizens which depends directly upon the amount of each product consumed and the amount of work done by each individual citizen; but this sum is subject to two qualifications, first, the addition to or subtraction from the citizens’ welfare due to the social costs or benefits (i.e., the external economies or diseconomies) associated with various economic activities and, second, the contribution to welfare due to governmental expenditure upon goods for public consumption.

Let me first consider these qualifications. $U_2(C_x, C_y, \bar{X}, \bar{Y})$ expresses the valuation of the external social costs and benefits which may be associated with the private consumption and production of the two commodities. It is only illustrative, since external
social costs and benefits might well be associated with other variables, such as for example as the number of persons employed in a certain productive activity. Such elements could readily be added. \( U_d(G_x, G_y) \) expresses the valuation placed upon the public services fed by the governmental purchases of the two goods.

As for the basic elements, namely the individual citizens' individual utilities from their individual consumptions and leisure, it would be unreasonable to ask any government to assess every individual's welfare separately and to add up the result—to assess the happiness of Mr. John Smith, of Mr. Tom Jones, of Mr. Richard Brown, and so on separately and add them all up. The thing clearly must be done in broad classes of persons. In equation 2 I have, again solely for illustrative purposes, divided the population into three groups. There are the unemployed, namely \((1-e)N\), who are unwillingly debarred from working and whose private utility depends solely upon their private consumption, utility per head of the unemployed being thus \( U(C_{xu}, C_{yu}) \). The employed, namely \( eN \), are then divided into two classes, the rich, namely \( aeN \) and the poor, namely \( beN \), where \( a+b=1 \). Clearly it would be possible to divide the population up into any number of gradations of wealth and poverty. For example if one divided them up into 100 one-per-cent classes (i.e. \( a_1+a_2+\ldots+a_{100}=1 \)), where \( a_i \) denoted the 1 per cent richest citizens, \( a_2 \) the 1 per cent next richest citizens, and so on, one would obtain an almost continuous description of the distribution of wealth and poverty. The two classes \( a+b=1 \) are chosen solely for purposes of simple illustration.

The \( \hat{U} \)'s of the social welfare function must be valuations made by the government in so far as they involve interpersonal comparisons, but for the rest they are valuations which respect individual choice in the market. Thus, simply to illustrate the principle, let us suppose that all citizens have the same tastes (i.e. the same indifference map) and have only to choose between the consumption of \( X \) and \( Y \) (i.e. \( \hat{C}_x \) and \( \hat{C}_y \)). Then the individual's indifference map might be as follows:

![Indifference Map](image)

If the income of a citizen in each of our three classes allowed consumption at points \( a, b, \) and \( u \) respectively, then the \( \hat{U} \)'s in the social welfare function would be based on the citizen's indifference map, but with governmental cardinal numbers alloted to the indifference curves \( \hat{U}_a, \hat{U}_b, \) and \( \hat{U}_u \).

The simple classification of the citizens into the three classes \( a, b, \) and \( u \) skates over
a number of difficulties of which I will mention only two. First, how does one
measure the degrees of wealth and poverty in order to order the citizens according to
their wealth? Is it by income after tax? Or is it solely by the amount spent on
consumption? In either case does one take into account the amount of work done? Is
a man who chooses a relatively low income from earnings and thus a relatively low
level of consumption because he greatly prefers leisure, in a real sense "poorer" than
one who, with the same opportunities, chooses to work more and earn more? Is the
ownership of property itself, quite apart from the income from property, to be counted
as making a man richer for the purposes of our classification? I am afraid that I
shall simply assume that these considerations are in fact relatively unimportant.

Second, there are demographic facts of the utmost importance in the real world
which my formulation entirely ignores. My formulation would be correct if the whole
population was comprised of male fully adult workers. But the age and sex composi-
tion of the population is, of course, of fundamental importance for the standard of
living. Workers do not spring fully trained from their mothers' wombs, and they
grow old and incapable of work. Governmental policies which affected fertility, nup-
tiality, and mortality rates are of immense importance; and the first effect of a change
in fertility is, of course, to increase the number of dependent children, later to increase
the number of workers, and still later to increase the number of the dependent old
aged in retirement. These dynamic demographic developments are of fundamental
economic importance and are disgracefully neglected by economic theorists interested
in economic growth and dynamic developments. I shall myself incur the shame of
neglecting them in this paper and assume that governmental policies may affect the
number of adult males of working age who comprise the whole population.

Let us suppose then that the government has a three-day plan for the development
of its controls over the four environmental paths illustrated in my diagram. That is
to say, it knows what levels of motor vehicle taxation and road building it will impose
Today; and it has an idea of the levels of road building and of motor vehicle taxation
which it intends to impose Tomorrow if it is at point <12> and the levels which it
intends to impose tomorrow if it is at point <34>, and similarly for the alternative
levels which it plans to impose The Day after Tomorrow if it is then at points <1>,
<2>, <3>, or <4> on the possible environmental paths. It then considers whether a
small change of a particular kind in this pattern of planned controls — e.g. a little
more to be spent on the roads Tomorrow if at point <12> — is an improvement or not.

Let me first pause to consider whether this is a sensible sort of question to ask.
One can see the point of the procedure if the question is whether a small change in
the controls actually to be imposed Today would improve things or not, always taking
into account the direct and indirect effects of the change in Today's controls on
possible events Tomorrow and the Next Day. But is there any point in asking Today
whether a small change in the controls planned for Tomorrow would improve matters?
After all Tomorrow's controls need not be actually determined till Tomorrow comes.
What is the point then of fussing one's head with the question whether a slight rise in the amount planned for road expenditure Tomorrow if Tomorrow turns out to be Fine will improve social welfare or not?

The answer is, I think, twofold.

In the first place, there is much to be said for publishing the government’s control plan. To obtain the best informed decisions by private citizens today it is necessary that they should have the most accurate information available about future conditions. The potential purchaser of a car Today may properly be influenced by the question whether or not a particular road will be built tomorrow. If, therefore, the government’s control plan is published, Today’s events (e.g. the number of cars bought Today) may be influenced by a change in the government’s plans for its future controls (e.g. the amount of road building Tomorrow). Future plans can thus have an actual effect on Today’s events and so on social welfare.

But, secondly, there is a much more fundamental point than this. Suppose that a change in the plans for the government’s future controls are not made public so that they can have no effect upon Today’s events in the private sector of the economy. Nevertheless there may be point in the government considering whether a small change in its controls planned for the future would increase or decrease the social welfare expected over the period of the plan, for the simple reason that an improvement in the pattern of Tomorrow’s planned controls (given Today’s controls) will enable the government to consider whether there might not then be a possible improvement in Today’s controls (given the new and improved level of the controls planned for Tomorrow). Thus given the actual level of Today’s motor-vehicle taxation and road building programme, the question may be asked whether it would not increase the total of social welfare to be expected over the three-day plan period if there were not some increase in the road expenditure planned for Tomorrow if Tomorrow is Fine (i.e. for point <34> on my diagram). Suppose the answer is Yes. Then the question may be raised whether a reduction in Today’s motor vehicle taxation would increase the social welfare expected over the planned period. The answer to this question might have been No if the old plan for low expenditure on road building Tomorrow had been maintained, since to stimulate the demand for cars Today would merely add to Tomorrow’s congestion. But the answer might be Yes if Tomorrow’s plan for road expenditure had been raised to the improved and higher level. Thus to improve Tomorrow’s plan (given Today’s controls) may be a useful way of finding out whether Today’s actual controls might not be improved.

This procedure is, of course, merely a gradient method of climbing the hill of social welfare. If a marginal change in Today’s actual controls or in Tomorrow’s planned controls is made only if it would increase the total social welfare expected over the period of the plan, then any series of such marginal changes will move one continually up hill until one reaches the summit.1)

The procedure is subject to two serious limitations.
In the first place, there is the well-known point that one may be climbing the wrong peak in the sense that nearby on the mountain range there may be a still higher peak. Some structural change might improve the final prospects for social welfare, although this might involve in the first place moving one's plan of controls in a direction which, if the changes were kept to small marginal adjustments, would decrease expected welfare. One must first descend into the valley in order to start on the ascent on the higher peak.

Secondly, the procedure may, of course, be an extremely roundabout way of getting near the top of the peak. Consider, for example, two commodities X and Y which are good substitutes for each other in demand. Consider two tax controls, a tax rate of $R_x$ on the consumption of X and of $R_y$ on the consumption of Y. The best course might be to reduce both taxes to zero. But damage may in fact be done not only by the average height of $R_x$ and $R_y$ (which undesirably diverts demands away from these two goods in general) but also by the difference between $R_x$ and $R_y$. If $R_x$ is much greater than $R_y$, then high-cost Y will be being produced which could better be replaced by low-cost X. A rise in $R_y$ will help to remove this particular inefficiency. Thus with $R_x$ considerably greater than $R_y$, some rise in $R_y$ might improve things. But when $R_y$ has been raised as near $R_x$ as is necessary to attain the maximum benefit, $R_y$ will still be less than $R_x$, since a low level of $R_y$ has advantage in not diverting demand away from Y onto goods other than X. It will then, of course, improve matters to reduce $R_x$ since a reduction in $R_x$ will attract demand from high cost Y and also from the other goods. $R_x$ will then be reduced below $R_y$ until the advantage of attracting demand on to X from other goods is offset by the disadvantage of attracting demand from the now relatively low cost Y. Then it will be desirable to reduce $R_y$ and so on, until $R_x$ and $R_y$ have both been reduced to zero. Clearly it would be a roundabout way to reach this final state by starting, as we did in this example, with a rise in $R_y$, even though it remains true that every step in the above sequence marked an improvement.

The moral of this is that the government in considering a marginal change in its control plan should enlist what help it can from its economic advisers to start on an initial set of exploratory changes which is likely to provide a direct route towards the best attainable position with the existing structure of the economy.

In any case I intend to devote the rest of this paper to the very limited question how the general philosophy behind a social welfare function of the kind indicated in equation 2 might be used to judge whether any small given change in the pattern of controls planned over the plan period would or would not be an improvement, given that the economic and statistical advisers of the government could say how the change in the pattern of controls would affect the arguments of the welfare function. The

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1) Assuming that if there is any point of inflexion one can perturb the system so as to find out whether there is any remaining direction in which one can still climb.
first step is, of course, to differentiate the expression for $U_q$ in equation 2 in respect of the variables $e, N, \hat{C}_{xa}, \hat{C}_{ya}, \hat{L}_{a}, \hat{C}_{xb}, \hat{C}_{yb}, \hat{C}_{xu}, \hat{C}_{yu}, C_x, C_y, X, Y, G_x$ and $G_y$, all of which are direct causes of change in the amount of social welfare ($U_q$) which the community will clock up on its meter of social welfare if and when it passes through point $\langle q \rangle$.

We shall obtain an expression for $dU_q$ after the introduction of a certain number of new constraints and new definitions which we will introduce in a number of stages.

Let us first write

$$\mu_{aq} = \left\{ \frac{\partial \hat{U}_a}{\partial \hat{C}_{xa}} + P_x \right\}_q = \left\{ \frac{\partial \hat{U}_a}{\partial \hat{C}_{ya}} + P_y \right\}_q = -\left\{ \frac{\partial \hat{U}_a}{\partial \hat{L}_a} + \hat{W}(1 + \delta_{ta}) \right\}_q$$

and similarly for $\mu_{bq}$, where

$$\mu_{bq} = \left\{ \frac{\partial \hat{U}_b}{\partial \hat{C}_{xb}} + P_x \right\}_q = \left\{ \frac{\partial \hat{U}_b}{\partial \hat{C}_{yb}} + P_y \right\}_q$$

where $\mu_{aq}, \mu_{bq},$ and $\mu_{uq}$ represent the marginal utility of money income to our three classes of citizens — rich, poor and unemployed, $-P_xq$ and $P_yq$ represent the market prices ruling for the two commodities, and $\hat{W}_q$ represents the money wage rate fixed for the employment of a unit of labour at the beginning of day $q$, and payable at the beginning of day $q$.

As far as the purchase of the two goods is concerned, equations 3 simply assume that all consumers can purchase any amount of any good for final consumption at the same current market price (which must therefore include any rate of tax levied on that good) without the price being affected by the purchases of any individual consumer. But this assumption is not made about the purchase of leisure by the employed citizens in classes $a$ and $b$. $\hat{W}$ represents the price paid by the employer of labour for a unit of labour. The worker may add less than $\hat{W}$ to his income for an additional unit of work provided by him. This would happen if the supplier of labour were a monopolist. But the direct taxation of earnings provides a much more certain and universal reason for this phenomenon, and the more progressive the system of direct taxation the greater the divergence between the wage paid by the employer and the amount added to his disposable income by the worker. If the marginal rate of tax were 20 per cent, then out of a $\hat{W}$ of $1$ only $.8$ would be received by the worker as a net addition to his income. The net wage would be $(1 - .2) \hat{W}$. This divergence is represented by the terms $\delta_{ta}$ and $\delta_{tb}$ in equation 3.

2) $G_x$ and $G_y$ are, of course, themselves controls. They may change as a direct part of the change in the planned pattern of controls. The other variables given above may change as an indirect effect of the changed pattern of controls.

3) I use the notation $\hat{W}$ instead of $W$ for the wage rate to distinguish it from the usage in my book The Growing Economy where I assumed that the wage rate for work done during day $t$ ($W_t$) was fixed at the beginning of day $t$, but payable at the beginning of day $t + 1$. If $i_t$ were the rate of interest ruling for a day's loan made at the beginning of day $t$, then $\hat{W}_t(1 + i_t) = W_t$.
Let us introduce a variable $\mu_q$ representing the marginal utility of income to the average citizen at point $q$. We will discuss later what sort of average of $\mu_{aq}$, $\mu_{bq}$, and $\mu_{uq}$ is best used for $\mu_q$, but for the next stage of our analysis any arbitrary value of $\mu_q$ may be chosen. We can then write

$$\frac{\partial U_x}{\partial C_x} = \left\{ \mu P_x \delta_{scx} \right\}_q$$

with similar expressions for $\frac{\partial U_y}{\partial C_y}$, $\frac{\partial U_z}{\partial X}$, and $\frac{\partial U_z}{\partial Y}$.

and

$$\frac{\partial U_x}{\partial G_x} = \left\{ \mu P_x (1+\delta_{gx}) \right\}_q$$

with a similar expression for $\frac{\partial U_y}{\partial G_y}$.

In equation 4 the term $\delta_{scx}$ measures the divergence between marginal social and marginal private utility in the private consumption of $X$, since the addition to total social utility of an extra unit of private consumption of $X$ is $\mu P_x + \frac{\partial U_x}{\partial C_x}$ (i.e. the sum of private utility and the valuation of any external benefit), so that

$$1 + \delta_{scx} = \frac{\text{marginal social utility}}{\text{marginal private utility}}.$$  \hspace{1cm} \text{In equation 5 the term $\delta_{gx}$ represents the proportion by which one should raise the market value of the goods bought by the government for public purposes in order to get the marginal social valuation of the use of these goods for government purposes. If $\delta_{gx} > 0$ then — apart from other indirect repercussions — to transfer a unit of $X$ from private to public consumption would add to social welfare.}

We may also note that the total amount of private consumption of each good is the sum of the consumption by the three classes of citizens so that

$$C_x = N(ae\hat{C}_{xa} + be\hat{C}_{xb} + [1-e]\hat{C}_{xu})$$

and similarly for $C_y$. From these equations we can get an expression for $dC_x$ in terms of $dN$, $de$, $d\hat{C}_{xa}$, $d\hat{C}_{xb}$, and $d\hat{C}_{xu}$; and similarly for $dC_y$.

Moreover if we write $L$ for the total amount of work done we have

$$L = eN(a\hat{L}_a + b\hat{L}_b)$$

From this expression we can obtain an expression for $dL$ in terms of $dN$, $de$, $d\hat{L}_a$, and $d\hat{L}_b$.

We may next consider the technical, production constraints on the economy. We assume production functions of the form

$$\hat{X}_x = F_x(L_{xq}, X_{xq}, Y_{xq})$$

and

$$\hat{Y}_x = F_y(L_{yq}, X_{yq}, Y_{yq})$$

where $\hat{X}_x$ and $\hat{Y}_x$ are the gross outputs of goods produced at the end of the day $q$ (including all the stock of capital equipment handed over from day $q$ to the next day and all the consumption goods produced during day $q$ and available for consumption on the next day) while $L_{xq}$, $X_{xq}$, and $Y_{xq}$ are the amounts of labour and intermediate products and capital goods put at the beginning of day $q$ into the industry producing
\[ X, \text{ and similarly for } L_{qq}, X_{qq}, \text{ and } Y_{qq}. \] We also have
\[
C_{q} = X_{q-1} - X_{qq} - X_{q} - G_{qq}
\]
and
\[
C_{qq} = Y_{q-1} - Y_{qq} - Y_{q} - G_{qq}
\]
where \( q = 1 \) represents the point on our diagram from which the point \( q \) must have proceeded—for example if point \( q \) is point \( 1 \) on our diagram then point \( q-1 \) is point \( 12 \), and so on. These equations state that the amount of any good available for private consumption on day \( q \) is the gross amount produced as a result of the productive operations of day \( q-1 \) less the amounts set aside on day \( q \) as inputs into the productive processes of day \( q \) and less the amounts consumed by the government for public services on day \( q \).

Let us also write
\[
L_{q} = L_{xq} + L_{yq}
\]
\[
X_{q} = X_{xq} + X_{yq}
\]
\[
Y_{q} = Y_{xq} + Y_{yq}
\]
Let us write
\[
1 + \delta_{l_{xq-1}} = P_{xq} \frac{\partial F_{xq-1}}{\partial L_{xq-1}} + W_{q-1}(1 + \frac{i}{1 - i})
\]
In this case \( \delta_{l_{xq-1}} \) measures the divergence between the value at time \( q \) of the wage fixed and paid at time \( q-1 \) for a unit of labour employed at \( q-1 \) and the value of its marginal product (namely, \( \frac{\partial F_{xq-1}}{\partial L_{xq-1}} \)) to be sold at time \( q \) for a price \( P_{xq} \). And similarly for \( \delta_{l_{yq-1}} \)

Let us also write
\[
1 + \delta_{x_{xq-1}} = P_{xq} \frac{\partial F_{xq-1}}{\partial Y_{xq-1}} + P_{yq}(1 + \frac{i}{1 - i})
\]
In this case \( \delta_{x_{xq-1}} \) measures the divergence between the cost of a unit input of \( Y \) at point \( q-1 \) into the \( X \) industry reckoned at its value at point \( q \) when the additional output of \( X \) will be sold (i.e., \( P_{yq}([1 + \frac{i}{1 - i}] \)) and the value at point \( q \) of that additional product when it is sold (i.e., \( P_{xq} \frac{\partial F_{xq-1}}{\partial Y_{xq-1}} \)). And similarly for \( \delta_{x_{yq-1}}, \delta_{y_{xq-1}}, \) and \( \delta_{y_{yq-1}} \).

Then differentiating equations 8, 9, and 10, using equations 11 and 12, and rearranging the terms we get
\[
(P_{x}[dC_{x} + dG_{x} + dX] + P_{y}[dC_{y} + dG_{y} + dY])_{q} = [(1 + i)P_{x}dX + P_{y}dY + \hat{W}dL + P_{s}[\delta_{x}dX_{s} + \delta_{y}dX_{y}] + P_{s}[\delta_{y}dY_{s} + \delta_{x}dY_{x}] + P_{s}[\delta_{x}dL_{x} + \delta_{y}dL_{y}]]_{q-1} (13)
\]
If we now differentiate the expression for \( U_{q} \) in equation 2 and make use of equations 3, 4, 5, 6, 7, and 13 we obtain the following expression which breaks down the change in social welfare clocked up as the economy passes through point \( q \) into a number of component parts:
\[ dU_i = \mu_q \left\{ \frac{dN}{N} \left( \frac{U_p}{\mu} - PC + \bar{WL} \right) \right\}_q + \mu_q \left\{ \frac{de}{e} \left( \frac{U_p - N \bar{U}_a}{\mu} - [PC - PNCu] + WL \right) \right\}_q + \mu_q \left\{ \left[ \frac{(1 + \delta)PdI}{\gamma} - \mu_q \left[ PdI \right] \right]_q \right\} \]

where \( U_p = aeN \bar{U}_a + beN \bar{U}_b + (1 + \delta)N \bar{U}_u \) thus measuring the total utility obtained from the personal consumptions of private individuals; \( PC = P_xC_x + P_yC_y \) thus measuring the total money expenditure on private consumption; \( PNCu = N[P_x \bar{C}_x + P_y \bar{C}_y] \) thus measuring what the total expenditure on personal consumption would be if everyone lived on the standard of an unemployed man; \( Pd\bar{C}_a = P_xd\bar{C}_a + P_yd\bar{C}_y \) thus measuring the increase in the total quantity consumed by an individual in class \( a \) valued at the current prices of consumption goods and similarly for \( Pd\bar{C}_b \) and \( Pd\bar{C}_u \); \( PdI = P_xdX + P_ydY + \bar{W}dL \) thus measuring the additional quantity of resources, valued at their current prices, invested at the beginning of the productive process on any day; and \[ *a/* = \frac{\mu_a}{\mu} \] or the ratio of the marginal utility of money income to an individual in class \( a \) to the general average marginal utility of money income and similarly for \[ *b/* = \frac{\mu_b}{\mu} \] and \[ *u/* = \frac{\mu_u}{\mu} \].

It is to be observed that all the items on the RHS of equation 14 are increments of social welfare clocked up at point \( <q> \) measured in money terms at the money prices ruling at point \( <q> \),\(^4\) which are then multiplied by \( \mu_q \), which measures the marginal utility of money income at point \( <q> \), a measure which we will discuss below.

We will now comment in turn on the various components of equation 14.

**Component I** This component is positive if \( \frac{U_p}{N} > \mu \frac{PC - \bar{WL}}{N} \). This corresponds to the criterion for judging whether an increase in total population would increase total welfare which I gave on page 91 of my *Trade and Welfare*. \( \frac{U_p}{N} \) measures the average utility or 'happiness per head' of a representative citizen, and therefore

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4) Thus \( \frac{U_p}{\mu N} \) measures the 'money value of a man' in the sense of the total utility enjoyed by a representative citizen valued in terms of current dollars.
measures what would be the addition to social welfare from having one more citizen to enjoy life, if the existence of the new citizen did not affect anyone else's welfare. But $\frac{PC}{N}$ is a measure of the consumption which a new representative citizen will take out of the national product, while $\frac{WL}{N}$ is a measure of the marginal product of a representative citizen and thus (apart from any divergences between the wage rate and the marginal product of labour which will appear in component V) measures the contribution of a new representative citizen to the national product. Thus $\mu \frac{PC-WL}{N}$ measures the social valuation of the net burden of a new citizen upon the national product.

**Component II** There are two alternative commonsensical ways of thinking about the Full Employment component.

1. Given a small increase in employment out of a given population, namely $Nde$, social welfare will be increased to the extent that $\frac{1}{\mu} \left\{ \frac{Up-N\hat{U}_u}{eN} - \mu \left( \frac{PC-PN\hat{C}_u-\bar{WL}}{eN} \right) \right\} > 0$. $Up-N\hat{U}_u$ is the total excess of the private utility of the total population over what it would be if all the members were at the unemployed's standards. Since the unemployed are in any case at the unemployed’s standards, this is the same as the excess of the total private utility of the employed population over what it would have been if they had been at the unemployed standards, so that $\frac{Up-N\hat{U}_u}{eN}$ is the increase in utility for a man moving from the unemployed to the employed sector of the economy. But by a similar reasoning it can be seen that $\frac{PC-PN\hat{C}_u}{eN}$ is the increase in consumption for a man so moving. If this excess consumption had to come at the margin from others’ consumption, it would impose a loss on others equal to $\mu \frac{PC-PN\hat{C}_u}{eN}$. But, in so far the employed man’s wage is equal to the value of his marginal product (and in so far as this is not so, the divergence is accounted in component VII), there is an addition to the social value of output of $\mu \bar{W} \frac{L}{eN}$ where $\frac{L}{eN}$ is the amount of work done per employed man. The net gain from the transfer of one man from the unemployed to the employed sector is therefore

$$\frac{Up-N\hat{U}_u-\mu(FC-PN\hat{C}_u-\bar{WL})}{eN}$$

2. Another way of looking at the matter is to say that $\bar{W} \frac{L}{eN}$ is the contribution to additional output due to having one more man in employment (apart from any divergences between marginal products and rewards which appear in component VII) and that $\mu \bar{W} \frac{L}{eN}$ is, therefore, the ‘efficiency’ contribution of fuller employment. The remaining element $\frac{Up-N\hat{U}_u-\mu[PC-PN\hat{C}_u]}{eN}$ is a change in social welfare due to the alteration of the ‘distribution’ of income as a result of paying more to the newly employed man and so much less than would otherwise be the case to other members of the population. We will return to this interpretation when we consider component IV.
Component III \( \{PdI\}_q = (P_x dX + P_y dY + W dL)_q \) is the value at current prices of the increment of productive resources on which capital funds are laid out at the beginning of day \( q \) to produce goods for sale at the beginning of the next day; that is to say it is the value of the increment of real resources invested in the productive process at the beginning of day \( q \). An inspection of component III indicates that this element, which appears with a negative sign at point \( q \), will reappear with a positive sign at all points to which the economy could move on the day after \( q \). Suppose point \( q \) is point \( 12 \) on our diagram; then it is clear that \( \{PdI\}_{12} \) will reappear with a positive sign at points \( 1 \) and \( 2 \). An inspection of equation 1 together with component III of equation 14 then shows that the net contribution of \( \{PdI\}_{12} \) to expected social welfare is \( E_{12} \{PdI\}_{12} \{(1+i_{12})(E_1\mu_1+E_2\mu_2)-\mu_{12}\} \) which is positive if

\[
i_{12} > \frac{\mu_{12} - (E_1\mu_1 + E_2\mu_2)}{E_1\mu_1 + E_2\mu_2}
\]

i.e., if the rate of interest is greater than the expected rate of decline of the marginal utility of income. This is the familiar formula for expressing the condition on which an increment of savings can add more to expected future utility through increased future consumption than it subtracts from present utility through decreased present consumption.  

Two comments on this expression may be useful.

First, there may be divergences between the rate of interest and the expected value on the subsequent day of the net marginal product of the additional real resources invested at the beginning of day \( q \). The formula without modification would be true only if there were no such divergences; but these divergences are accounted for in the \( \delta \) terms in component VII of equation 14.

Second, the \( i \) used in the formula is the money not the real rate of interest. But this does not present us with any difficulty. Suppose that all prices are rising by a proportion \( p \) between one day and the next. Then the marginal utility of income will be falling for two reasons, first because a unit of money will buy less consumption goods and second because a unit of consumption goods will 'buy' less 'utility'. The real decline in the marginal utility of consumption is thus \( \mu_{12} - (E_1\mu_1 + E_2\mu_2) - p \). But the real rate of interest is now \( i - p \). The balance between \( i \) and \( \frac{\mu_{12} - (E_1\mu_1 + E_2\mu_2)}{E_1\mu_1 + E_2\mu_2} \) remains unchanged.

5) If \( E_1 = 1 \) so that \( E_2 = 0 \), the condition would become \( i_{12} > \frac{\mu_{12} - \mu_1}{\mu_1} \) or \( i_t > \frac{\mu_t - \mu_{t+1}}{\mu_{t+1}} \).

For any individual the elasticity of substitution \( \sigma \) between consumption at \( t \) and consumption at \( t+1 \) can be expressed as (the proportionate increase in consumption) \( \div \) (the proportionate fall in the marginal utility of consumption). If money prices were constant \( \frac{\mu_t - \mu_{t+1}}{\mu_{t+1}} \) would measure the proportionate fall in the marginal utility of consumption and \( i_t \) would measure the real rate of interest. If \( \delta \) were the proportionate increase in an individual's consumption, then the expression becomes \( i > \frac{\delta}{\sigma} \) which is the form used on page 205 of *The Growing Economy*. 

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Component IV The algebra by which equation 14 is reached would be correct for any arbitrary value of $\mu$, which we might have cared to choose at equations 4 and 5. In fact in a second-best problem of the kind with which we are at present concerned any division of the effects of a policy change between their 'efficiency' effects and their 'distributional' effects is arbitrary. The change may make some people better off by a large amount and others worse off by a smaller amount; and that is that. Any attempt to say how much of this change is an 'efficiency' change and how much a 'distributional' change is arbitrary. Yet it may in my opinion be useful for a policy maker to make some arbitrary distinction of this kind. But how might one proceed to make a useful distinction of this kind?

The basic need is to compare the marginal utility of income to different groups both within a given generation (i.e., on any one day) and as between generations (i.e., as between one day and another). For this we need a benchmark measure. We might, for example, take the additional utility obtained from $1$ more to spend on consumption by an unemployed man at point $<1234>$ on my diagram as our measuring rod, i.e., $\mu_{1234}=1$. At point $<q>$ the basic measures would then be $\mu_{aq}$, $\mu_{bq}$, and $\mu_{uq}$. These are what in my Trade and Welfare I called the distributional weights. They depend upon value judgements made by the policy makers; and they imply, for example, that the policy makers would be equally satisfied by a policy change which gave $1$ more to a man with the standard of living of an unemployed man at point $<1234>$ faced with the cost of living of point $<1234>$ and by a policy change which gave $\frac{1}{\mu_{aq}}$ to a rich man with the standard of living of a rich man at point $<q>$ and with the cost of living ruling at point $<q>$.

But the formulation of these distributional weights is not sufficient to say whether a given change has had any, and if so what, distributional effect. In order to make any statement of this kind, it is necessary to formulate an arbitrary definition of what sort of distribution of any given improvement between the various classes is said to be such as to leave the distribution of welfare unchanged. This is an arbitrary definition, though it has no doubt the overtones of a value judgment. Suppose $1$ more was available for expenditure on consumption at point $<q>$ at the prices current at point $<q>$. The policy makers must define a distribution of this $1$ among the various groups in society (namely $ae_N, be_N$, and $(1-e_N)N$) which is such as to leave distribution unchanged. Let $\lambda_{aq}$, $\lambda_{bq}$, and $\lambda_{uq}$ (where $\lambda_2+\lambda_3+\lambda_4=1$) be the fractions of the $1$ which, according to the chosen definition, must go to the three groups if the distribution of welfare is to be unchanged. Then $\mu$ is defined as $\lambda_{aq}\mu_{aq}+\lambda_{bq}\mu_{bq}+\lambda_{uq}\mu_{uq}$ and it measures the increase in private utility which would be gained by society having $1$ more to spend on consumption, if that $1$ were distributed so as to have what has

6) When I wrote Part I of my Trade and Welfare I was not, I think, clear about the arbitrariness of this distinction. What follows should be read in this respect as an addendum to that volume.
been defined as no distributional effect.\(^7\)

If a proportion \(\lambda_a\) of any increase in total expenditure on consumption \([PdC]\) did in fact go to group \(a\), then \(\frac{\lambda_a[PdC]}{aeN}\) would be the increase in consumption of an individual in group \(a\). It can then be seen from equation 14 that component IV of that equation would be zero if the distribution of \([PdC]\) among the three groups were in the proportion \(\lambda_a, \lambda_b, \lambda_u\).

Since \(\alpha = \frac{\mu_a}{\mu}\), \(\beta = \frac{\mu_b}{\mu}\), and \(\gamma = \frac{\mu_u}{\mu}\), the total component would become

\[ [PdC] \left[ (\mu_a - \mu)\lambda_a + (\mu_b - \mu)\lambda_b + (\mu_u - \mu)\lambda_u \right] \]

which is equal to zero since \(\lambda_a + \lambda_b + \lambda_u = 1\) and \(\mu = \lambda_a \mu_a + \lambda_b \mu_b + \lambda_u \mu_u\).

We may now revert to the distributional element in the Full Employment component which was mentioned in connection with component II above, namely

\[ \frac{U_p \cdot N}{eN} \cdot \frac{\mu(\text{PC} - \text{PNC}_u)}{eN} \]

If we substitute the values for \(U_p\), \(\text{PC}\), and \(\text{PNC}_u\) given after equation 14 above, we can see that this distributional component will be positive if \(\mu < \frac{\dot{U}_a - \dot{U}_u}{P(a \text{Ca} + b \text{C}_b - \text{C}_u)}\).

In order to simplify the exposition of the commonsense which lies behind this formula, let us suppose that all the employed are similar in their standards (i.e., \(a = 1\) and \(b = 0\)) so that we have solely a distinction between employed \((a)\) and unemployed \((u)\). Then our condition becomes

\[ \mu < \frac{\dot{U}_a - \dot{U}_u}{P(a \text{Ca} - \text{C}_u)} \]

We may write the RHS of this expression as \(\Delta(\dot{U}) / \Delta(\text{PC})\) to represent the ratio of the increase in utility to the increase in expenditure resulting from an unemployed man finding employment. Now a citizen who moves from unemployment to employment will have a high marginal utility of income when he is unemployed \((\mu_u)\) and a low marginal utility of income when he is employed \((\mu_a)\). We may assume, therefore, that

\[ \mu_a > \frac{\Delta(\dot{U})}{\Delta(\text{PC})} > \mu_u. \]

But \(\mu\) is a weighted average of \(\mu_u\) and \(\mu_a\), namely \((1 - \lambda_u)\mu_a + \lambda_u \mu_u\).

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7) There are many different possible definitions of the absence of distributional effect. (i) If it involves giving an equal amount of additional income to every citizen, then \(\lambda_a = ae\). (ii) If it involves dividing the $1 in the ratio of citizens' existing consumption levels, then

\[ \lambda_a = \frac{[\text{PC}]_a}{[\text{PC}]_a + [\text{PC}]_b + [\text{PC}]_u}. \]

(iii) If it involves dividing the $1 in the ratio of citizens' existing total utilities, then \(\lambda_a = \frac{ae \dot{U}_a}{ae \dot{U}_a + be \dot{U}_b + [1-e] \dot{U}_u}\).

(iv) If it involves dividing the $1 so as to raise every individual's utility by the same proportion, then

\[ \lambda_a = \frac{ae \dot{U}_a / \mu_a}{ae \dot{U}_a / \mu_a + be \dot{U}_b / \mu_b + [1-e] \dot{U}_u / \mu_u}. \]

(v) If it involves dividing the $1 so as to give each individual the same absolute increase in utility, then

\[ \lambda_a = \frac{ae / \mu_a}{ae / \mu_a + be / \mu_b + [1-e] / \mu_u}. \]
If $\lambda_u$ is sufficiently small, then $\mu$ will be $<\lambda(\bar{U})/\lambda(P\bar{C})$. Let us take a numerical example. Suppose $\lambda(\bar{U})/\lambda(P\bar{C})=\frac{1}{2}\mu_u+\frac{1}{2}\mu_a$. Then, with $\mu_u>\mu_a$, the condition $\mu=(1-\lambda_u)\mu_u+\lambda_u\mu_a<\lambda(\bar{U})/\lambda(P\bar{C})=\frac{1}{2}\mu_u+\frac{1}{2}\mu_a$ is satisfied if $\lambda_u<\frac{1}{2}$.

The commonsense of this is clear. When one man shifts from the unemployed to the employed class, his standard of living is increased. Apart from what he produces (and this is already taken account of in the 'efficiency' element of the Full Employment), this increase in his consumption must be taken from the community in general. His marginal utility was $\mu_u$ in his previous position; but he has had a structural change in his position and his marginal utility is now $\mu_a$. Giving him an extra income is (with our assumption that $\lambda(\bar{U})/\lambda(P\bar{C})=\frac{1}{2}\mu_u+\frac{1}{2}\mu_a$) equivalent to giving $\frac{1}{2}$ to an employed man with marginal utility $\mu_a$ and $\frac{1}{2}$ to an unemployed man with marginal utility $\mu_u$.

The loss of utility to the community in general assumes a proportion $\lambda_u$ to have been raised from the existing unemployed. If $\lambda_u<\frac{1}{2}$ (as will almost certainly be the case) the next result is, as it were, a net redistribution from the 'rich' employed to the 'poor' unemployed.

Component V This component needs little comment. If as a result for example, of direct taxation the net wage received by the worker is less than the wage paid by the employer, then an incentive which induces an increased effort has a positive efficiency effect.

Component VI The $\delta$s here may be positive measuring the external social benefits, or negative measuring the external social costs, of increases in the private or governmental consumption or in the production of the various goods.

Component VII The $\delta$s here measure divergences within the productive system between the price paid to a factor by an employer and the price paid by the purchaser of the product for the marginal product of that factor. The most obvious causes of

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8) If the employed and the unemployed had the same utility function with a diminishing marginal utility of money income and if the only difference in their situation were that the employed had more income to spend on the consumption of products than the unemployed, this would be certain. $\lambda(\bar{U})/\lambda(P\bar{C})$ would be the average utility of the increment of consumption of products and this would lie somewhere between the high marginal utility of the first additional units of consumption ($\mu_u$) and the low marginal utility of the last additional units of consumption ($\mu_a$). But, as Professor Inada has pointed out to me, this argument overlooks the fact that there is a further distinction between the situations of the employed and the unemployed in that the former can choose how much work and how much leisure to enjoy, whereas the latter are constrained without choice to do no work. Nevertheless it would seem reasonable to make the assumption that $\lambda(\bar{U})/\lambda(P\bar{C})$ in fact lies somewhere between $\mu_u$ and $\mu_a$.

9) This would be the case if all citizens had the same utility function dependent only on the amount of products consumed with the marginal utility of income a linear function of the amount spent on consumption. But this neglects the qualification noted in the previous footnote.
such divergences are monopolistic or monopsonistic factors and rates of indirect taxation. Subsidisation of a product could make such a divergence negative in sign. It will be observed that these divergences are in respect of the productive operations carried out during day \( q-1 \) but they must be evaluated with the marginal utility of income of day \( q \) which is when the product will be available for use.

So far we have said nothing about the terminal conditions of the government’s control plan, that is to say about the elements \( U_{T1}, U_{T2}, U_{T3}, \) and \( U_{T4} \) in equation 1. This is in fact a particularly difficult matter in the sort of control planning which we are discussing. At first sight a normal procedure would seem to be to say that the government must place some social valuation, however arbitrary, upon the total goods which will be carried over from the last day of the plan to the beginning of the next day and to leave the matter there. In this case we could write the terminal utility elements of equation 1 as

\[
U_{Tq} = U_{Tq}(\bar{X}_q, \bar{Y}_q) \tag{15}
\]

where the point \( \langle q \rangle \) represents point \( \langle 1 \rangle, \langle 2 \rangle, \langle 3 \rangle, \) or \( \langle 4 \rangle \) in our diagram, since \( \bar{X}_q \) and \( \bar{Y}_q \) represent the total gross outputs which will be produced during day \( q \) and left over till the beginning of the next day which is outside the plan period. If we differentiate this expression, using equations 8, 11, and 12, and write

\[
\frac{\partial U_{Tq}}{\partial \bar{X}_q} = \mu_{Tq} P_{xTq}(1+\delta_{xTq})
\]

and similarly for \( \frac{\partial U_{Tq}}{\partial \bar{Y}_q} \) we get

\[
dU_{Tq} = \mu_{Tq} \{1+iPDl\} q + \mu_{Tq} \{1+i \} \{P_x[\delta_{xTq}dX_x+\delta_{xTq}dX_y] + P_y[\delta_{yTq}dY_x+\delta_{yTq}dY_y] + W[\delta_{xTq}dX_+\delta_{yTq}dL_y]\} q + \mu_{Tq} \{P_{xTq}\delta_{xTq}d\bar{X} + P_{yTq}\delta_{yTq}d\bar{Y}\} q \tag{16}
\]

where \( \mu_{Tq} \) represents the policy-makers’ attribution of a marginal utility of income at the beginning of the first day after the end of the plan, where \( P_{xTq} \) and \( P_{yTq} \) represent the econometricians’ estimates of the market prices then ruling, and where \( \delta_{xTq} \) and \( \delta_{yTq} \) (which, as we shall see, must cover a multitude of sins) represent any divergences between the marginal social utilities to be attributed to \( \bar{X}_q \) and \( \bar{Y}_q \) and the marginal utilities of their prices, \( \mu_{Tq} = \mu_{xTq} \) and \( \mu_{Tq} = \mu_{yTq} \).

From equation 1 it can be seen that for the end points of the possible environmental paths during the period of the plan we need to evaluate not simply \( dU_q \) (as is done in equation 14) but \( d(U_q+UT_q) \) which means adding the terms in equation 16 to those in equation 14. Equation 14 is then modified in the following three ways.

First, the ‘optimum savings’ component has added to it the term \( \mu_{Tq}\{1+iPDl\} q \) which means that the element \( \{PDl\} q \) is now multiplied by \( \mu_{Tq}(1+i_\delta)-\mu_\delta \). On the basis of the argument given above in discussing component III this means that the optimum savings criterion can be carried on until the termination of the plan.
Second, the ‘internal divergences’ component has added to it a term giving the internal divergences of the last day of the plan period evaluated at the marginal utility attributed to the income of the day following the end of the plan period.

Thirdly, there is the addition of the new and last item in equation 16, namely any divergences between the social values of the additional goods carried over from the end of the plan period and their market values.

This last item is, of course, vague and arbitrary in the extreme, as it is not known whether these goods will be used for consumption, for governmental purposes, or for investment for future production nor, if they are used for consumption whether they will go to the rich or the poor, nor in what conditions they will be consumed privately or by the government or invested. The period is ex hypothesi unplanned and uninvestigated.

But the trouble in fact goes much deeper than that. Suppose that it were known that the small change in controls the effect of which we are examining would have no effect on the amount of goods carried over from the end of the plan period so that $dX_q=dY_q=0$. It would still, alas, not be true that $d(U_q+UT_q)$ as reckoned from equations 14 and 16 would account for the whole of the story. The change in the controls might have caused a rise in population ($dN_q>0$) which would continue to live into the post-plan period and thus affect future welfare.

An attempt might perhaps be made to include $N_q$ as well as $X_q$ and $Y_q$ in equation 15 and then in one way or another to evaluate $\frac{\partial U_{T_q}}{\partial N_q}$ in terms of its probable future implications. But even that is not the end of the story. Even if $dX_q=dY_q=dN_q=0$, so that the small change in controls which is under study would have no direct effect upon the material or demographic resources of the post-plan period, nevertheless it might affect social welfare after the close of the plan period. A change in a control at one time may have a delayed as well as an immediate effect. Thus a change in income tax on day $t$ (even if the old rate of tax were restored on day $t+1$) may affect peoples expenditures on day $t+1$ since the purchases of one day may be affected by the tax-free disposal income of a previous day. In other words changes in controls during the plan period may affect not only the initial material and demographic resources with which the post-plan period is endowed, but also behaviour during the post-plan period. In a hunch-like manner policy-makers will no doubt try to take such possibilities into account; but I know of no precise way of quantifying them.

Does the type of analysis developed in this paper in fact have any practical significance? Does it point at all to ways in which governments in the sort of mixed economies of which I have been speaking might consider the implications of their policy objectives?

What I have been saying has relevance only to marginal decisions, i.e. only to the question whether small changes in control policies would make things better or worse. But subject to this very important limitation it does perhaps help in four ways.
First the analysis suggests that it is legitimate to consider the effects of a given small change in a control plan separately under the different component heads of equation 14 and to add up the results to obtain the net effect on social welfare. This could be of importance. A great deal of the analysis of the causal effects of different policy changes and of the consequential policy making is bound to remain very rough and ready. Some policy changes will have very significant effects on some of the components of social welfare while their effects on other components are probably relatively small and in any case almost impossible to estimate. The analysis in this paper suggests that it may not be totally illegitimate to treat the effects of such controls in a partial manner and to add up these partial, but important results of the various controls on the components which they are most likely to affect significantly.

Second there is the closely related consideration that this procedure can facilitate the inevitable, and indeed positively desirable, decentralisation of modern government. The problem here is that there are many controls (e.g. the rate of income tax) which affect many different objectives of policy (e.g. the distribution of income, incentives to save, incentives to work, etc.) but the level of which must in the end be determined by the central governmental authority after weighing up the pros and cons of its different effects. Each decentralised department which is concerned with one particular effect of the control can, however, independently report to the central government on the effect of the control on its own particular social objective, and the central government can be left to consider the reports from all the departments and by a process of addition to draw up the final balance sheet of social costs and benefits.

Third, it may help to draw a line between the ‘positive’ and the ‘normative’ elements in policy making. The policy makers must put to their administrators, statisticians, economists, and econometricians all questions of positive causal relationships explaining how the relevant and directly quantifiable variables (population, inputs, outputs, prices, amounts of work done etc.) would be affected by any given control plan along the different environmental paths. It is also a positive factual question to estimate a number of the divergences due to taxation, monopolistic conditions, etc. between the values of effort and rewards and of inputs and their marginal products (see components V and VII of equation 14). But it is for the policy makers themselves to form value judgements on many of the social benefits and costs associated with various activities (see component VI of equation 14), on all the factors which involve interpersonal comparisons of utility, and, in particular, on the distributional weights (the $\mu$’s) to be ascribed in different conditions to marginal changes in the incomes of various groups. As a series of small changes in control plans are considered (moving always from a lower to a higher total social welfare), so, of course, the policy makers must be prepared to revise these distributional weights. If, for example, a series of small policy changes is adopted which has the effect of raising progressively the income per head of one group and reducing that of another, so the policy makers at each stage may want to reduce the $\mu$ attached to the former, and to raise the $\mu$ attached to the latter group.
Fourth, the method of considering only small marginal changes in the controls enables a large number of the components in the social welfare function to be judged solely in terms of *marginal*, and not of *total*, interpersonal comparisons, i.e. in terms of the $\mu$'s and not of the $\hat{U}$'s (see components III, IV, V, VI, and VII of equation 14). Questions of the kind "How many dollars would you have to give to a man in situation I in order to bring about the same increase in social welfare as giving one dollar to a man in situation II?" would, I suspect, be much more meaningful to a policy-maker than questions of the kind "How much would total welfare be increased by moving a man from situation II to situation I?"

Unfortunately, as we have already seen in discussing components I and II of equation 14 the latter type of question cannot always be avoided even in the case of small marginal changes in the controls. One more citizen in the population (component I) is a small marginal change in the population, but it marks a structural change for the individual who comes out of the nowhere into here. Similarly one more citizen in employment is a small marginal change in the total labour force but marks a structural change in the standard of living of the man himself. One cannot avoid measuring the total difference between the 'money value of the happiness of a representative citizen' \(\left(\frac{U_p}{\mu N}\right)\) and the 'money value of the happiness of an unemployed man' \(\left(\frac{\hat{U}_u}{\mu}\right)\), if one is adding up the contribution to social welfare in dollars.

There is one further case of great importance which necessitates comparisons of total rather than of marginal utilities and which has so far been neglected in this paper. A change in a control may affect the probability with which a government expects one of the environmental paths. Thus some governmental expenditure on research and development may increase the probability with which a particular technological breakthrough is expected. In terms of our simple example suppose that a given change would increase the probability of tomorrow being Wet by \(dE_{12}\). Then remembering that \(E_{34}=1-E_{12}\), the effect on \(E(U)\) in equation 1 would be

\[dE_{12}\{U_{12} - U_{24} + E_1(U_1 + U_{12}) + E_2(U_2 + U_{24}) - E_4(U_4 + U_{44}) - E_4(U_4 + U_{44})\}.

A glance at equations 2 and 15 is sufficient to remind one that this would involve in an extreme form the comparison of the total utilities of citizens on one environmental path with their total utilities if they could be shifted on to another environmental path. The difference between one environmental path and another may well involve structural, rather than merely marginal, changes for the individuals concerned.

There is, moreover, one great remaining problem which I have not even mentioned in this paper. Consideration of equations 1 and 14 suggests that the proper procedure would be to estimate (equation 14) the effect of a change in a control plan on the social welfare at every point \(<q>\) on every conceivable environmental path and then (equation 1) to weight this result with the probability (which the policy makers must

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10) I owe this point to Professor Inada.
estimate in consultation with their relevant advisers) of passing through that point. This procedure in its purity is, however, obviously impracticable simply because of the immense number of possible environmental paths which the economy might take. At most one or two representative paths could be considered. This means that there will remain a vast element of 'residual uncertainty' in any control plan. The authorities must devise a plan, and have a set of flexible policy weapons, specifically designed to deal with those unexpected events, about which no precise calculations have been made. This raises far-reaching issues which I cannot discuss in this paper; but it does mean that it is not desirable to design rigid plans which are constructed solely in the light of the components of economic welfare enumerated in equation 14. General weapons for stabilisation against unexpected shocks and general flexibility to change the course of all policies in the face of unexpected events are additional desiderata.

It is in this connection, namely in dealing with all those myriad possible eventualities for which it is impossible to make precise probability calculations, that in the governmental sphere special stabilisation policies to deal quickly and promptly with general inflations and deflations of demand are most relevant and in the private sector the general need for liquidity becomes important. The cost of being liquid (i.e. of holding money) is obviously intimately connected with the prospect of price inflation. Price stability is, therefore, specially closely connected with methods of coping with 'residual uncertainty'. This is the basic reason why price stability as such did not appear directly in the components of equation 14 which are designed to deal with what we may call 'calculated uncertainty' and are not well designed to deal with 'residual uncertainty'. The whole problem of the undesirability of price inflation in a closed economy has thus, alas, fallen outside the scope of this paper, though I do not wish thereby to imply that it is unimportant.

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