INVOLUNTARY UNEMPLOYMENT AND MARKET IMPERFECTION

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Introduction

There are three major scandals, as Arrow called them, of recognized problems that the neo-classical general equilibrium theory has not succeeded in dealing with effectively, i.e., the failures to provide the integration of microeconomics and macroeconomics, to incorporate imperfect competitions properly and to take account of the cost of making transactions.1) These three failures are, in our opinion, not independent but mutually related. Our main concern is microeconomic foundations of macroeconomics. Micro and macro remain two distinct theories and the propositions in the latter are not derivable from those of the former. Keynesian macroeconomics particularly aims to explain "what determines the actual employment of the available resources" in underemployment equilibrium2) while the neo-classical microeconomics presents a beautiful picture of full employment equilibrium with fully flexible prices. Therefore, a microeconomic foundation of macroeconomics must be the explanation of unemployment. Of course, a neo-classical model modified by some elements of wage rigidity will yield the Keynesian-type equilibrium. But the question of the first importance is why the wage remain rigid or sticky in the face of unemployment. If the unemployment to be explained is of the frictional type, explanation may be partly based on the solution of what Arrow called the third failure, i.e., the consideration of the cost of making transactions.3) Involuntary unemployment, on the other hand, seems to be explained by the solution of the second failure, i.e., the consideration of the imperfection of the competitive market.

Even in the so-called perfectly competitive market, individual demand and supply curves perceived by each seller and buyer cannot be perfectly elastic unless total demand and supply are in equilibrium.4) When total demand exceeds supply, buyers

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2) Keynes [8], p. 4.
3) See Phelps [18], particularly p. 125, and Phelps [19], particularly p. 4, which seem to be mainly concerned with the left part of Phillips curve.
4) See Arrow [1]. We differ, however, from Arrow in several respects. Firstly, Arrow considers that supply (demand) curves perceived by buyers' (sellers') are also not perfectly elastic when total supply exceeds (falls short of) demand, which we do not for the sake of simplicity. Secondly, Arrow does not mention, at least explicitly, the possibility of kinks in demand and supply curves. Thirdly, Arrow does not recognize, at least explicitly, the possibility of a disequilibrium of perfect competi-
cannot perceive perfectly elastic supply curves beyond the quantity currently supplied since no one can buy larger quantity at the going price in the market. Similarly, perceived demand curves of individual sellers' cannot be infinitely elastic beyond the quantity currently demanded when total supply exceeds demand and no one can sell larger amount at the price currently prevailing in the market. A disequilibrium in a perfectly competitive market implies the inconsistency among individual plans of buyers and sellers made under the assumption of the infinite elasticity of demand and supply curves. Price must be changed under such an assumption. Consistency among individual plans may be recovered, however, if the assumption of the perfect elasticity is modified. Then, there will be no price changes. A situation viewed as disequilibrium from the view point of perfect competition may be considered as an equilibrium from the point of view of imperfect competition. It must be emphasized, however, that this imperfection is not due to any element of monopoly, even in the sense of Chamberlin's monopolistic competition. An individual perceives an inelastic demand or a supply curve, not because of his individual power to change market prices, but because he has to observe and take into consideration, in the barometric sense, the relation between the level of price and the total demand or supply.5)

Involuntary unemployment is simply defined as the situation in which there are men unemployed who would be willing to work at the existing or slightly lower real wage.6) If demand curves are assumed not to be perfectly elastic, however, it may be converted to an equilibrium situation and wage remains rigid. The riddle of how to reconcile competition and unemployment disappeared.7) Wage rigidity is not needed to be imposed exogenously, but may be explained endogenously in disequilibrium situation of a perfectly competitive market. The first aim of this article is to explore the possibility of an explanation of underemployment equilibrium in this line.

As is described by Phillips curves on British data, the relation between unemployment and the rate of change of wage rate is non-linear in the sense that wage rate falls only very slowly in the presence of involuntary unemployment.8) The average level of unemployment, for example, from 1925 to 1929 was 10.94% while the average rate of the change in wage rate merely −0.60% per year, which is in close agreement with the results obtained from the curve fitted to the 1861-1913 data of levels of unemployment and rates of change of wage. This may seem to imply the validity

5) The word "barometric" here implies the same implication as in the definition of the barometric price leadership. See Stigler [23].
6) Keynes [8], p. 15.
7) Haberler [7], pp. 167-176. See also Leijonhufvud [10], p. 53.
of underemployment disequilibrium theory rather than our underemployment equilibrium theory. However, what is more important is the relation between the rate of change of wage rate and the rate of change of unemployment, which is clearly seen in the presence of involuntary unemployment. Reduction of wage occurs only when unemployment is increasing. On the other hand, in the thirties with high but diminishing level of unemployment, we find wage rigid or even increasing slightly. According to the underemployment disequilibrium theory, wage must always fall, however slowly, in the presence of involuntary unemployment, whether the level of unemployment is increasing or diminishing. Phillips' data shows, however, that the wage is almost rigid when the high level of unemployment is decreasing while the rate of wage falls when the level of unemployment is increasing. A micro-economic foundation must be given to these empirical findings, which is our second aim in this paper. This can be done by considering the changes of the price of wage goods caused by the changes in the level of business activity.

Model I of Unemployment

Consider a very simple model of the household of a worker which supplies homogeneous labor service so as to demand wage goods. In Figure 1, demand for the wage goods is measured horizontally, while the amount of leisure (labor service) is measured vertically. It is assumed that initially available amount of labor service is AO. Real curves passing A, B and C are indifference curves between leisure and the wage goods. If the real wage is given by the reciprocal of the absolute value of the slope of AF, full employment equilibrium in perfectly competitive labor market is at B, where labor supplied is AD and the leisure is OD. The second postulate of classical employment theory is observed at B. Since the desired amount of labor is supplied, the assumption of perfect competition that any amount of labor can be supplied at the going rate of wage is consistent at B and demand curve for labor perceived by this household can be perfectly elastic. However, if there is deficiency of demand for labor and only AE amount of labor service can be supplied, the assumption of perfect competition cannot be maintained and the demand curve perceived by the household cannot be perfectly elastic beyond AE since larger amount of labor supply is desired but impossible at the given rate of wage. Then, the budget constraint is no longer the line AF but is a curve beyond AC, indicating that the reduction of wage rate must be expected if the amount of labor larger than AE is to be supplied. If such a curve happens to be the dotted one starting from C, it is an underemployment equilibrium of the household, with involuntary unemployment ED. This curve need not be tangent to indifference curve at C. Its slope may be


10) Lipsey [11]'s explanation of the effect of the rate of change of unemployment is based on the aggregation bias and can be applied only if the level of unemployment, at least in some sectors, is low.
The wage is determined by the negotiation between individual employer and employee, both sides being fully aware of the existence of competitors. At the underemployment equilibrium, workers are reluctant to offer their services currently supplied at less than the prevailing rate of wage even if they wish to offer larger amount of their services at the prevailing or slightly lower rate of wage. The second postulate of the classical economics is violated at C.12)

Analytically, our model I can be described as follows. When $l^*$ amount of labor is supplied at the rate of wage $w^*$, inverse demand function for labor perceived by the household is

$$w = f(l, w^*, l^*)$$  \hspace{1cm} (1)

where $w$ is the rate of wage expected if $l$ amount of labor is planned to be supplied and

$$w^* = f(l^*, w^*, p^*).$$ \hspace{1cm} (2)\textsuperscript{13}

At the level of $l$ smaller than $l^*$ and at the full employment equilibrium, $w$ is independent of $l$ and equal to $w^*$, i.e., wage elasticity of demand for labor is infinite. The condition for the underemployment equilibrium is obtained by maximizing the utility $U(x, 1-l)$ with respect to $x$ and $l$, being subject to budget constraint

$$px = wl$$  \hspace{1cm} (3)

and perceived demand function for labor service (1), and by imposing the condition of the realization of supply plan of labor

$$l = l^*,$$  \hspace{1cm} (4)

where $x$, $p$ and $l$ signify respectively the amount of the wage goods, the price of the wage goods and the $OA$ amount of labor services. If the inverse of the wage elas-

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\textsuperscript{11) The implications of dual decision hypothesis (Clower [6] and Morishima [14], pp. 24-26) is that this curve is a vertical line. However, Clower does not seem to consider that C is an equilibrium with unchanged wage rate.\textsuperscript{12} Keynes [8], pp. 15 and 128.\textsuperscript{13} For the perceived (subjective) demand curve, see Arrow-Hahn [3], pp. 151-168, Bushaw-Clower [5], p. 181, Negishi [15], and [16] pp. 103-115.
ticity of demand is denoted by $e$, i.e.,

$$e = -\frac{l^*}{w^*} f_i.$$  

(5)

for $l \geq l^*$, then the condition is

$$U_i/U_2 \leq p/w^*(1-e), \quad U_i/U_2 > p/w^*$$  

(6)

where $U_i$ and $f_i$ signify the partial derivative of $U$ and $f$ with respect to the $i$-th variable. The equality in the first part implies the tangency of the dotted curve and an indifference curve at $C$ in Figure 1.

The trouble with model I is that no simple relation among the rate of wage, the price of wage goods and unemployment seems to be derived. More fundamental difficulty is, however, that unemployment generally does not imply the shorter working time for each worker. It means the situation where some workers are completely unemployed while the rest of workers are fully employed.

**Model II of Unemployment**

Suppose all the households of workers are identical. If part-time employment is assumed not to be allowed institutionally, then, we must assume that those who are fully employed and those who are completely unemployed are chosen randomly, with the probability of employment for each worker being equal to the ratio of the number of employed workers to the total number of workers.

Let $k$ signify the probability or ratio of employment, i.e., $0 \leq k \leq 1$. Supposing that the total number of workers is very large, we assume $k$ is a real number. When $k^*$ per cent of workers are employed at the wage rate $w^*$, inverse demand function for labor

$$w = f(k, w^*, k^*)$$  

(7)

such that

$$w^* = f(k^*, w^*, k^*)$$  

(8)

is perceived by workers. As in the case of model I, $w = w^*$ irrespective of $k$, if $k \leq k^*$. When the rate of wage and the price of wage goods are $w$ and $p$, the utility of workers, if employed, is indirectly indicated by $U(w, p)$, which corresponds to the indifference curve passing $B$ in Figure 1. On the other hand, the utility of being unemployed is denoted by $\bar{U}$, which corresponds to the indifference curve passing $A$ in Figure 1.

The condition for underemployment equilibrium is obtained by the maximization of the expected utility

$$kU(w, p) + (1-k)\bar{U}$$  

(9)

with respect to $k$, being subject to given $p$ and the perceived demand function (7), when the condition of realization of expectation of employment ratio,

$$k = k^*$$  

(10)

is satisfied.  

14) Alternatively we may suppose that the length of working hours is institutionally fixed.
15) In fact workers choose $k$ indirectly be choosing $w$ (i.e., accept the wage reduction or not).
for $k \geq k^*$, the condition is

$$U(w, p) - U - ew(\partial U(w, p)/\partial w) \leq 0$$

(12)

at $(w^*, k^*)$, since $U(p, w) \geq U$. If the equality holds in (12), the second order condition

$$\left(\frac{\partial U}{\partial w}\right)(\frac{\partial w}{\partial k}) - \frac{\partial e}{\partial k} - ew\left(\frac{\partial^2 U}{\partial w^2}\right)(\frac{\partial w}{\partial k}) - w(\partial U/\partial w)(\partial e/\partial k) < 0$$

(13)

is assumed to be satisfied at $(w^*, k^*)$.

Let us assume that wage elasticity of demand for labor is perceived constant, i.e., both $\frac{\partial e}{\partial k} = 0$ and $\frac{\partial e}{\partial k^*} = 0$ are assumed. Then, conditions (12) and (13) on $w^*(=w)$ are independent of $k^* (=k)$. In Figure 2, employment ratio $k$ is measured horizontally, rate of wage $w$, vertically. $D_1$ and $D_2$ are perceived demand curves for labor, both having equal and constant elasticity for downward sloping portion. With rising business activity, involuntary unemployment equilibrium shifts from $A$ to $B$, with $k^*$ increased but $w^*$ constant. Furthermore, from (13),

$$\left(\frac{\partial U}{\partial w}\right) - \frac{\partial e}{\partial k} - ew\left(\frac{\partial^2 U}{\partial w^2}\right) > 0$$

(14)

is derived, since $\frac{\partial w}{\partial k} < 0$.

If the household of workers have no monetary assets as in the case of Figure 1, the left hand side of (12) is homogeneous of degree zero with respect to $w$ and $p$. Then, in view of (14) and the Euler’s theorem, we have

$$\left(\frac{\partial U}{\partial p}\right) - ew\left(\frac{\partial^2 U}{\partial p \partial w}\right) < 0.$$  

(15)

Even if (12) holds in inequality form, workers are reluctant to accept the reduction of $w^*$. However, starting from such a situation, changes in $p$ do not reflect to the changes in $w^*$ as far as (12) remains satisfied. In view of (15), reduction of real wage caused by the rise of price is always accepted by workers. If demand for labor is increased by such reduction of real wage, employment will be increased. The unemployment explained by our model II is, therefore, involuntary in the sense of Keynes.16 If, on the other hand, (12) holds with equality, changes in $p$, which violate (12) with constant $w^*$, must change $w^*$. In view of (14) and (15), reduction of $p$ must be accompanied by the fall of $w^*$.

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Figure 2
Unemployment and the Rate of Wage

The rate of change of wage rate, as was just seen, depends not on the level of unemployment but on the rate of change of $p$, i.e., the price of wage goods. Now we may explain the Phillips' finding on the relation between rate of change of wage rate and rate of change of unemployment by considering the relation between the rate of change of $p$ and the rate of the change in unemployment or in business activities in general.

Now consider a firm producing wage goods. The elasticity of the demand curve for output perceived by this firm is assumed not to be infinite, either because of the commodity differentiation or because of the deficiency of demand in the perfectly competitive market. In the latter case, the shape of the demand curve is similar to that of $D_1$ in Figure 2, with $w$ and $k$ replaced by $p$ and $y$, the price and amount of the output. In the case of differentiation, the price elasticity in the left of point $A$ is assumed to be not infinite, though still larger than in the right of $A$.\(^\text{17}\) As in the case of model II, these elasticities are assumed to be constant and the demand curve is assumed to be shifted by the change of current demand $y^*$ with elasticities unchanged.

Analytically, the model of the firm is as follows. When $y^*$ amount of output is being sold at the price of $p^*$, the firm perceives demand function for its output as

\[ p = f(y, \; p^*, \; y^*) \tag{16} \]

such that

\[ p^* = f(y^*, \; p^*, \; y^*). \tag{17} \]

The condition for equilibrium of firm and market for wage goods is derived from maximization of profit

\[ py - g(y, \; w) \]

with respect to $y$, being subject to the given wage rate $w$ and perceived demand function (16), and from the condition of realization of the supply plan of the output

\[ y = y^*, \tag{18} \]

where $g(y, \; w)$ signifies the total cost function, such that $\partial g/\partial y > 0$ and $\partial g/\partial w > 0$. The inverse of demand elasticities are defined as

\[ e^+= -\frac{y^*}{p^*} f_i^* \tag{19} \]

\[ e^- = -\frac{y^*}{p^*} f_i^- \tag{20} \]

where $f_i^*$ and $f_i^-$ signify respectively the right hand side and left hand side derivative of $f$ with respect to $y$ at $(p^*, \; y^*)$. Our assumption implies that $0 \leq e^- < e^+$. In terms of (19) and (20), the condition is reduced to

\(^{16}\) Keynes [8], pp. 14–16, and Leijonhufvud [10], p. 93. On the other hand, unemployment considered by Lucas-Rapping [12], [13] is voluntary, since unemployment is assumed there to be a decreasing function of real wage, even though real wage is assumed to be an increasing function of effective demand. See also Rees [21].

\(^{17}\) See Sweezy [24].
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\[ \dot{p}^* (1 - \epsilon^*) - \frac{\partial g}{\partial y} \leq 0 \]  
\[ \dot{p}^* (1 - \epsilon^*) - \frac{\partial g}{\partial y} \geq 0 \]

at \((\dot{p}^*, \dot{y}^*)\).

When both of (21) and (22) are satisfied with inequality, \(\dot{p}^* (=\dot{p})\) is independent of \(\dot{y}^* (=\dot{y})\). The price of wage goods and therefore the rate of wage remain unchanged when the level of business activity is changed slightly.\(^{10}\) However, large reduction (increase) in \(\dot{y}^*\) brings about the equality in (21) ((22)), if marginal cost is assumed to be increasing, i.e., \(\frac{\partial g}{\partial y} > 0\) and \(\epsilon^*\), less than 1. Further decrease (increase) in \(\dot{y}^*\) must be accompanied by the decrease (increase) in \(\dot{p}^*\). When the level of business activity falls, i.e., \(\dot{y}^*\) and \(\dot{k}^*\) are decreased, eventually \(\dot{p}^*\) must fall from (21). Then, \(\dot{w}^*\) falls from (12), which keeps the equality form in (21) and reduces \(\dot{p}^*\) further. Formally, this can be seen by differentiating the equality form of (12) and (21) with respect to \(\dot{y}^*\) and solving for \(\dot{w}^*/\dot{y}^*\). In view of (14) and (15), we have for \(\dot{y}^* < 0\),

\[ \frac{\dot{w}^*}{\dot{y}^*} > 0 \]

by the use of Euler's theorem, since the left hand side of (12) is homogeneous of degree zero with respect to \(\dot{p}\) and \(\dot{w}\) while the left hand side of (21) is homogeneous of degree zero with respect to \(\dot{p}\), \(\dot{w}\) and the price of other inputs, i.e., non-wage goods, which is a parameter in \(g\).

The rate of wage is reduced via the reduction of the price of wage goods when unemployment is increasing, though it does not fall merely because there exists involuntary unemployment.

**Concluding Remarks**

We have shown firstly that an equilibrium with involuntary unemployment is possible without assuming wage rigidity from the beginning. In other words, in such an equilibrium the rate of wage, even though it is flexible in general, is not reduced in the presence of unemployment. Secondly, we explained empirical findings made by Phillips that the rate of wage decreases when unemployment is increasing. Followings are some additional fragmentary remarks.

For the sake of simplicity, we did not consider the possibility that the elasticity of supply curves of labor perceived by individual employers is not infinite for the quantity less than the current employment when there exists excess supply in the market. If we replace labor demand inelasticity by such labor supply inelasticity, the resultant unemployment, if any, should be called voluntary. Furthermore, equilibrium of an employer is impossible unless the perceived demand curve for output has a kink. On the other hand, the introduction of such considerations, in addition to labor demand inelasticity, greatly complicates the model, but no essentially different results seem to be derived.

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18) Although their models are disequilibrium ones, the case of Patinkin [17], pp. 313-328, Barro-Grossman [4], and Solow-Stiglitz [22] corresponds to the assumption that \(\epsilon^* = \infty\).

19) This suggests the possibility of Fixprice method, the method of post-Keynesian macroeconomics. See Hicks [9], pp. 76-83.
The classical one to one relationship between the unemployment and the real wage rate is removed from our model. With the increase of employment, the real wage rate is eventually decreased since the price of wage goods increases while money wage rate is kept unchanged. This accords with Keynes who had adhered to the classical doctrine of countercyclical variation in real wage rate. When unemployment is increased, however, the money wage rate is eventually reduced by the reduction of the price of wage goods but real wage is kept unchanged in view of the homogeneity of the left hand side of (12).

Arrow is critical to Samuelson's neoclassical synthesis that neoclassical theory is valid when full employment is achieved by the Keynesian intervention. The reason is that the results of comparative statics in the system with price rigidity are different from those in the neoclassical system with flexible prices. Only if it is assumed that the price rigidity will soften in the beneficient sun of prolonged full employment, Arrow concludes, one can derive neoclassical results in the long run. Neoclassical synthesis can, however, work at least partly in our system, since the rigidity or imperfection, which is the cause of unemployment, disappears when full employment is achieved by the Keynesian intervention. For example, we may consider the partial equilibrium comparative statics of a perfectly competitive system composed by the household of workers and wage goods industry to determine real wage and output of wage goods, when full employment is assured by Keynesian policies.

To some the problem of the price rigidity might seem to be obsolete in the day of inflation. However, one must remember that an important cause of current inflation is the downward price rigidity in certain sectors where the demand is diminishing or the productivity is rising. The arguments in this paper suggest that such sectors need not be monopolistic.

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REFERENCES


20) Arrow [2].
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