DYNAMIC ADJUSTMENTS UNDER BOND FINANCE: AN EXTENSION OF THE BLINDER AND SOLOW MODEL*

By TOSHIHIRO IHORI

1. Introduction and Summary

In their well-known analysis of fiscal stabilization policies [1], Blinder and Solow clarify the meaning of the government budget constraint. However, the transitional, dynamic process by which government expenditures are transmitted is not explicitly considered. The purpose of this paper is to explore implications of the adjustment process of income in the analysis of a bond-financed increase in government expenditures.

In the IS–LM framework, income is instantaneously determined at the intersection of the IS and the LM curves, while stock variables such as bond balances, money balances, and physical capital are assumed to be constant. As the result of public expenditures, income increases instantaneously. If the impact government spending multiplier, $1/(1 - \text{marginal propensity to spend})$ is sufficiently large, the instantaneous increase in tax revenues will create the government budget surplus at the initial moment. The government budget constraint implies that the surplus is turned to the redemption of bonds. Since we are concerned with the effect of bond-financed budget deficits on income, the case in which government spending leads to a budget surplus at the initial moment is neither interesting nor relevant. Blinder and Solow assume away this troublesome case by omitting income from the investment function in order to lower the marginal propensity to spend. We shall present a more generalized framework and consider the case where induced investment is allowed.

We shall explicitly introduce an adjustment process in the commodity market, so that fiscal expenditures increase tax revenues only with delay. In this way we can avoid the complications noted above, and at the same time explicitly consider the time development of the system. Contrary to the conclusion by Blinder and Solow that the additional changes in bonds have a positive net effect on income if the system is stable, we shall show in our generalized framework that the subsequent effects may be contractionary even if the system is stable. Thus, Blinder and Solow's conclusion holds on narrower grounds than they seem to claim; that is, only when the parameter of policy is government purchases of goods and services

* The author wishes to thank K. Asako, K. Hamada and M. Nakamura and the referees of this paper for their valuable suggestions and comments. He is also very grateful to A. Blinder and J. Niehans for their useful comments and suggestions on earlier drafts.
and the marginal propensity to spend is sufficiently weak.

As regards the steady-state multiplier considered, it will be shown that the positive wealth effect on consumption is a necessary condition for the effectiveness of fiscal policy. The question as to whether or not the wealth effect is positive is not at all out of place here. However, the long-standing Keynesian-monetarist controversy concerning whether or not the impact effect of the increase in bonds on effective demand, $E_B$, is positive has no bearing on the analysis of the steady-state multiplier, because the sign of $E_B$ is not relevant to the long-run effectiveness of fiscal policy. In Model II (the parameter of policy is government expenditure including interest payments), even if $E_B$ is negative, it is still possible for the system to be stable and for fiscal policy to be effective.

As regards the dynamic adjustments under bond finance considered, we shall show in our consideration of the adjustment process that fiscal policy is normally effective during the initial phase where the government budget remains at a deficit, as long as the impact effect of the increase in bonds is positive. Fiscal policy in depression mostly corresponds to the situation where the government budget is in deficits. Therefore, in most of the relevant cases to be encountered, fiscal policy can be deemed to be effective in spite of some less favorable implications to be found in the paper.

2. Analytical Framework

Before going into the main arguments, we shall reproduce the government budget constraint. Any expenditures not financed out of current receipts (tax revenues) must be financed by issuing money and/or bonds.

\[ M + \frac{B}{r} = G + B - T, \]

where $M$ represents money balances, $B$ the number of bonds, $r$ the rate of interest, $G$ government purchases of goods and services, and $T$ tax revenues. A dot denotes the derivative with respect to time. For simplicity, it is assumed that each bond is a perpetuity paying $1$ per year and that the level of price is exogenously given.

Since the effect of changes in bonds continues to work as long as the government budget is not balanced, we shall define "equilibrium" to be a situation with a balanced budget.\footnote{This definition seems appropriate to a static model of the type being considered here. But for a growing economy, it would be more plausible to define fiscal equilibrium as a situation where real stock variables were growing at the same rate of product.} The efficacy of fiscal policy crucially depends on how the government budget is balanced. Blinder and Solow assume that tax revenues depend on disposable income and hence interest payments by the government on bonds are explicitly introduced as a separate express item,

\[ M + \frac{\dot{B}}{r} = G + B - T (Y + B) \]
where the marginal tax rate $T'$ is taken to lie between 0 and 1. Combined with the IS–LM apparatus or the adjustment equation of income, eq. (2) will determine the equilibrium values of $Y$, and $B$ (or $M$), for a given level of $G$. If $G$ is reduced to its initial level again, the equilibrium levels of $Y$ and $B$ (or $M$) will not be changed. The appropriate fiscal policy in this case is to maintain $G$ at its new level indefinitely. Even if $B$ is initially increased under bond financing, $B$ in the new steady state is not necessarily higher than it was at the initial level.\(^2\)

As for the fiscal policy options, we shall also consider the case where the policy variable is $G' = G + B$, government expenditure including interest payments in Model II. Blinder and Solow deal exclusively with our Model I (the parameter of policy is $G$). Model II seems to be better suited for examining the long-run consequences of a one-step increase of government expenditures because it is concerned with the size of government total expenditure. This procedure assumes that if bond interest increases, purchases are curtailed correspondingly. When fiscal authorities are anxious about the burden of bonds or heavy interest payments, Model II would be a relevant approach. On the other hand, Model I might be regarded not as a constant fiscal policy but as an ever more expansionary policy. If a Keynesian fiscal policy were to be pursued, it is this procedure that would be adopted.

We shall develop a more generalized form of the IS–LM framework, in order to avoid the shortcoming we have noted above in the Blinder and Solow model. We restrict our attention to the short run in the sense that capital stock is constant, and ignore the role of capital as an asset which must be absorbed into private portfolios.\(^3\) The relevant wealth variable is net financial asset as defined by

\[
(3) \quad A = M + \frac{B}{r}
\]

where $A$ is the value of net private wealth.

First, we assume that the asset markets are always in stock equilibrium:

\[
(4) \quad M = L(r, Y, A),
\]

where $L_r < 0$, $L_Y > 0$, and $1 > L_A > 0$. Here the symbol $L$ with a subscript means the partial derivative of $L$ with respect to that subscript. From eqs. (3) and (4), the equilibrium rate of interest is determined instantaneously as functions of $Y$, $B$, and $M$:

\[
(5) \quad r = r(Y, B, M),
\]

where $r_r > 0$, $r_B > 0$, and $r_M < 0$.

We now consider flow disequilibrium aspects, and apply the Keynesian adjustment assumption to the commodity market. Let $E$ be aggregate effective demand, which can be different in short-run disequilibrium from aggregate output, $Y$. $E$ is the sum of consumption, $C$, investment, $I$, and government purchases, $G$.

\(^2\) Blinder and Solow [1] were unclear about this point. See also Blinder and Solow [2], Infante and Stein [5], and Tobin and Buiter [8].

\(^3\) The stock of real capital can hardly change by more than a few percent a year, but the stock of money and bonds is capable of faster variation. We adhere to this convention, solely to facilitate understanding.

-155-
\( E = C(Y, A) + I(Y, r) + G, \)

where
\[
\hat{Y} = Y + B - T, \\
1 > C > 0, \quad C_A > 0, \quad I_T > 0, \quad \text{and} \quad I < 0.
\]

Substituting eq. (5) into eq. (6) and considering the government budget constraint, we can derive
\( E = E(Y, B, M; G, G'), \)

where
\( Z = 0 \) (in Model I) or 1 (in Model II)
\( E_G = E_G' = 1. \)

The signs of \( E_T \) and \( E_B \) cannot be assessed on a priori reasoning in general. But, the marginal propensity to spend, \( E_T \), is taken to lie between 0 and 1 on usual Keynesian grounds. Substantial controversy exists as to whether \( E_B \) is positive or not. Friedman [4] stresses the possibility that \( E_B < 0 \). Blinder and Solow seem to devote a great deal of their efforts to arguing \( E_B > 0 \). As a larger outstanding bond will require greater interest payments and increase disposable income, on this point the effect of \( B \) on aggregate demand is the same as the effect of \( Y \). Thus, we shall take the view that \( E_B > 0 \) in Model I. In Model II if interest payments increase, government purchases are curtailed correspondingly. As is shown by eq. (9), \( E_B \) in Model II is less than \( E_B \) in Model I by unity. Thus, we shall consider the possibility that \( E_B < 0 \) in Model II.

We now formulate the adjustment process of income:
\( Y = \beta(E - Y), \)

where \( \beta \) is the positive constant adjustment coefficient. Eq. (12) states that product, \( Y \), moves in response to the discrepancy between \( E \) and \( Y \). This is usually regarded as the basic criterion for a Keynesian approach.4) In the state of unemployment, firms would intend to produce exactly the quantity \( E \), the actual effective demand. If firms gradually perceive the level of \( E \) with some delay, then we can derive eq. (12).5)

3. The Dynamics of Fiscal Policy

4) Eq. (12) corresponds to the Walras-Keynes-Phillips model in Tobin [7]. A similar formulation concerning the dynamic properties of fiscal policy is found in Niehans [6].

5) In disequilibrium, discrepancies of \( Y \) and \( E \) are absorbed in unintended inventory investment or forced saving. The general analysis of the behavior of each economic unit under disequilibrium conditions is, however, not the main purpose of our paper.
The earlier literature evaluates the efficacy of fiscal policy only in the steady state, where the economy fully adjusts to the initial change in government expenditures and the associated financial changes. Since we have introduced the adjustment process of income, we can explicitly consider the time path of the system during fiscal policy. First, we shall examine Model I.

**Model I**

Let us now study the trajectories of our system in the \((B, Y)\)-plane. We have from eqs. (2) and (12)

\[
\begin{align*}
\frac{dY}{dB}_{B=0} &= \frac{T'}{1-T'}, \\
\frac{dB}{dY}_{Y=0} &= \frac{1-E_Y}{E_0}.
\end{align*}
\]

\[\text{Figure 1}\]

The \(B=0\) curve in Figure 1 represents the locus of points where the government budget is balanced, and the \(Y=0\) curve represents combinations of \(Y\) and \(B\) which correspond to zero excess demand for commodities. From eq. (13) the \(B=0\) curve is upward sloping, and from eq. (14) the \(Y=0\) curve is also upward sloping. At any point to the right (left) of the \(Y=0\) curve \(Y\) is decreasing (increasing), and at any point above (below) the \(B=0\) curve \(B\) is increasing (decreasing). The necessary and sufficient condition for local stability of the system is

\[
1 - E_Y < \frac{T'}{1-T'} E_B \quad \text{and} \quad r(1-T') - \beta(1-E_Y) < 0.
\]

Nothing in pure theory guarantees that condition (15) will hold, but a plausible argument can be made for its validity.\(^6\)

What happens to the government bonds and national income when an equilibrium is disturbed by an exogenous expansion of \(G\)? The \(Y=0\) curve will shift to the right and the \(B=0\) curve will shift to the right as well. At the initial equilibrium point \(E_0\), both \(Y\) and \(B\) increase. The government budget is balanced at point \(E_1\) before the commodity market is balanced, because income must be increasing in order to finance the increase in interest payments out of the increase in tax revenues. Since \(Y\) is further increasing at point \(E_1\), from eq. (2) \(B\) begins to be redeemed. The reduction of \(B\) suppresses effective demand so that eventually \(E=Y\) is

\[\text{Figure 1}\]

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\(^6\) Condition (15) is reduced to the stability condition in Blinder and Solow when \(\beta \to \infty\).
realized. However, due to the surplus of the government budget, $B$ decreases further and likewise $Y$. By the reduction in $Y$ and hence $T$, the government budget will be again balanced. But, since it corresponds to excess supply in the commodity market, $Y$ is bound to decrease yet resulting in a budget deficit. Then, the accompanying increase in $B$ will stimulate effective demand, and both $Y$ and $B$ will be increasing. Thus, the movement to the new equilibrium exhibits cycles. This is because deficits disappear in the state of excess demand, while surpluses disappear in the state of excess supply in the commodity market.7)

In Figure 1, we shall call the path from point $E_0$ to point $E_1$ the first phase and the path from point $E_1$ to point $E_2$ the second phase. In the first phase, since $G$ is increased and maintained at its new level indefinitely, the resulting deficit is financed in each subsequent period as well as the initial period. This can be regarded as the situation where the Keynesian fiscal policy is pursued to take the economy off the depression. In the second phase, the resulting surplus of the government budget is turned to the redemption of bonds. This can be regarded as the situation where the bond-redemption policy is pursued to reduce heavy interest payments. Figure 1 shows that crowding-out is in fact a phenomenon appearing in the second phase. From the point of view of policy implications, only the first phase is relevant. Since it is reasonable to interpret the first phase as the result of the Keynesian fiscal policy, we find that the Keynesian fiscal policy is effective.

To make our results easier to compare and contrast with the Blinder and Solow model, we next evaluate the efficacy of fiscal policy in the steady state. From the comparative static analysis, we have

$$\frac{dY}{dG} = \frac{1}{\Delta}(E_q + T' - 1),$$

$$\frac{dB}{dG} = \frac{1}{\Delta}(1 - T' - E_Y),$$

where $\Delta = T' - E_q - (1 - T')(1 - E_Y)$.

Inequality (15) means $\Delta > 0$, but the signs of $(E_q + T' - 1)$ and $(1 - T' - E_Y)$ are indeterminate. Since $1 - T' > C > 0$, from eq. (9) $E_q > 1 - T'$ requires the positive wealth effect on consumption. We know that when $1 - T' > E_Y$ ($\frac{dB}{dG} > 0$), inequality (15) means $E_q > 1 - T'$ ($\frac{dY}{dG} > 0$). However, when $1 - T' < E_Y$, the sign of $\frac{dY}{dG}$ cannot be assessed on a stable condition. The expansionary effect of government purchases of goods and services may be offset by the subsequent redemption of bonds.

We now consider an interesting special case, $E_Y = 1 - T'$. In this case, from eq.

7) If the system is unstable, the subsequent effects of changes in bond balances continue to accumulate and then dominate the effect of fiscal policy. The efficacy of fiscal policy in the long run is operative only when bond balances converge to a new equilibrium level, that is, when the system is stable.
(17) \( \frac{dB}{dG} = 0 \), and from eq. (16) \( \frac{dY}{dG} = \frac{1}{1 - E_Y} = \frac{1}{T'} \). That is, the \( \frac{dY}{dG} \) multiplier is given by the inverse of the marginal tax rate as Christ [3] has pointed out, and this value is equal to the pure multiplier effect of an increase in \( G \), the value obtained by ignoring the government budget constraint, \( \frac{1}{1 - E_Y} \). If the change in \( B \) is completely offset, the long-run effect of bond-financed fiscal policy is equivalent to the one obtained within the familiar IS–LM comparative static analysis.

In Blinder and Solow’s framework, which corresponds to the case \( \beta \to \infty \), even if \( I_T \) is sufficiently positive and therefore \( 1 - T' < E_Y \), this case is not relevant to fiscal policy. Actually, it is misleading to speak about bond-financed expenditures in such a case. \( 1 - T' < E_Y \) in their system implies that \( \dot{B} < 0 \) \( (dG - \frac{T'}{1 - E_Y} dG < 0) \) at the initial period, because each initial increase in \( G \) leads to a rise in income of \( \frac{1}{1 - E_Y} dG \) and a rise in tax revenues of \( \frac{T'}{1 - E_Y} dG \) but costs the government \( dG \). This is, so to speak, the bond-redemption policy, and under these circumstances the whole notion of bond-financed deficit spending loses meaning. The case that the impact spending multiplier is stronger than the inverse of the marginal tax rate is relevant only in our generalized framework where \( \beta \) is finite.\(^8\) In our framework, even if \( 1 - T' < E_Y \), government spending does not lead to a budget surplus at the initial period. We can, then, speak about bond-financed expenditures and our analysis has shown that fiscal policy in the long run may be invalid in this case.

**Model II**

As in Model I, we draw the phase diagram of the differential equations in Figure 2. Herein we must have

(18) \( \left( \frac{dB}{dY} \right)_{B=0} = -1 \).

At any point above the \( \dot{B} = 0 \) curve, \( B \) is decreasing in this model. The necessary

\[ \text{Figure 2-A} \]

\[ \text{Figure 2-B} \]

and sufficient condition for local stability is

\(^8\) As is stated in [2], in Blinder and Solow’s framework, the condition \( 1 - E_Y > T' \) is inevitable. Our generalized framework does not require such a condition.
Figure 2-A describes the case of $E_b > 0$ and Figure 2-B the case of $E_b < 0$.

Let us consider the effect of an exogenous expansion of $G'$. Both the $\dot{Y} = 0$ curve and the $\dot{B} = 0$ curve will shift to the right. At the initial equilibrium point $E_0$, both $Y$ and $B$ increase. In the case of Figure 2-A, the movement to the new equilibrium is almost the same as in Figure 1. Figure 2-A shows that in the first phase, where the resulting deficit is financed, fiscal policy is always effective. On the other hand, in the case of Figure 2-B the convergent process is monotonous and exhibits no cycle. If excess demand in the commodity market disappears before the government budget is balanced, the resulting increase in bonds will suppress effective demand and eventually overcome the initial expansionary effect.

From the comparative static analysis, we have

\begin{align}
(20) \quad \frac{dY}{dG'} &= \frac{1}{A}(T' + E_b), \\
(21) \quad \frac{dB}{dG'} &= \frac{1}{A}(1 - T' - E_r),
\end{align}

where $A = T'(E_b + 1 - E_r)$. Inequality (19) means $A > 0$. We find that $E_b > 0$ is not necessary for $\frac{dY}{dG'} > 0$. Even if $E_b$ is negative, it is still possible that the system is stable and fiscal policy is effective. It suggests that the Keynesian-monetarist controversy concerning whether or not $E_b$ is positive, is not relevant to the efficacy of bond-financed fiscal policy in Model II. However, from eq. (9) we know that the positive wealth effect on consumption is necessary for $\frac{dY}{dG'} > 0$.

Note that if $1 - T' < E_r$, then $\frac{dY}{dG'} > 0$. This is a great contrast to the result in Model I. This is because if $E_b$ is positive, the subsequent decreases in bonds offset the impact effect only partially, and the subsequent increases in $G$ stimulate effective demand. If $E_b < 0$, the subsequent decreases in bonds stimulate effective demand. Also, when $1 - E_r > T'$, the subsequent asset effect may be contractionary in Model II.

There are several remarks to be made in our analysis. First, let us compare the effect of bond-financed fiscal policy with that of money-financed fiscal policy from the point of view of the value of the income multiplier $\frac{dY}{dG'}$. It can be shown that in Model II if the impact multiplier is not sufficiently strong ($1 - E_r > T'$), the steady-state multiplier for money-financed deficit spending exceeds that for bond-financed deficit spending, contrary to Blinder and Solow's conclusion. It suggests that characteristics of the comparative static analysis crucially depend on how the policy system is formulated.

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9) Needless to say, in order to compare the income multipliers, the marginal tax rate should be fixed.
As a second observation, the highest income on the adjustment process under bond finance can be realized as the equilibrium income by altering the source of financing from bond finance to money finance at that point. Namely, since $B$ becomes exogenous under money finance, the government budget constraint can determine the highest income as the steady-state income. Under such bond or money finance, the crowding-out effect of fiscal policy can be completely avoided and the $\frac{dY}{dG}$ multiplier under such finance is obviously greater than under pure money finance.

Finally, we note that in order to explore further implications of fiscal policy, a analysis for a growing economy will be required. And in this case, recognition of price effects will be important.

(University of Tokyo)

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