TAX REFORMS IN A DEBT FINANCED ECONOMY: THE ROLE OF EXPECTATIONS*

By TOSHIHIRO IHORI

1. Introduction

One of the incidence effects through changes in the size of the stock of government debt is the so called chain-letter mechanism of debt finance where taxes are predetermined and government debt issuance is endogenously determined. In that case, as the deficit continues, the outstanding bond stock continues to grow. So, accordingly, does the interest that must be paid each period on the outstanding stock. The events of the 1970s and 1980s suggest that when governments become strapped for funds, they tend to rely more heavily on bond issuance.

In reality, however, it is argued that if the current deficits seem not sustainable, governments in such countries will be forced to in effect repudiate their debt, either explicitly through an introduction of new taxes or through inflation depreciation (inflationary taxes). We may call such a policy change the tax reform for debt repudiation. The more likely the current deficits seem not sustainable, the higher the subjective probability of the future tax reform. The consequent taxation postponement is not free from credibility problems: Will the additional debt be paid off in full, will the government find it optimal to resort to higher inflation or currency devaluation to diminish the burden of the debt, etc?

In the previous literature the chain letter problem has been investigated under the assumption that government debt and real capital are perfect substitutes.1) Accordingly, all incidence effects occur through changes in the size of the stock of government bonds rather than through changes in the relative prices. It should be stressed, however, that if the private sector recognizes such future possibilities of tax reforms for debt repudiation government bonds and real capital may no longer be regarded as perfect substitutes.2)

Such a situation might be relevant for the recent Japanese economy. A recent line of economic research suggests that private agents realize that current bond-financed deficits carry with them future tax obligations. Anticipating higher future taxes, private agents change current spending behavior to smooth consumption intertemporally. Although the econometric study of

---

* I am grateful to Raymond G. Batina, an anonymous referee, and the Editor for their useful comments and suggestions.
2) The inclusion of a fixed asset, such as land, is important in determining the incidence of fiscal policies. See Chamley and Wright (1987) and Ihori (1988b).

— 251 —
this issue is still in its infancy, some recent research indicates that private Japanese behavior has partially offset recent changes in fiscal policy (see Homma, et al. (1986) and Ihori (1987) (1989)).

Recently, economic theory has begun to catch up with political reality by not only studying the optimality of fiscal policy in a context where explicit account is taken of the government's budget constraint but it has gone a step further by examining the time consistency of optimal policy, that is, the issue of whether it is optimal to keep promises that were optimal to make in the past.\(^3\) The latter lies at the heart of the credibility dilemma faced by any serious politician.

Bearing these aspects in mind, we attempt to formalize one such psychological phenomenon: confidence. We consider a growing overlapping generations economy with capital in which the only nonphysical store of value and vehicle for intergenerational transfer is government bonds. Holding this debt to provide for old-age consumption requires confidence and trust as no one can guarantee to the young that the rate of return on debt is the same as that of real capital. Put another way, it is not sure to what extent the debt burden will be transferred to the next generation.\(^4\)

This paper is organized as follows. Section 2 develops the theoretical framework. Section 3 investigates the dynamic properties of our model. Finally section 4 concludes our paper. It is shown that including possible tax reforms for debt repudiation is important in determining the dynamic properties. A striking general result is that the future tax reforms may in some cases be not effective to reduce the possibility of bankruptcy.

2. Theoretical Framework

The theoretical framework is due to Diamond (1965). The economy consists of overlapping generations of identical, two-period lived individuals. Population, as measured by the number \(N_t\), of agents in each generation, grows at rate \(n\). The level of output during a given period \(t\) is defined by a twice differentiable neoclassical production function with positive decreasing first derivatives and constant returns to scale.

2.1. Consumers

Consumption and saving take place at the end of the period. The savings are invested partly as real capital which does not depreciate and is a perfect substitute for the consumption good, and partly in government bonds purchased from the older generation that is in the second period of life and consumes all of its accumulated wealth leaving no bequests.

Let us denote by \(1 - \theta\) a subjective probability that all of the debt burden will be transferred to the next generation. In other words, \(\theta\) is the subjective probability of the future tax reforms. We assume that there is no intrinsic uncertainty in the economy: neither tastes nor endowments are subject to any random shocks.

---

\(^3\) See Kydland and Prescott (1977) among others. Recently Calvo (1988) studies models in which debt repudiation is possible and shows that expectations may play a crucial role in the determination of equilibrium.

\(^4\) Valuation of an intrinsically useless and unbacked asset performing intergenerational transfers from the young to the old requires enough confidence that this asset will not be worthless in the future. Tirole (1985) and Weil (1987) examine deterministic and speculative bubbles in the overlapping generations framework which are, like government debt, intergenerational schemes based on trust.
T. Ihori: Tax Reforms in a Debt Financed Economy: The Role of Expectations

We explore the maximizing problem facing a consumer representative of generation $t$. We assume that utility is time separable, and that consumers are von Neumann-Morgenstern expected utility maximizers. A young born at $t$ chooses $c^t_1, c^t_2, s^t$, and $b^t$ to maximize

$$E_t\{u(c^t_1) + \beta u(c^t_2)\}$$

subject to

1. $c^t_1 = w_t - s^t - b^t$
2. $c^t_{i+2} = (1 + r_{i+1})s^t_i + (1 + i_{i+1})b^t_i$ for $p_b = 1 - \theta$
3. $c^t_{i-2} = (1 + r_{i+1})s^t_i + (1 + i_{i+1})b^t_i - \tau_{i+1}$ for $p_b = \theta$

where $c^t_1$ is the first period consumption, $c^t_2$ is the second period consumption, $b$ is per-capita debt issuance, $s$ is his savings of real capital, $\beta$ is the subjective discount rate, $E(\cdot)$ denotes the expected value conditional on information available to the young, $r$ is the rate of return on real capital, $i$ is the rate of return on government debt, and $\tau$ is a tax levied in the second period of his life under the tax reforms. Here superscript $t$ means generation $t$ and subscript $t$ means period $t$.

2.2. Government

The chain-letter mechanism of debt finance means that taxes are predetermined and government debt issuance is endogenously determined by the government budget constraint. The chain-letter mechanism is formulated by the following government budget constraint;

$$b^t_{i+1} = \frac{1 + r_{i+1}}{1 + n}b^t_i - \frac{q\tau}{1 + n}$$

Suppose the government introduces a random tax $\tau$; the fraction of $q$ of the old generation is imposed a tax $\tau$. If $q = 0$, the government does not actually introduce tax reforms for debt repudiation. If $q = 1$, the government introduces tax reforms to all the consumers. If $q = \theta$, the expectation of the private sector is fulfilled.

2.3. Market Equilibrium

Competitive profit maximization requires that firms hire labor and demand capital in such a way that

$$w_t = w(r_t), w'(r_t) = -k_t$$

where $w(\cdot)$ is the factor price frontier.

The bond and money markets clear:

$$b^t_i = b^t[\ ]$$

$$-(1 + n)w'(r_{i+1}) = s^t[\ ]$$

And we have

$$a^t[\ ] = -(1 + n)w'(r_{i+1}) + b^t_i$$

The capital stock at $t + 1$ consists of physical savings by the young at $t$. As the latter depend on the state of nature at $t$, by assumption known to agents born at $t$, the capital stock at $t + 1$ is predetermined as of $t$. In writing the first-order conditions, we implicitly impose the condition that agents be aware of this predetermination, as required in a rational expectations equilibrium.
3. Dynamics of Model

We now investigate how the anticipation about the future tax reforms is related to the dynamic properties. For the sake of exposition clarity, we characterize the dynamic properties of the lump-sum tax-reform economy before those of the debt-holding tax-reform economy.

3.1. lump-sum tax-reform case

saving behavior

When the tax reform is lump sum, there is no uncertainty about the net rate of return on government debt. Government debt and real capital are both safe assets and perfect substitutes. Portfolio composition would be indifferent. The rate of return on debt \( i \) is equal to the rate of return on capital \( r \).

Then, the first order condition is simply

\[
E_t u'(c_t) = (1 + r_{t+1}) \beta E_{t+1} u'(c_{t+1})
\]

The total saving \( a = s + b \) will be a function of \( w, r, \theta, \) and \( \tau \). It is easy to show that \( da/dw > 0, da/dr > 0, da/d\theta > 0, \) and \( da/d\tau > 0 \). An increase in \( \theta \) will reduce the expected disposable lifetime income and hence the first period consumption. It follows that the total savings will increase with \( \theta \).

As far as the portfolio choice between real capital and government debt is concerned, some distortionary taxes may have the same effect as the lump sum tax. Suppose, for example, general capital income taxation is imposed. There would be uncertainty about the net rate of return on government debt. However, the net rate of return on real capital would also be a random variable. Government debt and real capital become both risky assets and perfect substitutes. So long as the two assets are perfect substitutes, the total saving is a relevant variable.

phase diagram

The dynamic system will be summarized by

\[
a[w(r_t), r_{t+1}, \theta, \tau] = -(1 + n) w'(r_{t+1}) + b_t
\]

\[
b_{t+1} = \frac{1 + r_{t+1}}{1 + n} b_t - \frac{q \tau}{1 + n}
\]

Let us investigate dynamic properties of this economy using a phase diagram.\(^5\) To analyze the behavior of \( b_t \), we first find the locus of \((b, r)\) where \( b_{t+1} = b_t \). We call this locus the \( bb \) curve. From (7) this locus is given by

\[
b(r - n) = q \tau
\]

If \( q = 0 \), then this locus, illustrated in Figure 1, is given by \( r = n \) and \( b = 0 \). The locus of \( r_t = r_{t+1} \) is determined by analyzing (6). We call this locus the \( rr \) curve. The dynamic properties of the phase diagram will be investigated in Appendix.

Since we are concerned with the feasibility of chain-letter mechanism, we will concentrate on the case of \( b > 0 \). For given tastes and technology there are two kinds of steady states which we label as \( E_A \) and \( E_B \). At \( E_A r = n \) and at \( E_B b = 0 \). Here \( A \) subscript denotes variables in a Gale

---

5) Since our dynamic model is characterized by difference equations, strictly speaking, our dynamic analysis with the aid of a phase diagram is not appropriate and should be considered as a first order approximation. Tirole (1985), Ihori (1988a), Weil (1987), and Schmid (1988) use similar phase diagrams.
type-A equilibrium and B subscript denotes variables in a Gale type-B equilibrium.6)

From stability viewpoint, type A equilibrium is always a saddlepoint and hence unstable except only one path aa. There are three possibilities; (i) bankruptcy: Above this borderline (the aa curve), the economy eventually goes bankrupt; b and r will eventually approach to the infinite. As b increases, savings of real capital (excluding public debt holdings) will be reduced. The economy ends up in a vicious circle where the government borrows to finance the interest payments on public debt. As soon as public borrowing completely absorbs private saving, private investment becomes negative. That is to say, worn-out machines are no longer replaced. The stock of capital begins to decline, while labor continues to grow. This curbs the expansion of output. Ultimately, the stock of capital is exhausted, thus all economic activities stop. We may call such a situation bankruptcy of the economy. (ii) feasible chain-letter mechanism: If the initial condition \((b_0, r_0)\) is below the aa curve, the economy eventually approaches \(E_B\); the chain-letter mechanism may be feasible in the long run. (iii) saddle-point equilibrium: If the initial economy is on the aa curve, the economy eventually approaches \(E_A\); the chain-letter mechanism will also be feasible.

**Dynamic implication of \(\theta\)**

Let us now investigate the dynamic implication of \(\theta\). Since \(a_\theta > 0\), an increase in \(\theta\) will shift the rr curve upward. In other words, an increase in the subjective probability of an introduction of the lump-sum tax reform will shift the rr curve upward, and the critical borderline aa will shift

---

upward as well. An increase in the subjective probability of an introduction of a lump sum tax has a real effect. The real savings of the private sector is increased because the expected lifetime income and hence the desired first period consumption will be lowered. The more the value of $a_\theta$, the more likely that the chain-letter mechanism is feasible in the long run. The economy may avoid bankruptcy if $\theta$ is raised enough. An increase in the subjective probability of the future lump-sum tax reform has a desirable effect on the sustainability of the system and long run welfare.

**Dynamic Implication of $q$**

Let us then consider the situation where the economy is above the borderline $aa$ even if $\theta = 1$. How can this critical development be stabilized? In such a case the government will actually be forced to introduce a lump-sum tax so as to avoid bankruptcy. We now consider the consequence of the actual lump-sum tax reform (an increase in $q$).

If $q > 0$, the $bb$ locus is now the rectangular hyperbola, illustrated in Figure 2. In Figure 2 there exists only one type of equilibrium $E_A$. Above the $aa$ curve the economy eventually goes bankrupt. Below the $aa$ curve a permanently-maintained positive per-capita deficit is feasible in the long run.

Here an increase in $q$ will shift the $bb$ curve to the right. The critical borderline $aa$ will shift upward as in the case of an increase in $\theta$. Thus, an increase in $\theta = q$ in the rational expectation equilibrium will expand the region which the economy eventually avoids bankruptcy.

If an increase in $q$ is not sufficient, the economy may still be above the $aa$ line in the case of $q = \theta = 1$. Then the government is forced to use $\tau$. An increase in lump-sum taxation $\tau$ will
shift both the $bb$ curve to the right and the $rr$ curve upward. Comparing Figure 1 and Figure 2, we may say that the chain-letter mechanism is more likely to be feasible in Figure 2 than in Figure 1. An increase in the expected lump-sum tax on more consumers leads to higher private savings and more government revenues. The government can avoid bankruptcy if $q$ and $\tau$ are raised enough.

Hence, we have the following proposition.

Proposition 1: An increase in the subjective probability $\theta$, the objective probability $q$, or lump-sum taxes $\tau$ decreases the possibility of the bankruptcy of the economy in the lump-sum tax-reform case.

3.2. Debt-holding tax-reform case

saving behavior

When the tax reform means a debt-holding tax ($\alpha$), we have

$$\tau_{t+1} = \alpha b_t$$

If a tax ($\tau_t$) is levied on the interest of government bonds, $\alpha = \tau_t$. If $\alpha = i - n$, the perfect debt repudiation is realized. From now on for simplicity $\alpha$ is assumed to be fixed. So long as the differential tax on government bonds is imposed, (9) is relevant. Suppose, for example, differential capital income taxation is imposed but some of private capital (say, land) is untaxed. Then the resulting tax reform may essentially be described by (9).

Now consumers believe that the net rate of return $\tilde{r}$ is a random variable, depending on the subjective probability of the tax reform. The first order conditions for an interior maximum are given by

$$E[u'(c_t^1) - \beta(1 + r_{t+1})u'(c_t^2)] = 0$$

$$E[u'(c_t^1) - \beta(1 + \tilde{r}_{t+1})u'(c_t^2)] = 0$$

Table 1 summarizes the comparative statics results. All income effects are positive. What this means is essentially that it is now possible to increase both present consumption and future consumption from the levels enjoyed before the change in yield. As for substitution effects, direct substitution effects are positive, while the signs of the cross-substitution effects are indeterminate. If debt burden is more likely transferred to the next generation (i.e. the subjective probability of the introduction of tax reform is lowered), the demand for debt, the risky asset, will be increased.

phase diagram

The dynamic system will be summarized by

$$s[w(r_t), r_{t+1}, i_{t+1}, \theta; \alpha] = -(1 + n)w'(r_{t+1})$$

Table 1

<table>
<thead>
<tr>
<th>$w_t$</th>
<th>$r_{t+1}$</th>
<th>$i_{t+1}$</th>
<th>$\theta$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>?</td>
</tr>
<tr>
<td>$b$</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>$a$</td>
<td>+</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
</tbody>
</table>

* A note which derives the comparative statics results will be available upon request.
\begin{align*}
\text{(13) } & b[w(r_t), r_{t+1}, i_{t+1}, \theta; \alpha] = b_t \\
\text{(14) } & b_{t+1} = \frac{1 + i_{t+1}}{1 + n} b_t - \frac{qa}{1 + n} b_t
\end{align*}

$r_{t+1}$ and $b_{t+1}$ directly depend on $r_t$ and $b_t$, while $i_{t+1}$ does not directly depend on $i_t$. This dynamic system thus can be reduced to the system derived in Appendix.

Dynamic properties of the system in Figure 3 are qualitatively almost the same as in the lump-sum tax reform case 3-1. If $q = 0$, for given tastes and technology there are two kinds of steady states which we have labeled as $E_A$ and $E_B$ as in Figure 1. At $E_A$, $i = n$ and at $E_B$, $b = 0$.

**Dynamic implication of $\theta$**

In practice, debt repudiation is more likely if things go bad. We would expect $\theta$ to increase over time, as national debt threatens to inflate beyond bounds. Let us now investigate the dynamic implication of changes in $\theta$. As shown in Appendix, in the debt-holding tax-reform case the $bb$ curve is dependent on $\theta$. Since $i_b > 0$ and $i_\theta > 0$, an increase in $\theta$ will shift the $bb$ curve upward. In other words, if the subjective probability of an introduction of the tax reform (a debt holding tax) is increased ($\theta$ is increased), the gross rate of return on debt is increased, and hence the $bb$ curve will shift upward.

On the other hand, contrary to the lump-sum tax case, an increase in $\theta$ may not necessarily shift the $rr$ curve upward. The effect of an increase in $\theta$ on the $rr$ curve depends on the sign of

$$\frac{-s_\theta + s_i i_\theta}{s_i i_b}.$$
If $s_i < 0$ and $s_\theta + s_i \theta < 0$, then an increase in $\theta$ will shift the $rr$ curve downward. If $s_i = 0$, then an increase in $\theta$ will shift the $rr$ curve to the right.

An increase in $\theta$ will shift the $bb$ curve downward, while the effect on the $rr$ curve is ambiguous. The critical borderline $aa$ will shift downward if the effect of the shift of the $bb$ curve dominates the effect of the shift of the $rr$ curve. An increase in the subjective probability of an introduction of a debt tax has two real effects. First, the gross rate of return on government bonds is increased because debt is now perceived as more risky by the private sector. Second, the real savings of the private sector may or may not be increased because real capital is more attractive than government debt to the private sector and the nominal return on debt holding is increased. The more the value of $i_\theta$, it is more likely that the economy eventually goes bankrupt.

**Cobb-Douglas case**

It is interesting to consider a special case of the Cobb-Douglas utility function. Here (10) and (11) can explicitly be solved to derive the asset demand functions:

$$ s_t = \gamma_t h w_t $$

$$ b_t = (1 - \gamma_t) h w_t $$

$$ a_t = h w_t $$

where

$$ h = \frac{\beta}{1 + \beta} $$

$$ \gamma_t = \frac{\theta a (1 + r_{t+1}) - (1 + i_{t+1} - a)(i_{t+1} - r_{t+1})}{(i_{t+1} - r_{t+1})(r_{t+1} - i_{t+1} - a)} $$

The propensity to save of the total income ($h$) is independent of the asset returns, as income and substitution effects cancel out with logarithmic utility. $h$ is also independent of the probability of the future tax reform ($\theta$). Relative asset returns and the future possible tax reform only affect portfolio composition.

In this case the $rr$ curve is given by

$$ h w(r) = -(1 + n) w'(r) + b $$

which is independent of $\theta$. Hence, an increase in $\theta$ will shift only the $bb$ curve downward, and the critical borderline $aa$ will shift downward. When $\theta$ increases, it is more likely that the economy eventually goes bankrupt. Thus, we have

**Proposition 2:** An increase in the subjective probability of the future debt-holding tax reform reduces the sustainability of the system and long-run welfare in the Cobb-Douglas case.

**Dynamic implication of $q$**

Thus, in the debt-holding tax reform case, an increase in the subjective probability of the tax reform may well raise the possibility of bankruptcy. Even if the initial state is within the feasible region, an increase in $\theta$ may lead to bankruptcy. In such a situation the government will even-

---

7) This result is akin to Weil's result on bubbles. Weil considered a two-state model with real capital and a bubble. The bubble has probability $\theta$ of bursting every period. The main result in Weil is that the highest sustainable bubble (the equivalent of the highest sustainable debt in the present paper) decreases with the probability of bursting (debt repudiation).
tually be forced to introduce the actual tax reform so as to avoid bankruptcy. We now consider the economy where the government actually introduces the tax reform for debt repudiation.

First of all, let us consider the effect of an increase in \( q \) at the given level of \( \theta \). An increase in \( q \) will shift the \( bb \) curve upward. Hence, the borderline \( aa \) will also shift upward. By levying debt holding taxes on more consumers, the government may reduce new debt issuance and it is less likely that the economy eventually goes bankrupt. Although we do not intend to investigate the adjustment process of changes in \( \theta \) and \( q \), it seems reasonable to assume that an increase in \( q \) will lead to a further increase in \( \theta \). Sooner or later \( q = \theta = 1 \) will be realized.

We then consider the extreme case of \( q = \theta = 1 \). After the introduction of the tax reform there is no uncertainty about the net rate of return on government debt. We have \( i - \alpha = r \), and real capital and government debt become perfect substitutes. The system will be summarized by

\[
\begin{align*}
(15) \quad a[w(r_t, r_{t+1})] &= -(1 + n)w'(r_{t+1}) + b_{t+1} \\
(16) \quad b_{t+1} &= \frac{1 + r_{t+1}}{1 + n} b_t
\end{align*}
\]

It is interesting to note that the debt tax \( \alpha \) is not effective in the perfect tax reform economy. It is because the gross rate of return is increased to offset a debt tax. A change in \( \alpha \) will not affect real equilibrium. A size of a tax on government debt (for example an inflationary tax on debt) will not affect private savings or government revenues. In this sense the size of the debt holding tax reform (\( \alpha \)) does not matter once the reform has been fully introduced.\(^8\)

Compared with the \( q = 0 \) case, as the government actually collects debt taxes, the \( bb \) curve is at the right hand side of the original \( bb \) curve. The possibility of bankruptcy is lowered compared with the case of \( q = 0 \) discussed above. However, since the economy is essentially the same as the deterministic Diamond economy, the possibility of bankruptcy cannot be ruled out completely even if \( \alpha \) is raised enough. For example, as stressed by Ihori (1988a) and Schmid (1988), in the Cobb-Douglas economy for plausible values of parameters the economy will go bankrupt. The debt-holding tax reform will not avoid the bankruptcy completely.

4. Conclusion

Our dynamic analysis of the life cycle model without bequests shows that the inclusion of possible future tax reforms for debt repudiation is important in determining the incidence of fiscal policies on economic welfare. The central message that comes across this paper is that expectations of the future tax reforms may play a crucial role for the feasible chain-letter mechanism.

An increase in the subjective probability of an introduction of a debt-holding tax (\( \theta \)) has two real effects. First, the gross rate of return on government debt is increased because debt is now perceived as more risky by the private sector. Second, the real savings of the private sector may or may not be increased because capital is more attractive than debt to the private sector, while the gross rate of return on government debt is raised. The former effect is destabilizing, while

\(^8\) The perfectly anticipated tax induces an equal increase in the interest rate on national debt and is thus neutral. However, if \( \theta < 1 \), at the first period of imposition the tax reform has real effects. This may bring the initial economy below the \( aa \) curve. Therefore, it would be effective to introduce the actual tax reform at the early stage when \( \theta \) is low enough.
the latter effect may or may not be stabilizing. In the special case of the Cobb-Douglas utility function the latter effect is assumed away because the total savings are dependent only on the lifetime labor income. It follows that the higher the subjective probability of the debt-holding tax reform, the more likely the economy eventually goes bankrupt.

This paper suggests that the lump-sum tax reform is more effective than the debt-holding tax reform from the viewpoint of the sustainability of government budgets and long run welfare. The debt-holding tax reform is not effective in the long run because the gross rate of return on debt is adjusted to offset changes in the debt tax rate. It is shown that the size of the debt-holding tax reform does not matter once the reform has been introduced. Taxes on labor income and consumption, or general tax on capital will not have such an offsetting effect. Thus, if the lump-sum tax is not available, taxes on labor income and consumption or general tax on capital income may well be better than a differential tax on debt holdings so as to avoid bankruptcy.

Several extensions would be useful. An interesting extension would be to introduce transactions costs. Suppose transactions costs associated with debt repudiation is smaller than tax collection costs of taxation from labor income, consumption, or capital income. Then the government would have an incentive to repudiate debt even if such a differential tax on debt holding would eventually lead to a higher \( \theta \) and hence bankruptcy. Another extension would be to consider the adjustment process of \( \theta \) and \( q \) explicitly. It would seem that the qualitative results in the present paper hold in the situation containing the adjustment process. Finally, a useful extension would involve providing a reason for the government’s collection revenue. I think the analysis of these types of models should provide us with a deeper understanding of the incidence of recent Japanese tax reforms on economic welfare.

Appendix

**lump-sum tax reform**

By differentiating (7) partially with respect to \( r_t \),

\[
\frac{\partial b_{t+1}}{\partial r_t} = \frac{1}{1+n} b_t > 0 \text{ if } b_t > 0
\]

Hence, if \( b > 0 \) on the right hand side of the \( bb \) curve, \( b_{t+1} > b_t \), and on the left hand side of the \( bb \) curve, \( b_{t+1} < b_t \). If \( r \) were not changed, on the right (left) hand side of the \( bb \) curve \( b \) will increase (decrease).

Substituting \( r_t = r_{t+1} = r \) into (6), and totally differentiating (6), we have the slope of the \( rr \) curve as

\[
\frac{db}{dr} = (1+n)w'' + a w' + a_r
\]

where \( a_w = \partial a/\partial w \) and \( a_r = \partial a/\partial r \). (A.2) is likely to be positive when the elasticity of substitution between capital and labor is large.

By differentiating (6) partially with respect to \( b_t \),

\[
\frac{\partial r_{t+1}}{\partial b_t} = \frac{1}{(1+n)w'' + a_r}
\]
which is likely to be positive. Hence, above the $rr$ curve $r_{t+1} > r_t$, and below this locus $r_{t+1} < r_t$. If $b$ were not changed, above (below) this locus $r$ will increase (decrease).

debt-holding tax reform

From (13) $i_{t+1}$ may be solved as a function of $r_t, r_{t+1}, b_t, \theta$.

(A.4) $i_{t+1} = i[w(r_t), r_{t+1}, b_t, \theta]$  

where $i_w = \frac{\partial i}{\partial w} < 0$, $i_b = \frac{\partial i}{\partial b} > 0$, and $i_\theta = \frac{\partial i}{\partial \theta} < 0$. The sign of $i_r = \frac{\partial i}{\partial r_{t+1}}$ is ambiguous, but $i_r > 0$ if the substitution effect is stronger than the income effect. Substituting (A.4) into (12) and (14), the dynamic system will be reduced to time paths of $b$ and $r$.

If $q = 0$, the $bb$ curve is given by

(A.5) $i[w(r), r, b, \theta] = n$

Totally differentiating (A.5), we have the slope of the $bb$ curve as

(A.6) $\frac{db}{dr} = \frac{i_sw' + i_r}{i_b} < 0$

which is likely negative unless the income effect of $\partial b/\partial r$ is strong enough. By differentiating (14) partially with respect to $r_t$,

(A.7) $\frac{\partial b_{t+1}}{\partial r_t} = \frac{1}{1+n}b_t i_r > 0$ if $i_r > 0$

Hence, if $i_r > 0$, then $b_{t+1} > b_t$ on the right hand side of the $bb$ curve and $b_{t+1} < b_t$ on the left hand side of the $bb$ curve.

Considering (A.4), from (12) the slope of the $rr$ curve is given by

(A.8) $\frac{db}{dr} = \frac{(1+n)w'' + s_w w' + s_r + s_i(i_r + i_fw')}{s_l b}$

which is likely to be positive when the elasticity of substitution between capital and labor is large and the cross substitution effect of $\partial s/\partial i$ is stronger than the income effect ($s_i < 0$). If $s_i = 0$, the $rr$ curve is a vertical line. By differentiating (12) partially with respect to $b_t$, we have

(A.9) $\frac{\partial r_{t+1}}{\partial b_t} = \frac{s_l i}{(1+n)w'' + s_r + s_i i_r}$

which is positive if $s_i < 0$. Hence, above the $rr$ curve $r_{t+1} > r_t$, and below this locus $r_{t+1} < r_t$.

---

REFERENCES


T. Ihori: Tax Reforms in a Debt Financed Economy: The Role of Expectations


- 263 -