THE MEASUREMENT OF WASTE WITH INCREASING RETURNS TO SCALE*

By ATSUSHI TSUNEKI

1. Introduction

In the presence of increasing returns to scale in production, it is well-known that Pareto optimal equilibria may not be decentralized through perfect competition and moreover, imperfect competition prevails frequently. Therefore, both positive and normative analysis of resource allocation with increasing returns to scale becomes an important topic in applied welfare economics. The normative problem of developing mechanisms to support Pareto optima in the presence of increasing returns to scale has been discussed by many authors, including Arrow and Hurwicz (1960), Guesnerie (1975) and Brown and Heal (1980), where they revealed how difficult it is to obtain Pareto optimal allocations when production possibilities sets are nonconvex. Given this limitation, the second best pricing problem of public utilities facing a revenue constraint is discussed by the optimal pricing and taxation literature beginning with Boiteux (1956) and there have been numerous positive analyses of oligopolistic markets or Chamberlinian (1962) monopolistic competition. In contrast, the measurement of waste due to imperfect competition with increasing returns to scale is a relatively less developed area. The aim of this paper is to consider this measurement of waste problem.1)

In the literature on the measurement of deadweight loss, which includes Debreu (1954), Harberger (1964), Diewert (1981, 1983, 1985), and Kay and Keen (1988), it is assumed that all firms have a convex technology. Therefore, in order to compute the deadweight loss with increasing returns to scale, we must first characterize optimality in a nonconvex economy and extend the work of Debreu, Harberger and Diewert to increasing returns to scale technology to derive a measure of waste which can be evaluated using only local information on preferences and technology, so that the measure is useful in empirical research on the measurement of waste. In this paper, we show that these problems can be solved in a satisfactory way, at least in a simplified model.

Our findings in this paper may be summarized as follows. We can derive a Hotelling-
Harberger type general equilibrium approximate measure of the deadweight loss due to imperfect competition allowing for quite general differentiable functional forms for production and utility functions, including production functions that exhibit increasing returns to scale. This approximate measure can be implemented using local information up to the second order obtained at an observed distorted equilibrium.

In the next section, we construct a model employing the assumptions that production functions are quasiconcave, factor markets are competitive, and the number of firms in the imperfectly competitive sector is fixed. In section 3, we derive an Allais-Debreu-Diewert (ADD) measure of waste for an economy with increasing returns to scale and show that the corresponding optimum equilibrium is characterized by the marginal cost principle. In section 4, we compute a second order approximation to the ADD loss measure, discuss its informational requirements, and show how our measure generalizes previous works on deadweight loss. We also discuss various relaxations of our assumptions and limitations on applying our approach to empirical studies of various market imperfections. Section 5 gives a numerical example and a diagrammatic interpretation of our approximate measures in order to make clear the implications of our approach and the assumptions employed. Section 6 concludes.

2. The Model

We assume that there are \( N \) goods in the economy, where the corresponding price vector is \( p = (p_1, \ldots, p_N)^T \succ 0_N \), and that only sector \( n \) produces the \( n \)-th good for \( n = 1, \ldots, N \) by combining the other goods and \( M \) nonproducible factors. The vector of factor prices is \( w = (w_1, \ldots, w_M)^T \succ 0_M \).

Each production unit is assumed to have a quasi-concave production function \( f^n(x_1, \ldots, x_N, v_1, \ldots, v_M) \); where \( x_i (i = 1, \ldots, N) \) shows the level of intermediate goods and \( v_i (i = 1, \ldots, M) \) shows the level of nonproducible factors; that is, for a given level of output \( y_n \), marginal rates of technical substitution between inputs are diminishing.\(^2\) This assumption is weaker than global convexity in production; the possibility of increasing returns to scale is allowed for when we change the level of output in this characterization.\(^3\) We define the sector \( n \) cost function \( C^n(p, w, y_n) \) as

\[
C^n(p, w, y_n) = \min_{x \succ 0_N, v \succ 0_M} \{ p^T x + w^T v : f^n(x, v) \geq y_n \}, n = 1, \ldots, N.
\]

We assume that the regularity conditions listed in Diewert (1982, p. 554) are satisfied.

There are \( H \) households in this economy with utility functions \( f^h \) and utility levels \( u_h \). Their demands are characterized in terms of the expenditure functions

\[
m^h(p, w, u_h) = \min_{a, b} \{ p^T a + w^T b : f^h(a, b) \geq u_h, (a, b) \in \Omega^h \}, h = 1, \ldots, H,
\]

where \( \Omega^h \) is a (translated) orthant of \( R^{N+M} \) and \( a \) and \( b \) are the consumption of producibles and

\(^2\) For the production function \( f^n(x_1, \ldots, x_N, v_1, \ldots, v_M) \), we assume that \( \partial f^n/\partial x_n \equiv 0 \). Therefore, the cost function \( C^n \) dual to \( f^n \) has the derivative \( \partial C^n/\partial p_n \equiv 0 \).

\(^3\) For example, the increasing returns to scale technology obtained by combining a convex production possibilities set with a large fixed cost can be dealt with within our framework. (See Negishi (1962).) Aoki (1971) also used a similar technological assumption to the one adopted in this paper.
The Economic Studies Quarterly

nonproducibles respectively. We assume that the $h$-th household holds the vector of initial endowments $\bar{Y}^h \equiv (\bar{a}^h, \bar{b}^h)^T$.

To characterize the general equilibrium, we utilize the overspending function $B$ defined by:

$$B(y, p, w, u) = \sum_{h=1}^{H} [m^h(p, w, u_h) - p^T \bar{a}^h - w^T \bar{b}^h] - \sum_{n=1}^{N} [p_n y_n - C^n(p, w, y_n)],$$

where $y \equiv (y_1, \ldots, y_N)^T$ and $u \equiv (u_1, \ldots, u_H)^T$ (see Bhagwati, Brecher and Hatta (1983)).

Definition (3) may be simplified by defining $Q \equiv (p^T, w^T)^T$ as follows:

$$B(y, Q, u) = \sum_{h=1}^{H} [m^h(Q, u_h) - Q^T \bar{Y}^h] - \sum_{n=1}^{N} [p_n y_n - C^n(Q, y_n)].$$

We can derive the following properties for the overspending function: (i) $B$ is concave with respect to $Q$; (ii) if $B$ is once continuously differentiable with respect to prices, $\nabla_Q B(y, Q, u)$ equals the vector of excess demands; and (iii) $B$ is linearly homogeneous with respect to prices. Consequently, the following identities are satisfied:

$$Q^T B_{QQ} = 0_{N + M}'$$

and

$$Q^T B_{Q} = \nabla_y B(y, Q, u)^T,$$

where $B_{QQ} \equiv \nabla_Q^2 B(y, Q, u)$ and $B_{Q} \equiv \nabla_Q B(y, Q, u)$. Note that $-\nabla_y B(y, Q, u)$ is a vector whose $i$-th component is the difference between the price and marginal cost of the $i$-th good.

Now using the above relations, we characterize an imperfectly competitive general equilibrium $(y^1, Q^1, u^1)$ where $Q^1 \equiv (p^1^T, w^1^T)^T$ as follows:

$$m^h(Q^1, u^1_h) = Q^1^T \bar{Y}^h + \sum_{n=1}^{N} e^{hn} [p_n y^1_n - C^n(Q^1, y^1_n)] + g_h, h = 1, \ldots, H,$$

$$-\nabla_y B(y^1, Q^1, u^1) = t,$$

$$-\nabla_Q B(y^1, Q^1, u^1) = 0_{N + M},$$

where $e^{hn}$ is the share of the $n$-th firm held by the $h$-th individual: $\sum_{h=1}^{H} e^{hn} = 1$, for $n = 1, \ldots, N$). The number $g_h, h = 1, \ldots, H$, shows the net lump-sum transfer given to the $h$-th individual and $t \equiv (t_1, \ldots, t_N)^T$, where $t_n$ is the monopolistic mark-up imposed by firm $n$ on its sales.

We can show that (7) and (9) imply that the sum of the transfers $g_h, h = 1, \ldots, H$, equals zero. The equations in (7) are the budget constraints of the $H$ individuals. The equations in (8) state that the difference between the price of the $i$-th good and its marginal cost is equal to the mark-up $t_i$. For perfectly competitive firms $t = 0_N$, but with imperfect competition we expect $t \gg 0_N$. With increasing returns to scale, firms must charge prices larger than their marginal costs in order to attain nonnegative profits. If $t$ is a vector of commodity tax rates then the tax revenue is returned to consumers as a part of government transfers rather than profit dividends. All the results in this paper are valid in this ‘tax-distortion’ case, if appropriate changes are made in equation (7). Noting that $\nabla_Q B$ equals the vector of excess demands, the equations in (9) are the market clearing conditions for the equilibrium. Therefore, (7) to (9) characterize an imperfectly competitive general equilibrium, as elaborated by Negishi (1960-1), Arrow and Hahn (1971, Ch. 6) and Roberts and Sonnenschein (1977), or tax-distorted or publicly regulated general equilib-
3. The Allais-Debreu-Diewert Measure of Waste

Let us first take an \( N + M \) dimensional nonnegative reference bundle of goods and factors \( \lambda \equiv (\alpha^T, \beta^T)^T \geq 0_{N + M} \) and each consumer's utility level \( u^1_h, h = 1, \ldots, H \), in the imperfectly competitive equilibrium, and consider the following primal planning problem:

\[
\begin{align*}
\text{(10) } r^0 & \equiv \max_{r,a^h,b^h,y_n,x^n,v^n} \left\{ r : \right. \\
& \quad \sum_{h=1}^H a_h + \sum_{n=1}^N x^n + ar \leq y + \sum_{h=1}^H a^h, \\
& \quad \sum_{h=1}^H b^h + \sum_{n=1}^N v^n + r \beta \leq \sum_{h=1}^H b_h, \\
& \quad f^h(a^h, b^h) \geq u^1_h, (a^h, b^h) \in \Omega^h, h = 1, \ldots, H, \\
& \quad f^n(x^n, v^n) \geq y_n, n = 1, \ldots, N \}. 
\end{align*}
\]

The solution to (10) defines the Allais-Debreu-Diewert (ADD) measure of waste \( L_{ADD} = r^0 \).

Problem (10) may be interpreted as maximizing the number of multiples \( r \) of the given reference bundle \( \lambda \) that can be obtained while maintaining consumers' utilities at the observed equilibrium levels \( u^1_h, h = 1, \ldots, H \), and satisfying the materials balance and technology constraints. We assume that a finite maximum exists for (10).

We can also derive a dual expression to (10) as follows. First, let us fix \( y = (y_1, \ldots, y_N)^T \). From the definition of quasi-concavity, the sets \( f^n(x^n, v^n) \geq y_n, n = 1, \ldots, N \) are convex sets belonging to \( R_{N+M}^+ \). Then, the remaining programming problem is concave, so that we can rewrite (10) using the Uzawa (1958, p. 34)-Karlin (1959, p. 201, Theorem 7.1.1) Saddle Point Theorem\(^4\) as

\[
\begin{align*}
\text{(11) } r^0 &= \max_{y \geq 0_N} \left[ \max_{Q \geq 0_{N+M}} \min_r \left\{ r(1 - Q^T \lambda) - B(y, Q, u^1) \right\} \right] \\
& \text{using definitions (1), (2), and (4), where } Q \text{ is the vector of Lagrangean multipliers associated with the resource constraints, (i) and (ii). The max-min problem within the square brackets in (11) can be rewritten using the Uzawa-Karlin Theorem in reverse as} \\
& \quad \max_{Q \geq 0_{N+M}} \left\{ B(y, Q, u^1) \text{ s.t. } Q^T \lambda \geq 1 \right\}. \\
& \text{For the given level of } y, \text{ the solution of the max-min problem in (11) is } r(y) \text{ and } Q(y). \text{ Then (11) can also be written as} \\
& \quad \text{(13) } r^0 = \max_{y \geq 0_N} \left\{ r(y)(1 - Q(y)^T \lambda) - B(y, Q(y), u^1) \right\}. \\
\end{align*}
\]

The global programming problems (10) and (11) define the ADD measure of waste when the observed utilities are \( u^1 \), but it is difficult to compute \( r^0 \) using this approach since we need global information on preferences and technologies. To get more insight about the amount of waste in relation to the degree of imperfect competition, and bridge the gap between conventional deadweight loss measures and our ADD measure, we derive a second order approximation to the ADD measure of waste.

\[^4\text{To apply the theorem, we need to assume that the Slater constraint qualification condition holds; that is, we assume that a feasible solution for (10) exists that strictly satisfies the first } N + M \text{ inequality constraints.}\]
For this local analysis, we have to strengthen our assumptions as follows: (i) \((y^0, r^0, Q^0)\) solves (11) with \(y^0 \succ 0_N, Q^0 \succ 0_N + M'\), so that the first order conditions for (11) hold with equality; (ii) the expenditure functions \(m^h, h = 1, \ldots, H\), are twice continuously differentiable with respect to \(Q\) at \((Q^0, u^1_1); (iii) the cost functions \(C^n, n = 1, \ldots, N\), are twice continuously differentiable at \((Q^0, y^0_n); (iv) Samuelson's (1947) strong second order conditions hold for the two problems (12) and (13) when the inequality constraint in (12) is replaced by an equality.

The regularity condition (i) implies that there are no free goods and all firms are useful. Conditions (ii) and (iii) are differentiability assumptions, which are natural for a local analysis such as ours. Condition (iv) is an assumption which guarantees that the maximum of the planning problem (10) is locally unique. It follows from assumption (i) that an interior solution exists to (11).

The first order conditions are given by:

\[
\begin{align*}
\text{(14)} & \quad -\nabla_y B(y^0, Q^0, u^1) = 0_N, \\
\text{(15)} & \quad -\lambda r^0 - \nabla_Q B(y^0, Q^0, u^1) = 0_N + M', \\
\text{and} & \quad (16) \quad 1 - Q^{0T}\lambda = 0,
\end{align*}
\]

where (14) is a marginal cost pricing principle for monopolistic firms, (15) are resource balance equations for goods and factors with \(\lambda r^0 \succeq 0_N\) being the vector of surplus goods and factors, and (16) is a normalization rule for the optimal prices. Our regularity conditions for (12) imply the positive definiteness of the bordered Hessian

\[
C^0 \equiv \begin{bmatrix}
-B^0_{QQ}, & -\lambda \\
-\lambda^T, & 0
\end{bmatrix}
\]

where \(B^0_{QQ} = \nabla^2_y B(y^0, Q^0, u^1)\) and the superscript 0 means that the derivatives are evaluated at the solution \((y^0, Q^0)\) to (11). By defining

\[
\text{(18)} \quad A^0 \equiv -B^0_{yy}
\]

and

\[
\text{(19)} \quad B^0 \equiv [-B^0_{QQ}, 0_N],
\]

our assumption in (iv) that Samuelson's strong second order condition for (13) is met is equivalent to the following condition:

\[
\text{(20)} \quad A^0 - B^0(C^0)^{-1}B^{0T} \text{ is negative definite.}
\]

Note that the formula in (20) is the matrix consisting of the second order derivatives of the maximand of (13) with respect to \(y\) when \(r\) and \(Q\) are functions of \(y\). Condition (20) is much weaker than assuming marginal costs are increasing, which requires \(A^0\) to be negative definite, since \(C^0\) is positive definite. By merely requiring condition (iv), we are admitting the possibility of a downward sloping marginal cost curve, which follows the spirit of Hotelling (1938, pp. 255–

---

5) With increasing returns to scale, a local optimum that satisfies the first order conditions may not be globally optimal. We assume that \((r^0, y^0, p^0, w^0)\) is a global optimum.

6) To derive the formula within (20), first use (15) and (16) to get the change of \(Q\) and \(r\) when \(y\) is adjusted. Next, differentiate the maximand of (20) once with respect to \(y\) to get \(-\nabla_y B\) by applying envelope property. Finally, by differentiating \(-\nabla y B\) with respect to \(y\), and using the result above, we arrive at the required result.
Now comparing the market equilibrium conditions and the first order conditions for the optimum, we implicitly define a z-equilibrium, which depends on a scalar parameter \( z (0 \leq z \leq 1) \) as follows:

\[
\begin{align*}
(21) \quad -\nabla_y B(y(z), Q(z), u^1) &= tz, \\
(22) \quad -\nabla_Q B(y(z), Q(z), u^1) - \lambda r(z) &= 0_{N+M}, \\
(23) \quad 1 - Q(z)^T\lambda &= 0.
\end{align*}
\]

If we define \((y(0), Q(0), r(0)) \equiv (y^0, Q^0, r^0)\), then (21)–(23) coincide with the optimality conditions (18)–(20) when \( z = 0 \). In contrast, if we define \((y(1), Q(1), r(1)) \equiv (y^1, Q^1, 0)\), then (21) and (22) coincide with (8) and (9) respectively when \( z = 1 \). In this case, (7) is also satisfied for an appropriate choice of transfers \( g_h, h = 1, \ldots, H \). From condition (23) at \( z = 1 \), we can also assume that the market prices satisfy the normalization,

\[
(24) \quad 1 = Q^1T\lambda,
\]

by choosing the scale of \( \lambda \) appropriately. Thus we can conclude that the \( z \)-equilibrium (21)–(23) maps the optimal equilibrium into the imperfectly competitive equilibrium as \( z \) is adjusted from zero to one.

The main theorem in this section is:

**Theorem 1:** A second order approximation to the ADD measure of waste (10) is given by

\[
(25) \quad -\frac{1}{2}t^T(A^0 - B^0(C^0)^{-1}B^0T)^{-1}t > 0.
\]

**Proof:** Differentiate (21)–(23) with respect to \( z \) and we have

\[
\begin{bmatrix}
A^z, & B^z, & B^zT, & C^z
\end{bmatrix}
\begin{bmatrix}
y'(z) \\
Q'(z) \\
r'(z)
\end{bmatrix}
= \begin{bmatrix}
t \\
0_{N+M} \\
0
\end{bmatrix},
\]

where \( A^z, B^z, C^z \) are the matrices \( A^0, B^0 \) and \( C^0 \) defined by (18), (19) and (17) evaluated at \( z \), rather than 0. Premultiplying (26) by \([0_N, Q(z)^T, 0] \), we have

\[
(27) \quad -Q(z)^TB^0y'(z) - Q(z)^TB^0Q'(z) - Q(z)^T\lambda r'(z) = 0.
\]

Substituting (5), (6), and (23), and then (21) into (27), we obtain

\[
(28) \quad r'(z) = zT\dot{y}'(z).
\]

Noting that \( r(1) \equiv 0 \), by using a Taylor series expansion the ADD measure of waste \( L_{ADD} = r^0 = r(0) - r(1) \) can be approximated by

\[
(29) \quad r^0 - r^1 \equiv -r'(0) - \frac{1}{2}r''(0).
\]

However, from (28), \( r'(0) = 0 \) and \( r''(0) = t^Ty'(0) \). Therefore, we have

\[
(30) \quad r^0 - r^1 \equiv -\frac{1}{2}t^Ty'(0).
\]

---

7) Increasing returns to scale is usually defined as a more than proportionate increase of output when all the inputs are proportionately increased. Baumol, Panzar and Willig (1982, pp. 18–21) propose a weaker notion of increasing returns to scale, i.e., decreasing average cost, and showed that it is implied by decreasing marginal cost.
Evaluating (26) at $z = 0$ and inverting the left hand side matrix yields $y'(0) = (A_0 - B_0(C_0)^{-1}B_0^T)^{-1}t$. Substituting this expression into (30), the result (25) follows. The inequality in (25) follows from (20). (Q.E.D.)

The formula in (25) gives a general measure of deadweight loss applicable to either a convex or nonconvex economy. This formula is identical to the Debreu (1954)-Diewert (1985) approximate deadweight loss formula when the technologies are convex. However, the converse is not true, since the optimal shadow price $Q^0$ (or intrinsic price to use Debreu's (1951, 1954) term) may not exist with increasing returns to scale. This problem is overcome by our two-stage optimization procedure (11) for the characterization of the optimum, an approach which was suggested by Arrow and Hurwicz (1960) and Guesnerie (1975). Our resulting approximate loss formula (25) is calculated not from the derivatives of supply functions, but from the derivatives of restricted factor demand functions and marginal cost functions evaluated at the optimal level of output.

A drawback of our approach is that the derivatives in (30) are not observable at the distorted observed equilibrium. This shortcoming is somewhat overcome by the following corollary:

**Corollary 1:** The approximate ADD measure

$$-\frac{1}{2}t^TA^{-1} - B^1(C^1)^{-1}B^{1T} - t$$

is also accurate for quadratic functions as (25).

**Proof:** According to Diewert's (1976, p. 118) Quadratic Approximation Lemma, both $-(r'(0) + \frac{1}{2}r''(0))$ and $-\frac{1}{2}(r'(0) + r'(1))$ give the exact value of $r(0) - r(1)$ if $r$ is quadratic. The former was adopted to derive (25). Now using the latter approximation and using (28) we have

$$r^0 - r^1 \approx -\frac{1}{2}t^Ty'(1).$$

Evaluating (26) at $z = 1$, computing $y'(1)$ by inverting the left hand side matrix and substituting it into (32), we get (31). (Q.E.D.)

The remarkable property of (31) is that we can compute the deadweight loss of the economy from the local derivatives of demand and supply (cost) functions evaluated at the observed equilibrium. One important consequence of this observation is that (31) can be computed using flexible functional forms for utility and production functions, so that we need not assume restrictive functional forms to calculate the global optimum point, as is usual in numerical general equilibrium literature.

To derive our approximate loss formulae (25) and (31), we maintained several restrictive assumptions. The assumption of competitive factor markets can be dropped by introducing mark-up rates on factor prices, even though the resulting formulae become more complicated.

The assumption that the production functions must be quasi-concave was required to guarantee the optimality of the marginal cost principle. In section 3 we gave a proof based on programming that the marginal cost pricing principle is necessary for optimality if technologies are quasi-concave. As Arrow and Hurwicz (1960) showed, this condition is not necessary for general nonconvex technology. Suppose that we alternatively consider Guesnerie's (1975) type 3 firm; that is a firm's technology is convex if some input is given. By applying results in Guesnerie (1975, pp. 12-13), it is straightforward to show that optimality is characterized by the competitive maximization of 'restricted' profit given the level of the input which causes the increasing returns to scale, and by the equality of the marginal value product with the factor price. (See also
A. Tsuneki: The Measurement of Waste with Increasing Returns to Scale

Aoki (1971). It could be possible that our approach for the measurement of waste can be applied to an economy including type-3 firms by comparing the optimum with a market equilibrium which includes mark-up rates in either product or factor markets and deriving a deadweight loss measure using the methodology employed above. Though we do not pursue this line any further in this paper, it would be a fruitful area for future research.

We have assumed that each industry is monopolized. It is easy to extend our result to the case of an oligopolistic industry if we know the mark-up rates of firms and assume that the number of firms within each industry is fixed and that the firms have identical technologies. However, it is difficult to introduce entry-exit behaviour into our analysis which utilizes the Implicit Function Theorem, since the first order social optimality and market equilibrium conditions for incumbents and entrants are characterized by inequalities rather than equalities, and the number of firms changes discontinuously as equilibria are adjusted from the observed equilibrium to the optimum. The only case with entry that we can deal with within our framework is a Chamberlinian (1962) monopolistic competition with each product produced by homogeneous producers with respect to market shares, product quality and technology. Suppose also that the number of firms is continuous and no integer problem occurs. Then, the long-run equilibrium is characterized by the zero-profit conditions of firms, i.e., a set of equalities where the number of firms is also endogenous, and Chamberlinian excess capacities cause deadweight loss. The optimality conditions are characterized by the marginal cost pricing principle and the optimum number of firms is determined at the point where the marginal cost equals average cost. However, this model may be incomplete as a monopolistic competition model, since product diversity is exogenous. To make it endogenous, we must work with much more simplified models, as adopted by Spence (1976) and Dixit and Stiglitz (1977).

5. An Example

In this section, we present a simplified model which can be illustrated diagrammatically in order to make the preceding abstract discussion more concrete.

This model which is illustrated in Figure 1 is a single-consumer economy with one good $y$ and one non-producible production factor $v$, labour for example. Total endowment of labour is $v$ and the production function of the aggregate firm is given by $y \leq g(v)$. Assuming for simplicity, that $v = 0$, the production frontier of the economy is described by the curve $OA$ in Figure 1. Note that increasing returns to scale is always compatible with quasi-concavity of the production function in a one-good one-factor model. With uniform increasing returns to scale exhibited by $OA$, a competitive equilibrium cannot exist. However, an imperfectly competitive equilibrium $M$ can exist. The curve $BB$ which cuts $OA$ at point $M$ is the indifference curve for utility level $u^1$ of

---

8) Technological differences among firms creates the problem of differentiated mark-up rates among firms producing a same good, but it seems that appropriate extensions of our approach are possible. This is also delegated to future research.

9) According to the recent study of contestable markets by Baumol, Panzar and Willig (1982), these strategic aspects are immaterial when the fixed cost is not sunk. Since a natural monopoly must set the price equal to its average cost for a sustainable equilibrium, the mark-up rate $m$ equals the difference between the average cost and marginal cost, so that our approach is applicable.
The noncompetitive equilibrium corresponding to the utility function \( f(a, b) \) where \( a \) is the consumption of the single good and \( b \) is the consumption of leisure. The point \( M \) shows that the labour and the good market are cleared, but the marginal rate of substitution between the good and leisure in consumption is different from the rate of substitution in production. The ADD optimum point \( D \) is a point where surplus labour is maximized given production possibilities set and holding the consumer's utility level at \( u_M \). The point \( D \) is characterized by the equality of the marginal rates of substitution in consumption (or marginal benefit of the good) and the marginal rate of substitution in production (or marginal cost of the good). In Figure 2, \( MB \) shows the marginal rate of substitution (marginal benefit) as a function of \( y \). Similarly, \( MC \) shows the marginal rate of technical substitution (marginal cost) as a function of \( y \). Obviously, the optimal output \( y^0 \) is characterized by the intersection of the two curves, whereas the distorted equilibrium output level \( y^M \) is smaller than \( y^0 \).

We now show that the curvilinear triangle \( ABC \) in Figure 2 shows the amount of deadweight loss due to imperfect competition, whereas the two triangles \( ABC' \) and \( ABC'' \) correspond to the two approximations of the true deadweight loss (25) and (31) respectively. Let us first characterize the ADD optimal production plan \( D \) analytically. Given \( u^M \), the point \( D \) is characterized by the solution of the programming problem:

\[
(33) \quad \max_{a, b, y, v} \{ \bar{v} - b - v : y \geq a, f(a, b) \geq u^M, g(v) \geq y \},
\]

i.e., keeping the consumer's utility at \( u^M \) and satisfying both the production constraint and the resource balance of the good, maximize the surplus of labour. Using the Uzawa-Karlin Saddle Point Theorem, (33) may be rewritten as

\[
(34) \quad r = \max_{y \geq 0, p \geq 0} \bar{v} + py - m(1, p, u^M) - C(1, y)
\]

\[ -284 \]
Where $p$ is a Lagrangean multiplier associated with the resource balance constraint of the good. We assume that $y^0 > 0$ and $p^0 > 0$ solve (34) and that Samuelson's strong second order sufficient condition corresponding to (20) is satisfied. The associated first order conditions are

\begin{align}
\text{(35)} & \quad y^0 - \nabla_p m(1, p^0, u^M) = 0, \\
\text{(36)} & \quad p^0 - \nabla_y C(1, y^0) = 0.
\end{align}

The second order conditions are

\begin{align}
\text{(37)} & \quad \nabla_{pp} m(1, p^0, u^M) < 0, \\
\text{(38)} & \quad \nabla_{pp} C(1, y^0) - \{\nabla_{pp} m(1, p^0, u^M)\}^{-1} > 0.
\end{align}

Equation (37) says that the MB curve $\nabla_{pp} m(1, p, u^M)$ is negatively sloped at the point $y^0$, while (38) says that the MC curve $\nabla_{pp} C(1, y)$ cuts the MB curve from below at $y^0$ (see Figure 2). This example clarifies how we incorporated increasing returns to scale into our model.

In contrast, we characterize the distorted equilibrium by

\begin{align}
\text{(39)} & \quad y^M - \nabla_p m(1, p^M, u^M) = 0, \\
\text{(40)} & \quad p^M - \nabla_y C(1, y^M) = t.
\end{align}

Equation (39) says the good market is cleared and (40) defines the monopolistic markup $t$, which is a difference between the price and marginal cost (see Figure 2). We can now show that the curvilinear triangle $ABC$ is the amount of deadweight loss. From Figure 2, $ABC$ is defined as follows:
We are assuming that the firm's profit is transferred to the consumer. Then, from the budget constraints of the consumer at the distorted equilibrium,

\[ ABC = v + p^0 y_0 - C(1, y_0) - m(1, p^0, u^M). \]

This is nothing but the solution of the program (34).

We can also compute second order approximations to the waste \( ABC \) by constructing a \( z \)-equilibrium which corresponds to (35) and (36) when \( z = 0 \) and which corresponds to (39) and (40) when \( z = 1 \) as in the previous section. The results are as follows:

\[ ABC = \frac{t^2}{2} \left( \nabla_y C(1, y_0) - \nabla^2_{yy} m(1, p, u^M) \right)^{-1} \]

where \( (p, y) \) equals \( (p^0, y^0) \) or \( (p^M, y^M) \) for the two cases respectively. As \( AB = t \) and the height of the triangle \( ABC' \) is \( t \left( \nabla_y C(1, y_0) - \nabla^2_{yy} m(1, p^0, u^M) \right)^{-1} \), the area of the triangle is a second order approximation to \( ABC \) using the slope of \( MB \) and \( MC \) at \( y_0 \). An alternative approximation that uses the curvature at \( y_M \) corresponds to \( ABC'' \) in the same way.

We now illustrate our approach with explicit functional forms. We assume the utility function to be Cobb-Douglas, \( f(x, L) = x^c L^{1-c} \), so its associated expenditure function is \( c^{-c}(1-c)^{-1} p^c u^M \). We assume the production function is \( y = v^{1/d} \), \( d < 1 \), so that the associated cost function is \( y^d \). Using (34), (35), and (36) we can explicitly solve for \( r, p^0, y^0 \) as functions of \( \tilde{v}, u^M, c \) and \( d \).
observed equilibrium \((p^M, y^M, u^M, t)\). The values of these variables must be chosen so that they are consistent with the market equilibrium condition (39), (40), and labour market equilibrium. For the feasibility of computation, we fix \(u^M = y^M = 1\) and determine \(v, p^M, t\) endogenously given \(c\) and \(d\) using equations (39), (40), and the labour market equilibrium condition. We find 
\[ v = 2, \quad p^M = \frac{c}{1 - c}, \quad t = \frac{c}{1 - c - d}. \]
Using the data we can calculate both the true value of waste and its approximation. Tables 1–3 show some numerical results for several values of \(c\) and \(d\). Table 1 gives the value of \(y^0\). Comparing \(y^M = 1\) (which is fixed), this table illustrates whether the optimum and the distorted equilibrium are 'far' apart or not. Table 2 gives the true amount of waste while Table 3 gives its approximation (43). As far as this example is concerned, the approximations rather underestimate the true amount of waste, but it is between two to three times at most. When \(y^M\) and \(y^0\) are close, the approximation is especially accurate. In sum, therefore, we may conclude that the approximation is useful as an order of magnitude estimate.

6. Conclusion

This paper has reconsidered the methodology for the measurement of waste due to imperfect competition or taxation in the presence of increasing returns to scale. We have derived the optimality conditions under increasing returns to scale and defined the measure of waste. Then its second order approximations are calculated, which are extensions of Debreu (1954), Harberger (1964), and Diewert (1985) to nonconvex economies. In particular, for the loss measure defined by (31), only information observable at the distorted equilibrium is required to measure the deadweight loss.

Given the limitations and assumptions listed within the paper, we can apply our generalized deadweight loss measure to various models of imperfect competition and to publicly regulated markets when increasing returns to scale are present. We hope that the theoretical foundation provided here for the measurement of waste will stimulate future empirical research and policy evaluation using it.

\(\text{(Seikei University)}\)

REFERENCES


