PURCHASING POWER PARITY AND CURRENCY SUBSTITUTION*

By EISUKE SAKAKIBARA

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In the course of discussions on the monetary approach to exchange rate determination, particularly on the purchasing power parity models, strong attention has been paid to the interrelationship between relative price levels and exchange rates. Analysts in the monetary camp generally maintain that movements in relative price levels are fully offset by movements in exchange rates although such adjustments may take some protracted period. In most cases the failure of the purchasing power parity models to hold in the short-run is accounted for either by lags in the adjustments or problems associated with poor price indices. J. Frenkel, for example, has shown, in his recent empirical work, that exchange rates cause relative prices and the hypothesis of long-run unitary elasticity between the two cannot be rejected. The dynamic portfolio models of Dornbusch or J. Frenkel & C. Rodriguez are among models where such adjustment mechanisms are explicitly specified with the long-run purchasing power parity relationship.

Kravis and Lipsey, on the other hand, pointed out, in their important research work on the disaggregated price data, that real relative prices, even in the long-run, are not invariant as suggested by the PPP theorem. This Kravis & Lipsey conclusion is consistent with traditional trade models where real relative prices, or terms of trade, are essentially determined by the differential growth rates of productivity.

The purpose of this paper is to synthesize the two views while maintaining the basic characteristics of monetary approach. One distinguishing feature of monetary model as against traditional trade models is that the former assumes one homogeneous traded goods while the latter presupposes at least two. Although the one-goods assumption is often used in monetary models, it is by no means the essence of monetary approach. As has been pointed out by many, the salient characteristic of monetary approach lies in the fact that exchange rates are defined as "relative prices of different national monies rather than as relative prices of different national

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1) R. Dornbusch and D. Jaffee [11], J.F.O. Bilson [8].
2) S. P. Magee [25].
3) J. Frenkel [14].
4) R. Dornbusch [10], J. Frenkel & C. Rodriguez [12].
5) I. B. Kravis and R. E. Lipsey [21].
6) P. Samuelson [33], B. Balassa [3].
7) J. Frenkel [13], M. Mussa [28].
Although it is true that one-goods two money models emphasizes such feature as against one money two-goods world of traditional trade models, there is no reason why monetary approach cannot be maintained with two-goods world as long as two different monies are introduced in the model.

In the two-goods-two-money world, real relative price (terms of trade) between two goods is determined in the real sector, while two nominal prices and relative price of money (exchange rate) are determined in the monetary sector. At least in the equilibrium situation, the neutrality of money holds in a sense that monetary variables do not affect the real sector. The neutrality of money, in this sense, however, does not necessarily imply that movements in the relative prices (the ratio of two nominal price levels) is fully offset by the movements in the exchange rate. Suppose that the real relative price (terms of trade) in the real sector shows a secular upward trend due to productivity differentials while maintaining the equilibrium in the goods market. Then, the movement in the relative price is only partially offset by the movement in the exchange rate since real relative price or terms of trade ($E$) by definition is a product of relative nominal prices ($\frac{p^*}{p}$) and the exchange rate ($\frac{x}{p}$).

$$E \equiv x \cdot \frac{p^*}{p} \text{ or } \frac{\dot{E}}{E} \equiv \frac{\dot{x}}{x} + \left(\frac{p^*}{p}\right) \frac{\dot{p}^*}{p^*}.$$

Thus, it is our claim that the findings of Lipsey & Kravis that long-run real relative price is not invariant does not invalidate the monetary approach to the exchange rate determination but only points to the inadequacy of homogeneity assumption of traded goods.

Indeed, it is much more reasonable to assume that relative competitiveness (or real relative prices) of Ford vis-a-vis Toyota is not invariant but depends upon the relative productivity of the two companies. The movements in the real relative prices, in this sense, seems to underline the movements of monetary variables such as nominal price levels or exchange rate and it is our contention, in this paper, that the admission of such real influences does not invalidate monetary approach to the exchange rate determination.

The model of this paper, in this sense, provides a theoretical possibility for the long-run divergence of exchange rate from purchasing power parity without abandoning a basic proposition of the monetary approach in that the exchange rates are defined to be relative prices of monies rather than those of outputs and in that monetary variables are determined by the interaction of demand for and supply of respective monies.

The section 2 specifies a simple two goods-two money model while section 3 extends the model to a more general case. Section 4 presents some empirical evidence of Japan covering the period of floating exchange-rate regime.

In the seminal contributions to the monetary approach to the balance of payments with fixed exchange rate, both physical goods and money are assumed to be homogeneous. In a typical

8) M. Mussa, op. cit.
9) This concept of real relative price in two-goods-two-money world corresponds to that of terms of trade in one-money-two-goods world. It is sometimes called real exchange rate, as well.
10) H. Johnson [18], R. Mundell [27].
closed economy model, world price level for the homogeneous physical goods and the domestic output (supply of physical goods or demand for money) are given and the analysis is focused on the adjustment of money supply variables (domestic credit and foreign reserves) to these predetermined values of price and output. In extending this monetary approach to the floating regime, the homogeneity assumption of money is dropped and one physical goods and two money world is visualized. Typically, the purchasing power parity theorem is introduced and the equilibrium conditions for two money markets are solved out to obtain the exchange rate. In this one-goods-three-money world, the exchange rate ($x$) is, by definition, equal to the ratio of two nominal price levels ($\frac{p^*_2}{p}$), and the purchasing power parity theorem always prevails. ($x = \frac{p^*_2}{p}$). The validity of this model depends crucially on the assumptions of the homogeneity of physical goods and complete arbitration in the goods market (at least in the long-run). The purchasing power of two monies, here, depends upon their relative supplies vis-a-vis demands which are given by the predetermined outputs. In the absence of currency substitution, money, here, is only held for transaction purposes and not for portfolio reasons. Although there are two monies, residents of one country are constrained not to hold money of the other country since currency substitution is not allowed. This is, in a sense, a funny situation where the world is perfectly mobile with respect to goods transaction whereas it is completely closed in money transaction.

Traditional trade models, on the other hand, assume, at least, two different traded goods and restricts its attention on the analysis of real sector. Indeed, Richard's celebrated argument for comparative advantage would not have been advanced and refined, had economists always maintained the assumption of homogeneity of traded goods. In a classical world of dichotomy, this separation of real analysis from monetary considerations is only natural and as such, is simply the other side of the coin of traditional purchasing power parity approach, or conventional monetary approach.

Although two approaches are not mutually inconsistent, the link between the two is not that clear and this sometimes resulted in differences of opinions or confusions in the question of determination of exchange rate. The approach that could be appropriately termed “flow” approach extends the real trade analysis or elasticity version of real analysis to the question of balance of payments by simply adding capital account to the trade account. The adhoc nature of this flow approach has often been pointed out and the nature of equilibrium assumed has been critically scrutinized. What H. Johnson called “the elasticities approach,” “the Keynesian multiplier approach,” “the absorption approach” and “the economic policy approach” all fall into this category of “flow” models where monetary equilibrium is defined in terms of flows and in some extent of goods market analysis.

11) See, for example, S. Magee [25], L. Griton & D. Roper [15], N. Sargen [35].
12) P. Samuelson [32], B. Ohlin [30]. For the review of Heckscher-Ohlin-Samuelson model, see for example Bhagwati [5][6].
13) This terminology of “flow” versus “stock” models is due to Shinkai (Y. Shinkai [38]).
14) See, for example, B. Aghievli and G. Borts [1].
15) H. Johnson [19].
The "stock" analysis, or monetary approach, on the other hand, is more rigorous in that it is consistent with traditional monetary theory and fits in well with classical assumption of dichotomy. Although the approach is theoretically more satisfactory and more direct in analyzing the "monetary phenomenon" such as exchange rate determination, the practical question still remains whether it is proper to disregard the real factors completely, in view of the apparent effects that trade balance seems to bear on foreign exchange market.

The one answer to this practical question is to introduce the notion of disequilibrium dynamics or adjustment processes as was mentioned in the previous section. In this section, we follow another path by constructing a two-goods-two-money model, defining explicitly the real relative prices or terms of trade apart from the ratio of nominal price levels and the exchange rate. In this way, unlike in the purchasing power parity models, a possibility of divergence of the exchange rate from purchasing power parity is formally allowed.

To emphasize the essence of the approach, the model is simplified to the bare minimum. The world consists of two countries, say, the United States and Japan, and the former is denoted by the asterisk. There are four goods, U.S. physical goods \((Y^*)\), U.S. money \((M^*)\), Japanese physical goods \((Y)\) and Japanese money \((M)\). There are four relative prices, accordingly. Real relative price \((E)\), U.S. nominal price level \((p^*)\), Japanese nominal price level \((p)\) and the exchange rate \((x)\). The relationship among four goods and four prices are illustrated in the Figure 1 below.

Because prices are defined between two goods in relative terms, the following identity has to hold among four prices;

\[
E = x \cdot \frac{p^*}{p} 
\]

Note that the above identity is different from purchasing power parity relationship \(x = \frac{p^*}{p}\). Our basic assumption here is more or less classical in that we postulate semi-dichotomy or recursiveness in the determination of these four prices. First, real relative price, \(E\), is determined in the real sector, or by the interaction of \(Y\) and \(Y^*\) (or their sub-components). Monetary variables, at least in the equilibrium, cannot affect the real market.

The other three price variables, \(p\), \(p^*\) and \(x\) are determined in the money markets given real variables, \(Y\), \(Y^*\) and \(E\). In other words, given real variables \(Y\), \(Y^*\), \(E\) and real wealth \((W\) and \(W^*)\) the two equilibrium conditions in the money markets along with identities determine three monetary price variables, \(p\), \(p^*\) and \(x\).

Let us first specify the money markets. An important assumption here is that of currency sub-
In other words, depending upon the configuration of $p$, $p^*$ and $x$ and, in particular, the expectation of $x(x)$, both U.S. and Japanese money holders choose between dollars and yen. Demand for money, here, is the combination of transaction demand, affected by $Y$ and $Y^*$, and portfolio demand, affected by $W$ and $W^*$. Total real wealth, $W$ and $W^*$, assumed to be held only in the form of money, is predetermined in the real sector. It is this assumption of currency substitution that allows us to introduce portfolio consideration in the money market. As was pointed out by Bilson, this type of currency substitution model deemphasizes the role of relative real incomes, $Y$ and $Y^*$ because of the introduction of $W$ and $W^*$.

The equilibria in two money markets which are assumed to be attained instantaneously are given below;

\[(2) \ \frac{M}{p} = f(x, x, Y)W + g(x, x, Y*)E \cdot W^* \]
\[(3) \ \frac{M^*}{p^*} = \frac{[1 - f(x, x, Y)]W}{E} + \frac{[1 - g(x, x, Y*)]W^*}{E} \]

where $f \cdot W$ and $(1 - f)\frac{W}{E}$ are demands for real balances of yen and dollars by Japanese respectively and where $(1 - g)W^*$ and $g \cdot E \cdot W^*$ are demands for real balances for dollars and yen by American respectively, and where $f_y, g_y > 0, f_y^*, g_y^* < 0$.

Note that equations (2) and (3) are not mutually independent because of the wealth identity. The wealth identity is expressed as;

\[(4) \ \frac{M}{p} + E \cdot \frac{M^*}{p^*} = W + E \cdot W^* \]

Note that equation (2) is expressed in terms of Japanese physical goods whereas equation (3) is in terms of U.S. physical goods. Given real variables, $Y, Y^*, W, W^*$ and $E$, and policy variables $M$ and $M^*$ equations (2) and (3) along with identities (1) and (4) determine three price variables $p, p^*$ and $x$.

Solving these equations and identities for $x$,

\[(5) \ x = F(x, M, M^*, E, Y, Y^*, W, W^*) \]

where it can be shown; $F_x, F_M, F_{E}, F_{W^*} > 0, F_{M^*}, F_W < 0$.

Given $x, M, M^*, E, Y, Y^*, W$ and $W^*$, equation (5) determine the exchange rate. Although latter seen variables are exogeneous to the sector, $x$ has to be explained within the sector. Here,

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17) For models incorporating this assumption, see G. Calvo & C. Rodriguez [9], L. Girton & D. Roper [15], D. King, B. Putnam, D. Wilford [20], A. Laffer [23], M. Miles [26], K. Kudoh [22] and Y. Shinkai [38].
18) J.F.O. Bilson [8], op. cit.
19) For $F_W < 0, \frac{f}{M} > \frac{1 - f}{x \cdot M^*}$ has to be satisfied. Also, for $F_E > 0$ and $F_{W^*} > 0, \frac{1 - g}{x \cdot M^*} > \frac{g}{M}$ has to be satisfied. Both of the two inequalities are likely to hold in the real world. The sign conditions of $F_Y$ and $F_Y^*$ depends on the sign conditions of $f_y$ and $g_y^*$ as follows; $F_Y \equiv 0$ for $\frac{M^*}{E \cdot g_y} \equiv \frac{M^*}{E \cdot (1 - f_y)}$.
20) The equation (5) is obtained by deviding (2) by (3), then by substituting (1) for $\frac{p^*}{p}$, there.
we adopt the theory of rational expectation. That is, people in the market are assumed to know the functions (2) and (3) and the mechanism of determination of real variables to be specified later.

Thus;

$$x_t = \exp(x_{t+1}) = \exp(F_{t+1}) = \exp(F(x_{t+1}, Z_{t+1}))$$

where $Z$ is a vector of exogeneous variables, $Y$, $Y^*$, $E$, $W$, $W^*$, $M$ and $M^*$.

Recursive substitution above and substitution of $x_t$ in (5) yields;

$$(6) \quad x_t = H(Z_t, \exp(Z_{t+1}), \exp(Z_{t+2}), \ldots)$$

Thus, the exchange rate is a function of real variables, $Y$, $Y^*$, $W$, $W^*$ and $E$, policy instruments $M$ and $M^*$ and their expected future values.

Let us, now, turn to the analysis of the real sector. In this section, it is assumed that goods market are instantaneously cleared. This restrictive assumption is made in this section only to illustrate the essence of our model and will be dropped in the next section.

With the no savings assumption, the following income constraints holds;

$$(7) \quad Y_S = C_{JJ} + EC_{JU^*}$$


The equilibrium conditions in the goods markets are;

$$(8) \quad Y = Y_S = Y_D \equiv C_{JJ} + C_{UU}$$

$$(9) \quad Y^* = Y_{S^*} = Y_{D^*} \equiv C_{JU^*} + C_{UU^*}$$

where $Y_D$ and $Y_{D^*}$ are total demand for Japanese and U.S. goods, respectively.

Using the identity (7), the above two equilibrium conditions are reduced to the following;

$$(10) \quad \frac{C_{UU}}{E} = C_{JU^*}$$

The equation (9) is nothing but the condition of trade balance. In equilibrium, real relative price $E$ is determined so that supply for physical goods is equal to the demand in the two markets, or equivalently, so that trade balance is zero.

Assuming the following demand functions,

$$(11) \quad \frac{C_{UU}}{E} = C_{UU}(E, Y_{SP^*}), EC_{JU^*} = C_{JU^*}(E, Y_{SP})$$

where

$$\frac{\partial C_{UU}}{\partial E} > 0, \frac{\partial C_{UU}}{\partial Y_{SP^*}} > 0, \frac{\partial EC_{JU^*}}{\partial E} < 0, \frac{\partial EC_{JU^*}}{\partial Y_{SP}} > 0$$

and where $Y_{SP}$ and $Y_{SP^*}$ are permanent income of the two countries, then, $E$ can be expressed as follows;

$$(12) \quad E = G(Y_{SP}, Y_{SP^*})$$

where $G_{Y_{SP}} > 0$ and $G_{Y_{SP^*}} < 0$.

21) See, J. Muth [29], R. Barro [4], T. Sargent [36], T. Sargent & N. Wallace [37], K. Shirakawa [39] [40].
Note that in this no savings world, wealth, \( W \) and \( W^* \), are constant and denoted by \( \bar{W} \) and \( \bar{W}^* \).

Assuming that permanent income, \( Y_{SP} \) and \( Y_{SP}^* \), are functions of current and future expected income, \( Y_{St}, Y_{St}^*, \text{exp.} (Y_{St+1}), \text{exp.} (Y_{St+1}^*) \ldots \), equation (11) could be transformed to the following;

\[
(11)' \quad E = G(Y_{St}, Y_{St}^*, \text{exp.} (Y_{St+1}), \text{exp.} (Y_{St+1}^*) \ldots)
\]

Substituting (11)' into (6),

\[
(12) \quad x_t = H(\tilde{Z}_t, \text{exp.} (\tilde{Z}_{t+1}) \ldots)
\]

where \( \tilde{Z}_t \) is a vector of \( (Y_S, Y_S^*, \bar{W}, \bar{W}^*, M, M^*) \).

The mechanism of price determination described above can be described as following;

\[\text{Figure 2} \quad \text{Mechanism of Price Determinations}\]

The system, as can be seen from Figure 2, is recursive in this simple equilibrium model. Money is still a veil or neutral, in that neither \( M \) nor \( M^* \) can affect real variables.

\section{3}

A simple model of previous section abstracts from several important empirical facts, namely, existence of savings and security transactions and lags in adjustment in the goods market. In this section, these restrictive assumptions are dropped to modify the model for estimation.

First, relaxing the assumptions of no savings and instantaneous clearance in goods market, we introduce a dynamic adjustment equation where the change in real relative price (\( \Delta E \)) depends upon excess supplies in the markets. In addition to the actual excess supplies (\( Y_S - Y_D \) and \( Y_S^* - Y_D^* \)), the gap between potential and actual supplies (\( \bar{Y}_S - Y_S, \bar{Y}_S^* - Y_S^* \)) also affect \( \Delta E \).

\[
(13) \quad \Delta E = L(Y_S - Y_D, Y_S^* - Y_D^*, \bar{Y}_S - Y_S, \bar{Y}_S^* - Y_S^*, E)
\]

Actual excess supply, \( Y_S - Y_D \) (or \( Y_S^* - Y_D^* \)) is equal to undesired accumulation of inventories which is not observable. If actual excess supply can be approximated by trade surplus,\(^{22}\) the above equation is reduced to,

\[
(14) \quad E_t = q\left(\frac{C_{Uj}}{E} - C_{jU^*}\right)_{t-1}, (\bar{Y}_S - Y_S)_{t-1}, (\bar{Y}_S^* - Y_S^*)_{t-1}, E_{t-1}
\]

Alternatively, \( Y_D \) and \( Y_D^* \) in equation (13) can be assumed to be functions of permanent incomes \( Y_{SP} \) and \( Y_{SP}^* \) respectively. Then,

\[
(15) \quad E_t = q(Y_{St-1} , Y_{St-1}^*, Y_{SPt-1} , Y_{SPt-1}^*, (\bar{Y}_S - Y_S)_{t-1}, (\bar{Y}_S^* - Y_S^*)_{t-1}, E_{t-1})
\]

\(^{22}\) This assumption is quite ad-hoc. But in view of seemingly important role of trade balance in the exchange markets, this type of approximation may actually be made by traders in the market.
Monetary sector needs to be changed to incorporate security transactions. Here, we follow a rather ad-hoc procedure in assuming that demands for money are functions of interest rate differentials along with the following interest parity conditions,

\begin{equation}
(16) \quad i_t - i^*_t = \exp \left( x_{t+1} \right) - x_t
\end{equation}

where \( i \) is nominal interest rates.

With the above somewhat casual assumptions, our basic equation (5) remain unchanged. Or, for the purpose of estimation, independent variables of equation (5) could be augmented by the forward premium.\(^2\)

Indeed, the procedure outlined above is admittedly ad-hoc. Formally, a portfolio model involving securities needs to be specified and the four equilibrium conditions for two money markets and two securities markets have to be solved out for five price variables \( x, p, p^*, i \) and \( i^* \) with the identities for a given \( E \). The only reason why such a formal model is not shown here is empirical. That is, the empirical counterpart of the security market and the amount of the security outstanding would be hard to obtain. We would rather circumvent these empirical difficulties by adopting the simplest possible assumption of interest parity condition of equation (16).

Note that monetary variables could affect real variables in this dynamic version of the model. In particular, both potential and actual supplies and wealth \( (Y_S, Y^*_S, \overline{Y}_S, \overline{Y}^*_S, W, W^*) \) could be changed through monetary or fiscal policies. Monetary policies, for example, would change the level of investment expenditures through interest rates and affect potential supplies. However, since our interest is the determination of the exchange rate, \( x \), here, we would not go in details of this discussion and simply assume these supply variables are given.

### 4

In this section, the model presented in the previous section is estimated using the Japanese data (monthly) covering the period of floating regime, March 73 to May 78.

Since the model of the previous section is a two country model, we simply took the liberty of assuming that the world consists of the U.S. and Japan alone. Although the assumption is a gross simplifications, it is not unreasonable to represent the rest of the world for Japan by the U.S., particularly in the trade and international financial matters. The fact that U.S. dollar is a key currency would, at least, partially justify the above simplification.

Estimations of the equations (14) and (15) are presented below:\(^{2,4}\)

\begin{equation}
(17) \quad E_t = 48.313 + 0.815E_{t-1} + 0.851(\overline{Y}_S - Y_S)_{t-1} - 0.713(\overline{Y}^*_S - Y^*_S)_{t-1}
\end{equation}

\begin{align}
&= 4.772 \left( \frac{C^{ul}}{E} - C^{lu} \right)_{t-1} \\
&= 3.6
\end{align}

\begin{align}
\bar{R}^2 &= 0.8829 \\ D.W. &= 1.85 \\ S &= 5.4508
\end{align}

\(^{23}\) For the use of interest parity condition in monetary models, see, J. Bilson [7], M. Mussa [28] and K. Shirakawa [40].

\(^{24}\) Because of the serious multicollinearity problem, variables \( Y_{S,t-1} \) and \( Y^*_{S,t-1} \) are dropped from equation (15). Alternatively, variables \( (\overline{Y}_S - Y_S)_{t-1} \) and \( (\overline{Y}^*_S - Y^*_S) \) could be dropped. The results, however, are essentially the same.
and

\[(17)'\]
\[
E_t = 254.304 + 0.584 E_{t-1} + 1.391 (\bar{Y}_S - Y_S)_{t-1} - 1.012 (\bar{Y}_S* - Y_S*)_{t-1}
\]
\[
+ 1.136 Y_{SP,t-1} - 2.550 Y_{SP,t-1}*
\]
\[
(4.3) \quad (6.29) \quad (4.69) \quad (3.23)
\]
\[
\bar{R}^2 = 0.9018 \quad D.W. = 1.81 \quad S = 5.0334
\]

where

\[E_t = \text{relative monthly wholesale price index of U.S. and Japan (p*/p) multiplied by monthly averages of exchange rate (yen per dollar)}\]

\[
\frac{C_{UJ}}{E} - C_{UJ}^* = \text{Japanese trade surplus (in billions of dollars) deflated by U.S. wholesale price index 1975 = 1 (monthly, seasonally adjusted)}\]

\[Y_S^* = \text{U.S. industrial production index (whole industry), 1975 = 100 (monthly, seasonally adjusted)}\]

\[Y_S = \text{Japanese industrial production index (whole industry), 1975 = 100 (monthly, seasonally adjusted)}\]

\[\bar{Y}_S = \text{Assumed to be } A e^{\alpha t} \text{ where } \alpha \text{ is the average growth rate of } Y_S \text{ during Jan. 1970 and May 1978.} \]

\[\bar{Y}_S = \text{Assumed to be } A e^{\alpha t} \text{ where } \alpha \text{ is the average growth rate of } Y_S^* \text{ during Jan. 1970 and May, 1978.} \]

\[Y_{SP} = (Y_{St} + Y_{St-1} + Y_{St-2} \ldots Y_{St-23})/24. \text{ The arithmetic mean of } Y_S \text{ for the past two years.}\]

\[Y_{SP}^* = (Y_{St}^* + Y_{St-1}^* + Y_{St-2}^* \ldots Y_{St-23}^*)/24. \text{ The arithmetic mean of } Y_S^* \text{ for the past two years.}\]

The fit is better for (17)' and this is probably due to the fact that variables \(Y_{SP}\) and \(Y_{SP}^*\) could capture relative state of excess supplies (or demands) in the two countries better than \(\frac{C_{UJ}}{E} - C_{UJ}^*\).

The problem, however, may be in the representation of \(\frac{C_{UJ}}{E} - C_{UJ}^*\) in the equation (14). It could be argued that the demension of trade surplus variable in (14) is different from other variables and that physical quantities of trade transactions rather than real values of trade should be used. The equation below shows that use of physical quantities for trade does not change the substance of the equation (14). Because of the lack of recent data, the equation below is estimated using the data 73:3 to 78:2.

\[(18)\]
\[
E_t = 71.889 + 0.738 E_{t-1} + 1.101 (\bar{Y}_S - Y_S)_{t-1} - 0.826 (\bar{Y}_S^* - Y_S^*)_{t-1} - 0.094 QTB_{t-1}
\]
\[
(3.68) \quad (10.5) \quad (3.62) \quad (2.41)
\]
\[
\bar{R}^2 = 0.8666 \quad D.W. = 1.81 \quad S = 5.3747
\]

where

\[QTB = EX/E_t - IM_t, \text{ where } EX \text{ is Japanese export quantity index (monthly, 1970 average = 100, seasonally adjusted) and IM is Japanese import quantity index. (monthly, 1970 average = 100, seasonally adjusted)}\]

For other versions of the function, the reader is referred to Table 1. The use of ratio variable
### Table 1-(1) Equation for Real Relative Price

<table>
<thead>
<tr>
<th>Dependent Variable ($E_t$)</th>
<th>Constant ($E_{t-1}$)</th>
<th>Lagged Independent Variables</th>
<th>$\frac{C_{UJ} - C_{JU}^*}{E}</th>
<th>(Y_S - Y_{S^*})_{t-1}</th>
<th>(Y_{S^*})_{t-1}</th>
<th>Y_{SP_{t-1}}^*</th>
<th>QTB_{t-1}</th>
<th>RCUS_{t-1}</th>
<th>RCJ_{t-1}</th>
<th>RTB_{t-1}</th>
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<th>D.W.</th>
<th>S</th>
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<tbody>
<tr>
<td>73:3 to 78:5</td>
<td>48.313</td>
<td>0.815</td>
<td>0.851</td>
<td>-0.713</td>
<td>-4.772</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td>(3.02)</td>
<td>(13.8)</td>
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<td>to 78:2</td>
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<td>(10.5)</td>
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<td>to 78:2</td>
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Table 1-(2) Log Version

<table>
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<tr>
<th>log $E$</th>
<th>Constant</th>
<th>log $E_{t-1}$</th>
<th>log $Y_{St}$</th>
<th>log $Y_{d*}$</th>
<th>Time</th>
<th>log $Y_{SP_{t-1}}$</th>
<th>log $Y_{SP_{t-1}*}$</th>
<th>log RCUS$_{t-1}$</th>
<th>log RCJ$_{t-1}$</th>
<th>log RTB$_{t-1}$</th>
<th>$\bar{R}^2$</th>
<th>D.W.</th>
<th>$S$</th>
</tr>
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<tbody>
<tr>
<td>73:3 to 78:5</td>
<td>4.577</td>
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<td>0.378</td>
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<td>(4.57)</td>
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<td>(2.69)</td>
<td>(3.26)</td>
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<td></td>
<td></td>
<td>(0.92)</td>
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<tr>
<td>73:3 to 78:2</td>
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<td>(2.30)</td>
<td>(3.44)</td>
<td>(2.68)</td>
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</table>

where

$E_t$ = relative monthly wholesale price index of U.S. and Japan ($p*/p$) multiplied by monthly averages of exchange rate (yen per dollar)

$C_{UJ}/E - C_{JU*}$ = Japanese trade surplus (in billions of dollars) deflated by U.S. wholesale price index (1975 = 1) (monthly, seasonally adjusted)

$Y_{S*}$ = U.S. industrial production index (whole industry), 1975 = 100 (monthly, seasonally adjusted)

$Y_{S}$ = Japanese industrial production index (whole industry), 1975 = 100 (monthly, seasonally adjusted)

$\bar{Y}_{S}$ = Assumed to be $Ae^{at}$ where $a$ is the average growth rate of $Y_{S}$ during Jan. 1970 and May 1978. $\bar{Y}_{S}$ $\geq$ $Y_{S}$ for the whole period and $\bar{Y}_{S}$ = $Y_{S}$ for Jan. 1974.

$\bar{Y}_{S*}$ = Assumed to be $Ae^{a^*t}$ where $a^*$ is the average growth rate of $Y_{S*}$ during Jan. 1970 and May 1978. $\bar{Y}_{S*}$ $\geq$ $Y_{S*}$ for the whole period and $\bar{Y}_{S*}$ = $Y_{S*}$ for Sep. 1973.

$Y_{SP}$ = $(Y_{S_t} + Y_{S_{t+1}} + Y_{S_{t+2}} + \ldots + Y_{S_{t+23}})/24$. The arithmetic mean of $Y_{S}$ for the past two years.

$Y_{SP*}$ = $(Y_{S_{t*}} + Y_{S_{t+1*}} + Y_{S_{t+2*}} + \ldots + Y_{S_{t+23*}})/24$. The arithmetic mean of $Y_{S*}$ for the past two years.

$QTB = EX/E_t - IM_t$, where $EX$ is Japanese export quantity index (monthly, 1970 average = 100, seasonally adjusted) and $IM$ is Japanese import quantity index (monthly, 1970 average = 100, seasonally adjusted)

$RCUS = \frac{Y_{S*}}{\bar{Y}_{S*}}$

$RCJ = \frac{Y_{S}}{\bar{Y}_{S}}$

$RTB = EX/IM$
Table 2-(I)  Equations for the Exchange Rate

<table>
<thead>
<tr>
<th>$x$</th>
<th>Constant</th>
<th>$Y_{S_t}$</th>
<th>$Y_{S_t}^*$</th>
<th>$Y_{SP_{t}}$</th>
<th>$Y_{SP_{t}}^*$</th>
<th>$M_{t}$</th>
<th>$M_{t}^*$</th>
<th>$FP_{t}$</th>
<th>$E_{t}$</th>
<th>$E_{t}^1$</th>
<th>$E_{t}^2$</th>
<th>$C_{1U^*}$</th>
<th>$\tilde{R}^2$</th>
<th>D.W.</th>
<th>$S$</th>
</tr>
</thead>
<tbody>
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<td>73: 3 to 78: 5</td>
<td>478.130</td>
<td>-2.070</td>
<td>1.506</td>
<td></td>
<td></td>
<td>0.00408</td>
<td>-1.596</td>
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<td>0.8771</td>
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<td>(7.29)</td>
<td>(4.95)</td>
<td>(3.03)</td>
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<td>(5.22)</td>
<td>(6.20)</td>
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<tr>
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<td>(6.03)</td>
<td>(5.13)</td>
<td>(2.41)</td>
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<td>(5.97)</td>
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<td>73: 3 to 78: 5</td>
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<td>(3.70)</td>
<td>(2.45)</td>
<td>(2.92)</td>
<td>(4.10)</td>
<td>(3.57)</td>
<td>(5.85)</td>
<td>(3.37)</td>
<td>(4.59)</td>
<td>(5.38)</td>
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### Table 2-(2) Log Version

<table>
<thead>
<tr>
<th>( \log x )</th>
<th>Constant ( Y_{S1} )</th>
<th>( \log M_1^* )</th>
<th>( \log Y_{SPt} )</th>
<th>( \log Y_{SPt}^* )</th>
<th>( \log M_t )</th>
<th>( \log M_t^* )</th>
<th>( \log E_t )</th>
<th>( \log E_t^1 )</th>
<th>( \log E_t^2 )</th>
<th>( \log RTB )</th>
<th>( R^2 )</th>
<th>( D.W. )</th>
<th>( S )</th>
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<td>5.805</td>
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<td>0.761</td>
<td>(5.83)</td>
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<td>(6.81)</td>
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<td>6.400</td>
<td>(5.61)</td>
<td>0.836</td>
<td>(5.87)</td>
<td>-1.976</td>
<td>(6.68)</td>
<td>-1.464</td>
<td>(5.32)</td>
<td>0.415</td>
<td>(.8580)</td>
<td>1.59</td>
<td>0.032</td>
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<tr>
<td>73: 3 to 78: 2</td>
<td>7.255</td>
<td>(6.33)</td>
<td>0.572</td>
<td>(3.56)</td>
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<td>(5.90)</td>
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<td>(.9094)</td>
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<td>0.026</td>
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where

- \( x \) = end of month exchange rate (yen per dollar)
- \( M^* \) = end of month money supply \( (M_1) \) for the U.S. in billions of dollars (seasonally adjusted)
- \( M \) = end of month money supply \( (M_1) \) for Japan in billions of yen (seasonally adjusted)
- \( FP \) = forward premium for yen using one month forward market rate (end of month figure) in percentage point for definitions of other variables see the footnotes of Table 1.
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for \( QTB(EX_d/IM_d \text{ instead of } EX_d/E_t - IM_d) \), in particular, gives us essentially the same result as in equation (18).

According to equation (17), one billion dollar increase in Japanese trade balance in 75 U.S. price lowers the real relative price by 4.8 yen per dollar. Also, one percentage point increase of industrial production index of the U.S. and Japan raise the real relative price by 0.71 and -0.85 respectively. Also, permanent effects of \( \left( C_{UJ}^{E} - C_{UJ}^{E*} \right) \), \( Y_{S*} \) and \( Y_{S} \) are 25.79, 3.84 and 4.59 respectively. In other words, a billion dollar increase of Japanese trade surplus (in 1975 U.S. constant dollars) has a permanent appreciation effect of 25.79 yen per dollar. Other equations in Table 1 indicate the robustness of the above equation.

Also, one version of \( F \) function in equation (5) with forward premium is presented below:\(^{25/26}\)

\[
\begin{align*}
(19) \quad x_t &= 478.130 + 1.506 Y_{St}^* - 2.070 Y_{St-1} - 1.596 M_t^* + 0.00408 M_t - 4.080 FP_t + 0.459 E_t \\
& (7.29) \quad (3.03) \quad (4.95) \quad (6.20) \quad (5.22) \quad (5.97) \quad (4.38) \\
\bar{R}^2 &= 0.8771 \quad D.W. = 1.52 \quad S = 8.03
\end{align*}
\]

where

- \( x = \text{end of month exchange rate (yen per dollar)} \)
- \( M^* = \text{end of month money supply (} M_t^* \text{) for the U.S. in billions of dollars (seasonally adjusted)} \)
- \( M = \text{end of the month money supply (} M_t \text{) for Japan in billions of yen (seasonally adjusted)} \)
- \( FP = \text{forward premium for yen using one month forward market rate (end of month figure) in percentage points} \)

Note that the wealth variables \( W \) and \( W^* \) are suppressed in equations (19) because of the lack of appropriate proxies. Accordingly, variables \( Y_{St} \) and \( Y_{St}^* \) in the above equation play the double role of income variables and wealth proxies.

Note that equation (19) completely neglects future expectation variables in equation (5). The current variables in (19) not only represent their direct impact effects but also their indirect effects through expectation.

It might be argued that future expectations of \( E_t \), \( exp. (E_{t+1}), \text{ exp. (} E_{t+2} \text{) \ldots \ldots \ldots } \) are particularly important among others because of the lag in the \( E \) functions shown by equations, (17), (17)' or (18). That is, in the case of \( E \), people have better information in the prediction of its future values than others since current values of excess supply variables and \( E_t \) which determine \( E_{t+1} \) are given to them. Thus, it is reasonable to add the variable \( \text{exp. (} E_{t+1} \text{)} \) alone to the equation (19).

\[25\] For log versions of these equations, see Table 2.
\[26\] TSLS version of (19) with (17) and (17)' are as follows;

\[
(19)' \quad x_t = 560.850 + 1.526 Y_{St}^* - 2.213 Y_{St-1} - 1.801 M_t^* + 0.00468 M_t - 3.892 FP_t + 3.323 E_{t+1}^* \\
& (7.08) \quad (2.73) \quad (4.50) \quad (6.30) \quad (5.39) \quad (5.13) \quad (2.41) \\
\bar{R}^2 &= 0.8504 \quad D.W. = 1.67 \quad S = 8.8534
\]

where \( E_{t+1}^* \) is calculated from equation (17).

\[
(19)'' \quad x_t = 549.831 + 1.576 Y_{St}^* - 2.247 Y_{St-1} - 1.775 M_t^* + 0.00464 M_t - 4.186 FP_t + 0.336 E_{t+1}^* \\
& (6.88) \quad (2.86) \quad (4.71) \quad (6.19) \quad (5.37) \quad (5.57) \quad (2.53) \\
\bar{R}^2 &= 0.8519 \quad D.W. = 1.66 \quad S = 8.8105
\]

where \( E_{t+1}^* \) is calculated from equation (17)'.
Assuming the rationality in the expectation mechanism, we can derive the following from equation (14) or (15),

$$\exp(E_{t+1}) = Q\left(\left(\frac{C_{IJ}}{E} - C_{JV}^*\right)_t, (\bar{Y}_S - Y_S)_t, (\bar{Y}_S^* - Y_S^*)_t, E_t\right)$$

or $$q(\bar{Y}_S, Y_S, Y_{SP}, Y_{SP}^*, (\bar{Y}_S - Y_S)_t, (\bar{Y}_S^* - Y_S^*)_t, E_t)$$

In adding the above variable exp. ($E_{t+1}$) to equation (19), one statistical difficulty arises, that is, the collinearity of variables, $Y_S$ and $Y_S^*$ with $(\bar{Y}_S - Y_S)$ and $(\bar{Y}_S^* - Y_S^*)$. In the equations below, we have neglected variables $(\bar{Y}_S - Y_S)$ and $(\bar{Y}_S^* - Y_S^*)$ and represented them by $Y_S$ and $Y_S^*$.

$$x_t = 377.344 + 1.489Y_{St}^* - 1.926Y_{St} - 1.2488M_{t^*} + 0.00376M_t - 2.773FP_t$$

$$R^2 = 0.8979 \quad D.W. = 1.45 \quad S = 7.3816$$

$$x_t = 361.885 + 1.367Y_{St}^* - 1.167Y_{St} - 1.064M_{t^*} + 0.00425M_t - 2.992FP_t$$

$$R^2 = 0.9110 \quad D.W. = 1.40 \quad S = 6.9555$$

Note that the effects of exp. ($E_{t+1}$) on $x_t$ are quite substantial. In particular, one billion dollar increase in Japanese trade surplus (in 1975 U.S. price) results in the immediate appreciation of yen by 13 yen per dollar through this expectational mechanism. Since $(\frac{C_{IJ}}{E} - C_{JV}^*)$ works through $E_t$ variable in equation (20), added permanent effect of a billion dollar increase in Japanese trade surplus (in 1975 U.S. price) is 26.6 yen per dollar even when monetary effects are accounted for separately. Equation (21) indicates similar results in that effects of permanent income through expectation is very large. In particular, effects of permanent income are six to seven times as big as those of current income which work in the different direction through demand for money functions.

So much for the empirical estimation for the model. Although the results for one country and for one period are not sufficient to validate the model, these results, at least, would not contradict the theoretical approach adopted here. In particular, it should be emphasized that addition of variables $E_t$ and exp. ($E_{t+1}$) to a typical monetary model, increase the explanatory power of the equation quite substantially. Also, it is extremely interesting that the effects of expectation on $E_{t+1}$ is quite large. The movements of trade balance or permanent income, in this sense, is very important in the determination of exchange rate along with ordinary monetary variables. These results indicate that an eclectic view that both trade balance and monetary variables affect the exchange rate can be substantiated both on theoretical and empirical grounds.

(Saitama University)

27) We could include $(\bar{Y}_S - Y_S)$ and $(\bar{Y}_S^* - Y_S^*)$ and drop $Y_S$ and $Y_S^*$, instead. The results, though, are only marginally different.

28) For the comparable results of typical monetary model, see M. Shirakawa [40].
REFERENCES


