OPTIMAL MONETARY POLICY WITH FINITE LIFETIMES*

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In a two-period overlapping generations context, postulating the instantaneous social welfare function consisting of utilities of generations who live at that time and defining their discounted sum as the social objective, it is shown that optimal monetary policy depends on income distribution between generations within a period and on relative social marginal weights to the generations, and that, even when income distribution is optimized, i.e., even in the first-best optimum, the optimal money growth rate is not equal to the social discount factor unless the planner attaches to the working generation the relative weight equal to the social discount factor.

1. Introduction

Optimal monetary policy has been intensively analyzed, both theoretically and empirically, since Friedman's (1969) work. Recently Abel (1987) examined the optimal monetary policy in an overlapping generations model populated by two-period-lived individuals and showed that, in the first-best optimum, the optimal monetary policy requires a constant (contractionary) growth rate of money stock equal to the social discount factor. Yakita (1989), using the overlapping generations model, analyzed the effects of dynamic inefficiency on the optimal monetary policy. However, Abel and Yakita postulated the social objective function as the discounted sum of lifetime utility of a representative individual of each generation.

As was pointed out by Calvo and Obstfeld (1988), one of the problems facing the social planner is that of distributing consumption optimally on each date among individuals living in that time. Thus, it seems plausible that the planner would attempt to maximize the sum of instantaneous social utility discounted according to the time at which it is enjoyed. It is well-known that models with the discounted infinite sum of total utility as an objective function have two varieties; total utility per generation and total utility per period. The choice between them is a value judgement, as was shown in the discussion by Samuelson (1958, 1967) and Lerner (1959a, 1959b) and later Asimakopulos (1967, 1968).

In this study, we postulate the instantaneous social welfare function consisting of utilities of individuals who live in that time and define their discounted sum as the social objective function, which may be seen as a generalized form of Lerner's social utility function. The next section presents a model; and the optimal monetary policy is examined in the third section.

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2. Model

Individuals live for two periods, supplying one unit of labor inelastically in the first period and being retired in the second. The population grows at a constant gross rate \( G(>1) \).\(^1\)

The aggregate production function can be written, in intensive form, as \( y_t = f(k_t) \), where \( y_t \) is output/labor ratio, \( k_t \) is capital/labor ratio, \( f'(k_t) > 0 \) and \( f''(k_t) < 0 \).

The preference of a representative individual born in period \( t \) (generation \( t \)) is given as

\[
U_t = u^1(c^1_t, m_t) + u^2(c^2_{t+1})
\]

where \( c^1_t \) and \( c^2_{t+1} \) are his first-period and second-period consumption and \( m_t \) is the real stock of money he holds at the end of the first period.\(^2\)

At the beginning of the first period of his life the individual does not know the interest rate and the inflation rate realized in the next period. Then, based on expectation, he chooses an optimal allocation among immediate consumption, capital and money holdings so as to maximize his utility (1), subject to the budget constraints

\[
w_t = c^1_t + s_t + m_t \quad \text{and} \quad c^2_{t+1} = R^*_{t+1}s_t + \left( \pi^*_{t+1} \right)^{-1}m_t
\]

where \( s_t \) is his capital holdings; \( w_t \) is the wage rate paid in period \( t \); \( \pi^*_{t+1} \) and \( R^*_{t+1} \) are his expectations for the gross inflation rate and the gross interest rate from period \( t \) to period \( t + 1 \). The first order conditions are\(^3\)

\[
u^1_{ct} = u^2_{ct} R^*_{t+1}\quad \text{and} \quad \left[ R^*_{t+1} \pi^*_{t+1} - 1 \right] u^2_{ct} = \pi^*_{t+1} u^1_{m_t}
\]

Since the individual's expectations are not necessarily correct, the consumption plan for the retirement period may not be realized: Then the actual consumption is given as

\[
c^2_t = R_t s_{t-1} + \pi^{-1}_t m_{t-1}
\]

where \( R_t \) and \( \pi_t \) are the realized interest rate and the inflation rate.

The social planner imposes taxes on wage and interest income in each period, the rates of which are defined as the difference between the marginal product of labor and the wage rate paid, \((f(k_t) - k_t f'(k_t)) - w_t\), and the difference between the marginal product of capital and the interest rate paid, \(f'(k_t) - R_t\), respectively. These are lump-sum taxes in the sense that both capital and labor are inelastically supplied in the period in which the tax rates are determined. However, it is noted that the interest income taxes have distortionary effects in general and particularly if, as assumed below, individuals have naive expectations as to the net-of-tax interest rate which they receive in their retirement period. The public consumption expenditure \( g \), the benefit of which is neglected in this study, is financed by these taxes and money issue. Then the social planner's budget constraint is

\[
m_t - G^{-1}\pi^{-1}_t m_{t-1} = g - \left[ f'(k_t) - R_t \right] k_t - \left[ (f(k_t) - k_t f'(k_t)) - w_t \right]
\]

where \( g \) is fixed in per worker terms; and \( m_t \) satisfies that

\[
G^{-1}\mu_t m_{t-1} = \pi_t m_t \quad (\text{the equilibrium condition in the money market})
\]

where \( \mu_t \) denotes the gross growth rate of the nominal aggregate stock of money from period

\[1\] The term "gross" means the usual rate plus one.

\[2\] For the rationale for real balances in the utility function, see McCallum (1983).

\[3\] In condition (3), expectation operators are omitted for expositional simplicity since we will assume naive expectation formation.
t - 1 to period t; that is, \( \mu_t = M_t/M_{t-1} \).

3. Optimization

Define the instantaneous social utility function in period t as \( W(u^1(c^1_t, m_t), u^2(c^2_t)) \). Then the social planner’s objective is to maximize

\[
(7) \sum_{j=t}^{\infty} \beta^{t-j} W(u^1(c^1_j, m_j), u^2(c^2_j))
\]

where \( \beta \) is the social planner’s discount factor for the instantaneous social utility. The social planner’s optimization problem is to choose tax rates and the growth rate of nominal money stock so as to maximize (7) subject to the constraint (5).

We assume in this study that the formation of the individual’s expectation is naive; that is, \( R_{t+1} = R_t \) and \( \pi_{t+1} = \pi_t \).

Then the planner’s problem is to maximize

\[
\sum_{j=t}^{\infty} \beta^{t-j} [W(u^1(c^1_j, m_j), u^2(c^2_j)) + \lambda_j (m_j - G^{-1} \pi_{j-1} m_{j-1} + f(k_j) - w_j - R_j k_j - g)]
\]

where \( \lambda_j \) is the Lagrange multiplier attached to the planner’s budget constraint in period j; \( c^1_j \) and \( c^2_j \) are given in (2) and (4), respectively; and \( k_{j+1} \) is given as

\[
(8) k_{j+1} = G^{-1}s_j \quad \text{(the equilibrium condition in the capital market)}
\]

Assuming the existence and the convergence of the optimal path, the first order conditions require, on the steady state path, that

4) The social utility function is similar with “the direct analogue of Lerner’s model” based on a representative individual’s attitude toward current and future consumption in Gigliotti (1983); in our notation, \( \sum_{j=t}^{\infty} \beta^{t-j} \left[u^1(c^1_j, m_j) + G^{-1} u^2(c^2_j)\right] \). Although Samuelson (1967) assumed away both Böhm-Bawerkian subjective-time preference and social time preference in formulating Bentham-Lerner social utility function, we introduce them as he mentioned (1967, p.277). Okuno and Yakita (1983) examined a case in which the planner supplies public goods and public capital by postulating a similar social objective function, but without money.

On the other hand, under the assumption of time-separable utility function, Abel’s social utility function can be rewritten as

\[
\beta^t \sum_{j=t}^{\infty} \beta^{t-j} [u^1(c^1_j, m_j) + u^2(c^2_{j+1})] = \sum_{j=t}^{\infty} \beta^{t-j} [u^1(c^1_j, m_j) + \beta^{-1} u^2(c^2_j)]
\]

Therefore, Abel’s analysis may be considered as a special case of our model, \( W = u^1 + \beta^{-1} u^2 \), in which the first-best policies are such that \( \mu = \beta \) and \( f(k) = G/\beta \).

5) What we require is only that the expectation about the future is a continuous function of current values. Rogoff (1987) stated that the most implausible feature of the reputational equilibria is that the public’s expectations be discontinuous functions of current values. In a naive expectation context, whatever tax policies the planner announces, workers believe that they will receive interest at the rate that the retired receives. Fischer (1980) showed, using a two-period model, that under such expectations the time inconsistency problem would not arise. Under the assumption of rational expectation, it is known that the planner’s decision problem may exhibit a time inconsistency, which causes the difficulty in implementing the optimal policy rules.

6) We assume in this study that the planner’s tastes do not change over time. The social planner does not set \( c^2_t = 0 \), since the utility of the retired generation partly constitutes the social welfare in period t.
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\[(9a) \quad W_1u_1c - (s_w + R^{-1}\pi^{-1}m_w)(W_1u_1c - W_2u_2R\beta) + \lambda[-1 + \beta G^{-1}s_w(f'(k) - R) + (1 - \beta \pi^{-1}G^{-1})m_w] = 0 \]
\[(9b) \quad W_2u_2cGk - (s_R + R^{-1}\pi^{-1}m_R)(W_1u_1c - W_2u_2R\beta) + \lambda[-k + \beta G^{-1}s_R(f'(k) - R) + (1 - \beta \pi^{-1}G^{-1})m_R] = 0 \]
\[(9c) \quad -W_2u_2^{-2}m - (s_n + R^{-1}\pi^{-1}m_n)(W_1u_1c - W_2u_2R\beta) + \lambda[\pi^{-2}G^{-1}m + \beta G^{-1}s_n(f'(k) - R) + (1 - \beta \pi^{-1}G^{-1})m_n] = 0 \]

where subscripts denote the derivatives with respect to the variable.

We now consider the optimal monetary policy. From (9), we obtain the condition

\[(10) \quad \mu = \beta + R^{-1}\pi G \frac{W_1u_1c}{\lambda} \left(1 - \frac{W_2}{W_1} \right) + \pi GA \frac{W_1u_1c}{\lambda} \left(1 - \frac{W_2}{W_1} \right) \]

where \( A = (ks'_\pi + G^{-1}\pi^{-2}ms'_R)/(m'_s s'_\pi - m'_s s'_R); \ s'_\pi = s, \ s'_R = s_R - ks_w, \ m'_s = m + \pi^{-2}G^{-1}mm_w, \) and \( m_R = m_R - km_w. \) Here we assume that \( A > 0. \)

Then, if both the second and the third terms on the right hand side of (10) are zero, the optimal growth rate of money is the social discount factor, which is the first-best optimum policy obtained by Abel (1987). However, two distortionary factors are here relevant to the optimal monetary policy in general:

(1) While the individual’s marginal rate of substitution between first-period and second-period consumption is equal to the interest rate, i.e., \(-dc_{2t+1}/dc_{1t}(u_1c/u_2c) = R, \) the planner’s marginal rate of substitution is equal to the interest rate multiplied by \( W_1/\beta W_2, \) i.e., \(-dc_{2t+1}/dc_{1t} = R(W_1/\beta W_2). \) Only when the planner attaches to the retired individuals the relative marginal weight equal to the social discount factor, these two rates become equal and the second term on the right hand side of (10) becomes zero.

(2) In each period the social marginal utility of consumption of working individuals is \( W_1u_1c \) and the social marginal utility of consumption of retired individuals is \( W_2Gu_2c, \) which is equal to \( W_2Gu_1cR^{-1} \) in terms of working-period utility. If the distribution of income (consumption) between working and retired individuals within a period is such that \( W_1 = W_2GR^{-1}, \) then the third term on the right hand side of (10) is zero.

The intuitions are as follows: If the planner weights the working individual’s utility heavier and the marginal social utility of consumption of working individuals is greater, then it is optimal to inflate away some of the real income of retired individuals and the growth rate of money will exceed the social discount factor (and vice versa).

4. Concluding Remarks

We have assumed that each individual supplies one unit of labor inelastically, so that the tax on wage income is a lump-sum tax. If lump-sum transfers (taxes) to retired individuals are additionally available in each period, the first-best optimum can be achieved in which the income distribution between working and retired individuals within a period can be optimized, i.e., \( W_1 = W_2GR^{-1}. \) Then we can see that \( f'(k) = G/\beta \) (the Modified Golden Rule) and that \( \pi R = 1, \)

7) We can see that this is the case when \( u_1c \) is sufficiently large.
8) In the steady state, from (6), \( \mu = G\pi. \)
which implies, from (3), that $U_n = 0$ (Friedman's Full Liquidity Rule). Even in the first-best optimum, however, unless the planner attaches to working individuals the relative marginal weight equal to the social discount factor, the optimal growth rate of money does not equal the social discount factor, that is, $\mu = W_1/W_2$.

Finally, if we assume, following Lerner (1959), that $W(u_1, u_2) = u_1 + G^{-1}u_2$ and that $\beta = G$ with normalized initial population, then the condition $W_1 = W_2 GR^{-1}$ becomes $R = \pi^{-1} = 1$, that is, the zero (net) interest rate for the consumption allocation; the condition $f''(k) = G/\beta$ becomes $f''(k) = 1$, that is, Ramsey's Rule for capital accumulation; and $\mu = G$, that is, a constant expansionary growth rate of money stock.

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REFERENCES