DESIRABLE RULES OF MONETARY
COORDINATION AND INTERVENTION AMONG
A LARGE NUMBER OF COUNTRIES
—A New Method to Analyze the N-Country World—*

By SHIN-ICHI FUKUDA

The purpose of this paper is to analyze the appropriate stabilization rules among a large
number of countries. By decomposing the world system into one aggregate and \( N - 1 \)
difference systems, the paper calculates the desirable monetary rules in the \( N \)-country
model. We find that the optimal monetary rule is a combination of the global monetary
rule and the intervention rule. In general, the derived rule depends on the number of
countries and the relative magnitudes of exogenous variances. The paper first analyzes the
symmetric world economy and extends the results to the asymmetric world.

1. Introduction
Nowadays people often claim the existence of misalignments of exchange rates among major
currencies and propose the appropriate policy coordination. However, the present world
economy is confronted with various kinds of disturbances which are closely interlinked among
countries. As a result, the desirable monetary rules and intervention rules are not easily
designed for the policymakers in the actual multicountry world.

The purpose of this paper is to characterize the nature of appropriate stabilization rules among
a large number of countries. In the previous literature, not a few economists have argued that
countries experiencing large monetary disturbances would enjoy more output stability if interest
rates (or exchange rates) were fixed, while countries experiencing large goods market disturb-
ances would achieve more output stability if interest rates (or exchange rates) were flexible.
These arguments were usually made within the framework of a one-, two-, or three-country
model. However, the number of countries which have some significant effects on the actual
world economy is not so small. Even if we restrict our attention to major industrialized
countries, the coordination rules need to be designed among nearly ten countries. In particular,
it is quite vital to see how the optimal coordination rules will change when an additional country
joins in the coordination.

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referee, Nouriel Roubini and seminar participants at Yale University and University of Tokyo.
1) Among others, Boyer (1978) and Turnovsky (1983, 1984) considered the feedback intervention rules
for a small open economy. Their analyses were extended to a two-country model by Buiter and Eaton
(1985), and to a three-country model by Canzoneri (1982) and Marston (1985).

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Needless to say, the main reason why the previous studies neglected this kind of multicountry analysis was the cost of computation. That is, since multi-countries are closely related to each other in the world economy, calculating an explicit form of solution is almost impossible when the number of country is not small.

However, any nonsingular matrix with rank $N$ can be decomposed into $N$ orthogonal vectors. Thus, we can decompose the $N$-country world into $N$ independent subsets. Once the world economy is decomposed into their orthogonal subsets, the computation of solution for each subset is no more difficult. If an explicit form of solution is obtained, we can calculate the explicit form of solution of interdependent countries by transforming the solution of orthogonal subsets into the original matrix.

The method proposed by Aoki (1981) which decomposes two- or three-country model into average and difference systems is the special case of above orthogonalization. Miller (1982), Buiter (1986), Turnovsky (1986), Aoki (1986), and Jones (1987) have applied this Aoki's method to the policy design in the two-country world. This paper generalizes their idea to $N$-country case by decomposing the world system into one aggregate system and $N - 1$ difference systems. General desirable monetary rules are calculated by using this generalized method. It is shown that the optimal monetary feedback rule in each country is composed of two types of rules, that is, the feedback on the world average interest rate (i.e., the global money supply rule) and the feedback on the country specific interest rate or the effective exchange rate (i.e., the intervention rule). Both types of rules depend on the relative magnitudes of exogenous variances. They also depend on the number of countries in general. The analysis is extended to the case that the weight of one country is small and to the case that the economic structure is asymmetric among countries.

The paper proceeds as follows. After describing a basic $N$-country model, the next section presents the method which decomposes the $N$ country world into $N$ independent subsystems. By using this method, we calculate the solution and optimal monetary rule in each interdependent subsystem in Section 3. Transforming the $N$ independent subsystems into the original $N$ interdependent subsystems, we derive the explicit solutions in the interdependent world economy in Section 4 and the optimal monetary rules in Section 5. In Section 6, we extend the analysis to the case that economic structures are asymmetric among countries. In Section 7, we summarize our main results and refer to some possible applications of our method.

2. Basic Framework

A. Model

In order to illustrate the advantage of decomposing the system into one aggregate and $N - 1$
difference systems, we consider a simple $N$-country world. Until Section 6, every parameter is assumed to be symmetric among countries, although disturbances and policy variables may differ among countries. Variables in country $i$ are indicated by superscript $i (i = 1, 2, \ldots, N)$. Subscripts refer to time periods. The model of country $i$ is described as follows:

1. $m_{i}^{t} - p_{i}^{t} = ay_{i}^{t} - bi_{i}^{t} + v_{i}^{t}$
2. $y_{i}^{t} = c(p_{i}^{t} - p_{i}^{t-1}) + w_{i}^{t}$
3. $y_{i}^{t} = -\alpha(i_{i}^{t} - \nu_{i}^{t+1} + p_{i}^{t}) + \beta \sum_{j \neq i} (e_{ij}^{t} - p_{i}^{t} + p_{j}^{t}) + \gamma y_{i}^{t} + \delta \sum_{j \neq i} y_{i}^{t} + u_{i}^{t}$
4. $i_{i}^{t} = i_{i}^{t} + e_{i+1}^{t} - e_{i}^{t}$ for all $j \neq i$,

where $y = \log$ of real output, $p = \log$ of price level, $i =$ nominal exchange rate, $m = \log$ of nominal money supply, $e^{j} = \text{the value of } i\text{-country currency in terms of } j\text{-country currency}$, $u = \text{stochastic disturbance in IS equation}$, $v = \text{stochastic disturbance in money demand}$, $w = \text{stochastic disturbance in aggregate supply}$, and $t-hx_{t+k}$ = conditional expectation of $x_{t+k}$ based on the information set at time $t-h$. The term $\sum_{j \neq i} x_{j}^{t}$ in equation (3) is the summation of $x_{j}^{t}$ from $j = 1$ to $j = N$ except for $j = i$.

Equations of the model are standard. Equations (1), (2), and (4) respectively denote money demand equation, aggregate supply function, and uncovered interest rate parity condition between country $i$ and $j$. Equation (3) describes aggregate demand equation, which depends negatively on domestic real interest rate and positively on real exchange rates as well as domestic and foreign incomes. For simplicity, we assume that the elasticities of real exchange rates and foreign incomes are symmetric for all foreign countries. In Section 6, we examine how the results will change if this symmetric assumption is dropped.

There is an element of arbitrariness in what one assumes about the information set of private agents. For simplicity, we assume that the private agents at time $t$ possess full information on current variables. We denote the variances of $u_{i}^{t}, v_{i}^{t},$ and $w_{i}^{t}$ by $\sigma_{u_{i}^{t}}, \sigma_{v_{i}^{t}},$ and $\sigma_{w_{i}^{t}}$ for all $i = 1, 2, \ldots, N$. We also denote the covariances of $u_{i}^{t}$ and $u_{j}^{t}, v_{i}^{t}$ and $v_{j}^{t},$ and $w_{i}^{t}$ and $w_{j}^{t}$ ($i \neq j$) by $\text{cov}_{u_{i}^{t}}, \text{cov}_{v_{i}^{t}},$ and $\text{cov}_{w_{i}^{t}}$ for all $i = 1, 2, \ldots, N$. Throughout this paper, all stochastic disturbances are assumed to be serially uncorrelated and be independent of other types of disturbances.

The essential results in the following argument will not be altered even if we allow more general stochastic processes.

**B. System Decomposition**

Extending the method of Aoki (1981) to the $N$ country case, we decompose the $N$ interdependent subsystems into the $N$ independent subsystems. For any vector $X = (x_{1}, x_{2}, \ldots, x_{N})$, define

5a. $x^{a} \equiv \sum_{i=1}^{N} x_{i}$ (an aggregate variable of $x$)

5b. $x^{d(q)} \equiv \sum_{i=1}^{N} x_{i} - qx_{i+q+1}$ (a difference $q$ variable of $x$)

For an arbitrary variable $x$, we call a variable $x^{a}$ an aggregate variable of $x$, and a variable $x^{d(q)}$ a difference $q$ variable of $x$. 

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As an objective of the system, we take the minimization of the sum of unconditional output variance of each country, \(^4\) that is, 
\[
\min \sum_{i=1}^{N} \text{Var} (y_i')
\]
By the following proposition, it is shown that the object is reduced to respective minimization of the variance of aggregate output and the variance of difference \(q\) output for \(q = 1, 2, \ldots, N - 1\).

**Proposition.** Suppose that the variance of \(x_i'\) is equivalent for all \(i\). Suppose also that the covariance of two variables (or vectors) \(x_i'\) and \(x_j'\) is equivalent for all \(i \neq j\). Then, if policy instruments can symmetrically affect \(x_i'\) for all \(i\), then minimizing the sum of \(\text{Var} (x_i')\) \((i = 1, 2, \ldots, N)\) is equivalent to minimizing \(\text{Var} (x_a)\) and \(\text{Var} (x_d^{(q)})\) \((q = 1, 2, \ldots, N - 1)\) respectively.

**Proof.** Define:
\[
X_t = (x_{1t}' , x_{2t}' , x_{3t}' , \ldots, x_{Nt}' )^T,
\]
\[
Z_t = (x_{at}' , x_{d(1)}' , x_{d(2)}' , \ldots , x_{d(N-1)}' )^T,
\]
and
\[
A \equiv \begin{pmatrix}
1 & 1 & 1 & 1 & \cdots & 1 \\
1 & -1 & 0 & 0 & \cdots & 0 \\
1 & 1 & -2 & 0 & \cdots & 0 \\
1 & 1 & -3 & \cdots & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
1 & 1 & 1 & 1 & \cdots & 1 - N + 1
\end{pmatrix}
\]
where superscript \(T\) denotes the transpose of the vector or matrix.

Then, by definition, it holds that \(Z_t = AX_t\). Thus,
\[
X_t^T X_t = Z_t^T (A^{-1})^T A^{-1} Z_t = Z_t^T B Z_t,
\]
where \(B = (AA^T)^{-1}\). Since \(B = \text{diag} (1/N, 1/2, 1/6, \ldots , 1/\{q(q-1)\}, \ldots , 1/\{N(N-1)\})\) where \(\text{diag} (\cdot)\) is the diagonal matrix, it holds that:
\[
\sum_{i=1}^{N} \text{Var} (x_i') = \frac{1}{N} \text{Var} (x_a') + \sum_{q=1}^{N-1} \frac{1}{q(q+1)} \text{Var} (x_d^{(q)})
\]
Since the variance of \(x_i'\) is equivalent for all \(i\) and the covariance of \(x_i'\) and \(x_j'\) \((i.e., \text{Cov} (x_i', x_j'))\) is equivalent for all \(i \neq j\), the covariance of \(x_k'\) and \(x_l'\) \((i.e., \text{Cov} (x_k', x_l'))\) is always zero when \(k \neq l\) for all \(k = a\) and \(d(q)\) \((q = 1, 2, \ldots, N - 1)\). Because policy instruments can symmetrically affect \(x_i'\) for all \(i\), this is true after policy instruments are optimally chosen. Hence, minimizing the weighed average of \(\text{Var} (x_k')\) \((k = a\) and \(d(q)\)) is equivalent to minimizing \(\text{Var} (x_k')\) \((k = a\) and \(d(q)\)) respectively.

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\(^4\) Following Aizenman and Frenkel (1985), we could consider the minimization of output deviation from its frictionless level. We could also add the price stability as one of policy objectives. In any cases, the qualitative results in the following analyses would not be altered.
Therefore, minimizing the sum of $\text{Var} \left( x_i \right)$ ($i = 1, 2, \ldots, N$) is equivalent to minimizing $\text{Var} \left( x^a \right)$ and $\text{Var} \left( x^d(q) \right)$ ($q = 1, 2, \ldots, N - 1$) respectively. (Q.E.D.)

A similar decomposition has originally been proposed in Fukuda and Hamada (1988). However, their proof in the appendix was still incomplete and their decomposition could not orthogonalize the $N$-country model.

Because each country is symmetric, the condition that the covariance of $x^i$ and $x^j$ (i.e., $\text{Cov} \left( x^i, x^j \right)$) is equivalent for all $i \neq j$ is satisfied. In particular, since each monetary policy can symmetrically affect each output, the covariance of $x^i$ and $x^j$ (i.e., $\text{Cov} \left( x^i, x^j \right)$) is still equivalent for all $i \neq j$ even after money supply is optimally chosen. Hence, the above proposition states that we can consider the separate minimizations of the variance of aggregate output and the variance of difference $q$ output.

3. Optimal Feedback Rules in Decomposed Systems

A. Analysis of the Aggregate System

Define the system composed of aggregate variables ($\text{say, } x^a$) as the aggregate system. Under the assumption of symmetric parameters, equations (1)-(4) lead to the following aggregate system.

$$m_t^a - p_t^a = a y_t^a - b i_t^a + v_t^a$$
$$y_t^a = c(p_t^a - p_{t-1}^a) + w_t^a$$
$$\left(1 - \delta(N - 1)\right)y_t^a = -\alpha(i_t^a - \delta(N - 1)p_t^a + p_{t+1}^a) + u_t^a$$

Because $e^{ij} = -e^{ji}$, any exchange rate does not appear in the aggregate system. In addition, all endogenous and exogenous difference variables are cancelled out and have no effect on the aggregate system. This indicates that the world price and output levels are independent of exchange rates in our symmetric parameter set. Although Section 6 shows that this type of independence does not hold under the asymmetric parameter world, the above result may be viewed as giving support to McKinnon's (1984) proposal that each country should agree to provide a stable growth rate for the global money stock.

Based on the proposition in the last section, the loss function in the aggregate system is taken as:

$$V^a = \text{Var} \left( y_t^a \right)$$

If the monetary authorities were able to observe all current variables, then they could reduce the expression in (9) to zero. However, it would be unrealistic to assume that the monetary authorities can not only observe but also react to all current variables instantaneously. For example, price index data usually come with some lags, while interest rates are known readily. Thus, following Poole (1970), we consider the case where monetary authorities observe and jointly react to the sum of interest rates, or equivalently the world average interest rate. Specifically, nominal money stock in the aggregate system is assumed to be supplied by the following feedback rule:

$$m_t^a = \pi i_t^a$$

This aggregate money supply may be controlled jointly through a coordination of central banks.
Since \( t_{t+1}^p = t_{t-1}^p = 0 \), equations (6)-(8) and (10) lead to:

(11a) \[ p_t^e = (1/\Omega) \left[ (b + \pi)u_t^e - \alpha v_t^e - [\alpha a + (b + \pi)][1 - \gamma - \delta(N - 1)]w_t^e \right] \]

(11b) \[ y_t^e = (1/\Omega) \left[ c(b + \pi)u_t^e - \alpha c v_t^e + \alpha(1 + b + \pi)w_t^e \right] \]

(11c) \[ i_t^e = (1/\Omega) \left[ (1 + ac)u_t^e + [\alpha + c(1 - \gamma - \delta(N - 1))]v_t^e + [\alpha a - (1 - \gamma - \delta(N - 1))(b + \pi)]w_t^e \right] \]

where \( \Omega = \alpha(1 + ac) + [\alpha + c(1 - \gamma - \delta(N - 1))] (b + \pi) \).

Hence, noting that the variances of \( u_t^e, v_t^e, w_t^e \) are respectively \( N\sigma_u^2 - N(N - 1)\text{cov}_u, N\sigma_v^2 - N(N - 1)\text{cov}_v, \) and \( N\sigma_w^2 - N(N - 1)\text{cov}_w, \) we obtain:

(12) \[ V^a = \frac{N}{\Omega^2} \left[ c^2(b + \pi)^2[\sigma_u^2 + (N - 1)\text{cov}_u] + \alpha^2 c^2[\sigma_v^2 + (N - 1)\text{cov}_v] + \alpha^2(1 + b + \pi)^2[\sigma_w^2 + (N - 1)\text{cov}_w] \right] \]

Differentiating \( V^a \) with respect to \( \pi \) and setting the result to be zero, we obtain the optimal global monetary rule such that:

(13) \[ \pi = -b + \frac{\alpha c[\alpha + c(1 - \gamma - \delta(N - 1))][\sigma_v^2 + (N - 1)\text{cov}_v]}{c(1 + ac)[\sigma_u^2 + (N - 1)\text{cov}_u]} - \frac{\alpha[1 - \gamma - \delta(N - 1) - a\alpha][\sigma_u^2 + (N - 1)\text{cov}_u]}{a(1 - \gamma - \delta(N - 1) - a\alpha)[\sigma_w^2 + (N - 1)\text{cov}_w]} \]

There are two noteworthy characteristics in (13). The first is that the optimal rule depends on the relative magnitudes of variances and covariances. In particular, (i) when the worldwide money demand shocks are dominant, the optimal rule is to peg the world (average) interest rate; (ii) when the worldwide IS shocks are dominant and when the interest rate elasticity of money demand is small, then the global monetarism that proposes a constant world money supply is a proper prescription. Hence, as in Fukuda and Hamada (1988), the idea of Poole (1970) in the closed economy can be generalized to the policy design in stabilizing the global world economy.

The second characteristic is that the optimal global monetary rule \( \pi \) in (13) may depend on \( N \). This dependency can arise through two routes. The first is that the variance of aggregate variable (say, \( u^e \)) places more weight of \( N \) on covariance (say, \( \text{cov}_u \)) than the variance (say, \( \sigma_u^2 \)). This happens unless \( \text{cov}_u = \text{cov}_v = \text{cov}_w = 0 \). In particular, when the number of countries is large enough, variances are dominated by covariances. The second route can arise when an increase in the number of foreign country shifts the domestic demand to the foreign demand. That is, unless both \( \gamma \) and \( \delta(N - 1) \) are constant, an increase in \( N \) will lead to a decline of \( \gamma \) and an increase of \( \delta(N - 1) \) in (13). Hence, unless the propensities of consumption for domestic goods and foreign goods are independent of \( N \), the optimal global monetary rule \( \pi \) comes to depend on \( N \) through \( \gamma \) and \( \delta(N - 1) \).

B. Analysis of the Difference System

Define the system composed of difference \( q \) variable \( x^{d(q)} \) as the difference system \( q \). Recall the definition of difference \( q \) variable, i.e., equation (5). Then, since \( e^{d(q)} = -e^{d(q)} \), it holds that:

\[ \sum_{i=1}^{q-1} \sum_{j \neq q} (e_{t}^{ij} - p_{t}^{ij} + p_{q}^{ij}) = -q \sum_{j \neq q} (e_{t}^{ij} - p_{t}^{ij} + p_{q}^{ij}) = N(e_{t}^{d(q)} - p_{t}^{d(q)}) \]

\[ \sum_{i=1}^{q-1} \sum_{j \neq q} y_{t}^{ij} = -q \sum_{j \neq q} y_{t}^{ij} = -y_{t}^{d(q)} \]
and
\[ \sum_{i=0}^{q} \sum_{j=1}^{N} \left( i_t^q - i_t^q - \epsilon_{t+1}^q + \epsilon_{t}^q \right) - q \sum_{j=1}^{N} \left( i_t^q - i_t^q - \epsilon_{t+1}^q + \epsilon_{t}^q \right) \]
\[ = N \left( i_t^q - i_t^q + \epsilon_{t+1}^q + \epsilon_{t}^q \right), \]
where \( \epsilon_{t+1}^q \equiv \frac{1}{N} \left[ \sum_{h=1}^{N} \left( \sum_{j \neq h} \epsilon_{t+1}^j \right) - q \left( \sum_{j \neq h} \epsilon_{t+1}^j \right) \right] = \sum_{h=1}^{N} \epsilon_{t+1}^h q. \]

Hence, the model of the difference system \( q \) is described as follows:

(14) \( m_{t}^q = p_{t}^q = ay_{t}^q - bi_{t}^q + v_{t}^q \)

(15) \( y_{t}^q = c(p_{t}^q - t - 1 p_{t}^q) + w_{t}^q \)

(16) \( (1 - \gamma + \delta) y_{t}^q = -\alpha(i_{t}^q - \epsilon_{t+1}^q + p_{t}^q) + \beta N(e_{t}^q - p_{t}^q) + u_{t}^q \)

(17) \( i_{t}^q = \epsilon_{t+1}^q - e_{t}^q \)

The formal structure of the model is similar to that of a small open economy and is independent of \( q \). Because of this independency, we can treat \( N - 1 \) difference systems symmetrically in the following analysis.

Based on the proposition in the last section, the loss function in the difference system \( q \) is taken as:

(18) \( V^q = \text{Var} \left( y_{t}^q \right) \)

Here again, it would be unrealistic to assume that monetary authorities can not only observe but also react to all current variables instantaneously. Thus, we consider the case where the monetary authorities observe and jointly react to interest rate and exchange rate in the difference \( q \) system. Specifically, nominal money stock in the difference \( q \) system is assumed to be supplied by the following feedback rule:

(19) \( m_{t}^q = \rho i_{t}^q + \lambda e_{t}^q \)

However, since \( \epsilon_{t+1}^q = 0 \) when exogenous disturbances are serially uncorrelated, (17) leads to \( i_{t}^q = -e_{t}^q \). Hence, the monetary rule (19) can be written as:

(20) \( m_{t}^q = \mu i_{t}^q \)

where \( \mu \equiv \rho - \lambda \).

Since \( \epsilon_{t+1}^q = 0 \), equations (14)-(16) and (20) lead to:

(21a) \( p_{t}^q = (1/\Gamma)[(b + \mu)u_{t}^q - (\alpha + \beta N)v_{t}^q - (\alpha + \beta N)a + (b + \mu)(1 - \gamma + \delta)]w_{t}^q \)

(21b) \( y_{t}^q = (1/\Gamma)[c(b + \mu)u_{t}^q - (\alpha + \beta N)c v_{t}^q + (\alpha + \beta N)(1 + b + \mu)w_{t}^q] \)

(21c) \( i_{t}^q = (1/\Gamma)[(1 + ac)u_{t}^q + (\alpha + \beta N + c)(1 - \gamma + \delta)]v_{t}^q \)

where \( \Gamma = (\alpha + \beta N)(1 + ac) + ((\alpha + \beta N) + c(1 - \gamma + \delta))(b + \mu) \). Hence, noting that the variances of \( u_{t}^q, v_{t}^q, \) and \( w_{t}^q \) are respectively \( q(q + 1)(\sigma_u^2 - \text{cov}_u), q(q + 1)(\sigma_v^2 - \text{cov}_v), \) and \( q(q + 1)(\sigma_w^2 - \text{cov}_w) \), we obtain:

(22) \( V^q = \left[q(q + 1)/\Gamma^2\right]c^2(b + \mu)^2(\sigma_u^2 - \text{cov}_u) + (\alpha + \beta N)^2 c^2(\sigma_v^2 - \text{cov}_v) + (\alpha + \beta N)^2(1 + b + \mu)^2(\sigma_w^2 - \text{cov}_w) \)

Because the variance of \( x_{t}^q \) is \( q(q + 1)(\sigma_x^2 - \text{cov}_x) \) for all \( x = u, v, \) and \( w \), the variance term
Differentiating $V^q$ with respect to $\mu$ and setting the result to be zero, we obtain the optimal monetary rule such that:

$$
(\alpha + \beta N)[c((\alpha + \beta N) + c(1 - \gamma + \delta))(\sigma^2 - \text{cov}_u) + ((1 - \gamma + \delta) - a(\alpha + \beta N))(\sigma^2 - \text{cov}_u)]
$$

$$
(23) \quad \mu = -b + \frac{c(1 + ac)(\sigma^2 - \text{cov}_u)}{(1 + ac)(\sigma^2 - \text{cov}_u)}
$$

$$
- (\alpha + \beta N)((1 - \gamma + \delta) - a(\alpha + \beta N))(\sigma^2 - \text{cov}_u)
$$

The optimal rule (23) has three noteworthy characteristics. The first is that $\mu$ is independent of $q$. That is, the optimal rule is exactly the same for all difference system $q(q=1, 2, \ldots, N - 1)$. The second characteristic is that the optimal rule in the difference system depends on the relative magnitudes of variances of difference variables. In particular, (i) when variance of money demand shocks is dominant in the difference system, the optimal rule is to peg the difference interest rate; (ii) When variance of IS shocks is dominant in the difference system and when the interest rate elasticity of money demand is small, then no intervention is a proper prescription. Hence, as in Fukuda and Hamada (1988), the analogy of Poole (1970) holds for variances in the difference system. The third characteristic is that $\mu$ in (23) depends on $N$. This dependency happens because the number of countries may affect the elasticities of aggregate demand. That is, since an increase in $N$ generally leads to declines of $\beta, \gamma$, and $\delta$, the changes in the number of countries will affect the optimal rule through the changes in the relative price elasticities $\beta N$ and relative income elasticities $1 - \gamma + \delta$.

4. An Explicit Solution of the N-Country Model

Transforming one aggregate and $N - 1$ difference systems into the original $N$-country world, we can obtain the explicit solution in the $N$-country world. That is, since $X_t = A^{-1}Z_t$ where $X_t \equiv [x^1_t, x^2_t, \ldots, x^N_t]$ and $Z_t \equiv [x^{d(1)}_t, x^{d(2)}_t, \ldots, x^{d(N-1)}_t]$, it holds that:

$$
(24a) \quad x^1_t = \frac{1}{N} x^1_t + \sum_{q=1}^{N-1} \left( \frac{1}{q(q+1)} x^{d(q)}_t \right)
$$

$$
(24b) \quad x^k_t = \frac{1}{N} x^1_t - \frac{1}{k} x^{d(k-1)} + \sum_{q=k}^{N-1} \left( \frac{1}{q(q+1)} x^{d(q)}_t \right) \quad \text{when} \ 2 \leq k \leq N - 1,
$$

$$
(24c) \quad x^{N}_t = \left( \frac{1}{N} \right) (x^1_t - x^{d(N-1)}_t).
$$

Hence, equations (11a, b) and (21a, b) with the definition (5) lead to the explicit solution of the $k$-th country ($k=1, 2, \ldots, N$) such that:

$$
(25a) \quad p^{d(k)}_t = (1/\Omega)[(b + \pi)u^{d(k)}_t - acv^{d(k)}_t - [aa + (b + \pi)(1 - \gamma - \delta(N - 1))]w^{d(k)}_t
$$

$$
+ (1/\Gamma)[(b + \mu)u^{d(k)}_t - (\alpha + \beta N)v^{d(k)}_t - [(\alpha + \beta N)a + (b + \mu)(1 - \gamma + \delta)]w^{d(k)}_t],
$$

$$
(25b) \quad y^{d(k)}_t = (1/\Omega)[c(b + \pi)u^{d(k)}_t - acv^{d(k)}_t + \alpha(1 + b + \pi)w^{d(k)}_t]
$$

$$
+ (1/\Gamma)[c(b + \mu)u^{d(k)}_t - (\alpha + \beta N)v^{d(k)}_t + (\alpha + \beta N)(1 + b + \mu)w^{d(k)}_t],
$$

$$
(25c) \quad i^{d(k)}_t = (1/\Omega)((1 + ac)u^{d(k)}_t + [\alpha + c(1 - \gamma - \delta(N - 1))]v^{d(k)}_t
$$

$$
+ [aa - (1 - \gamma - \delta(N - 1))]w^{d(k)}_t) + (1/\Gamma)((1 + ac)u^{d(k)}_t + [\alpha + \beta N + c(1 - \gamma + \delta)]v^{d(k)}_t + [(\alpha + \beta N) - (1 - \gamma + \delta)]w^{d(k)}_t].
$$
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where \( u^{av}_t = u_t^i / N \), \( v^{av}_t = v_t^i / N \), \( w^{av}_t = w_t^i / N \), \( u^d_t = u_t^i - u^{av}_t \), \( v^d_t = v_t^i - v^{av}_t \), and \( w^d_t = w_t^i - w^{av}_t \). The optimal feedback rules \( \pi \) and \( \mu \) are those defined in equations (13) and (23).

Equations (25a, b, c) state that price level, real output, and interest rate in the country \( k \) are affected by two types of shocks. One is average worldwide shocks such as \( u^{av}_t \). The effects of these shocks are common for all countries and are quite similar to those in a closed economy model. However, they may depend on the number of countries because the structural parameters, especially the elasticities of aggregate demand, generally depend on the number of countries. The other types of shocks are country-specific shocks such as \( u^{dk}_t \). They are deviations from world average shocks. The effects of these shocks are quite similar to those in a small open economy. However, they also depend on the number of countries.

5. Optimal Monetary Rules in N-Country Model

A. An Explicit Form of Optimal Monetary Rules

Since \( m^i_t = \pi i^i_t \) and \( m^{d(q)}_t = \mu i^{d(q)}_t \), the retransformation formula in (24) lead to the optimal monetary rule of the \( k \)-th country (\( k = 1, 2, \ldots, N \)) such that:

\[
(26) \quad m^k_t = \pi i^{av}_t + \mu (i^k_t - i^{av}_t)
\]

where \( i^{av}_t \equiv i^i_t / N \).

Equation (26) states that the optimal monetary rule in the \( k \)-th country is composed of two types of rules. The first type of rule is the feedback rule on the world average interest rate \( i^{av}_t \). This type of rule contributes to the desirable supply of global money stock. An explicit form of \( \pi \) is given by equation (13), which minimizes the output variance in the aggregate system, that is, \( \text{Var} (y^i_t) \).

The second type of rule is the feedback on the country specific interest rate. When \( \mu \) is positive, the feedback rule is chosen so as to stabilize the deviation of the country-specific interest rate from the world average interest rate. When \( \mu \) is negative, the optimal feedback rule is to widen the deviation of the country-specific interest rate. In general, the optimal value of \( \mu \) can be positive or negative depending on the relative magnitudes of exogenous variabilities and parameters. An explicit form of \( \mu \) is given by equation (23), which minimizes the output variance in the difference system \( q \), that is, \( \text{Var} (y^q_t) \). It is noteworthy that this feedback rule is common for all difference systems \( q (q = 1, 2, \ldots, N - 1) \).

Since \( i^k_t - i^{av}_t = \sum_{j \neq k} e^j_t \), the optimal monetary rule (26) can be rewritten as:

\[
(27) \quad m^k_t = \pi i^{av}_t + \mu \sum_{j \neq k} e^j_t
\]

Equation (27) states that the second type of monetary rule (26) can be reinterpreted as the intervention rule on the effective exchange rate \( \sum_{j \neq k} e^j_t \). This optimal intervention rule on the effective exchange rate is formally equivalent to the optimal feedback rule on the country specific interest rate.

B. Optimal Rules for a “True” Small Open Economy

The previous sections have derived the optimal monetary rules in the \( N \)-country world. This
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section investigates how these rules will change when the weight of one country is negligible in the world economy. Since the effect of each country on the world economy is negligible in this case, the following analysis may have some analogy with the previous studies based on a small open economy. However, our analysis is distinct from the previous ones because our model endogenously determines all macro variables in the world economy.

Since the weight of one country becomes negligible when \( N \to +\infty \) in our model, the following object is to derive the optimal rules \( \pi \) and \( \mu \) when \( N \to +\infty \). However, since some structural parameters may change as \( N \) increases, the property of the optimal rules among an infinite number of countries may depend on the assumptions on how the structural parameters will change as \( N \) increases. Hence, we here assume that all parameters are bounded even when \( N \to +\infty \). We also assume that \( \delta(N - 1) \to D < +\infty \) as \( N \to +\infty \). The second assumption implies that the elasticity of domestic aggregate demand for the total foreign incomes is bounded because the elasticity for one foreign income (i.e., \( \delta \)) decreases as \( N \to +\infty \).

Under these assumptions, the optimal rules among an infinite number of countries depend only on how \( \beta N \) will change as \( N \to +\infty \). We first consider the case that \( \beta N \to +\infty \) as \( N \to +\infty \). This case will happen when the elasticity of aggregate demand for the domestic price goes to infinity as \( N \to +\infty \), or equivalently when foreign goods becomes perfectly substitutable for home goods as the number of foreign goods becomes infinite. In this case, the optimal monetary rule in each country converges to:

\[
(28) \quad m_t^k = \pi^o i_t^o + \mu^o (i_t^o - i_t^o)
\]

where

\[
\pi^o = -b + \{ac[a + c_1(1 - \gamma - D)] \text{cov}_v + a(1 - \gamma - D - a) \text{cov}_u\}/\{c(1 + ac) \text{cov}_u
\]

and

\[
\mu^o = -b + \{c(a^2_2 - \text{cov}_v) - a(a^2_2 - \text{cov}_u)\}/\{c(a^2_2 - \text{cov}_u)\}.
\]

The optimal rule (28) has two noteworthy characteristics. One is that the optimal feedback rule on the world average interest rate depends on the cross-country correlations of disturbances but is independent of the variances of disturbances. The reason is that since \( x^a = N \sigma_x^2 + N(N - 1) \text{cov}_x \) for all \( x \), the covariance term dominates the variance term when the number of countries goes to infinity. Hence, when the weight of one country is small enough, the contribution of each policymaker in stabilizing the world economy depends only on how the exogenous disturbances have correlations across countries. The second characteristic is that when the number of countries is large enough, the optimal feedback rule on the country specific interest rate becomes independent of IS shocks. The reason is that when the number of countries goes to the infinity, the effect of the real exchange rate in the IS equation dominates the IS disturbance \( u \).

The first characteristic still holds even if \( \beta N < +\infty \) as \( N \to +\infty \). That is, even if the elasticity of aggregate demand for the domestic income is bounded, the optimal policy in stabilizing the world economy is determined only through the cross-country correlation structure of exogenous disturbances for each small country. However, the second characteristic no longer holds when \( \beta N < +\infty \) as \( N \to +\infty \). In this case, the optimal feedback rule on the country specific interest rate becomes similar to that under the finite number of countries. Since \( \beta N < +\infty \) as
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\( N \to +\infty \) as long as each resident has a special preference for his home goods regardless of the number of foreign country, this implies that the feedback on the country specific interest rate depends on how the preferences for his home goods will change as the number of foreign countries increases.

6. The Effects of Asymmetry

A. Model

Until now, we have focused on the multi-country world when the structural parameters in each country are symmetric. The purpose of this section is to extend these analyses to the case that the structural parameters in each country are asymmetric. In order to make the analysis tractable, we here investigate the two country world where their structural parameters are different.

Consider the following two country world composed of home and foreign countries. Variables with asterisks over them are foreign variables.

\[
\begin{align*}
(29a) \quad m_t - p_t &= a_1 y_t - b_1 i_t + v_t \\
(29b) \quad m^*_t - p^*_t &= a_2 y^*_t - b_2 i^*_t + v^*_t \\
(30a) \quad y_t &= c_1 (p_t - t_{-1} p_t) + w_t \\
(30b) \quad y^*_t &= c_2 (p^*_t - t_{-1} p^*_t) + w^*_t \\
(31a) \quad y_t &= \alpha_1 (i_t - t_{-1} p_t) + \beta_1 (e_t - p_t + p^*_t) + \gamma_1 y_t + \delta_1 y^*_t + u_t \\
(31b) \quad y^*_t &= \alpha_2 (i^*_t - t_{-1} p^*_t) - \beta_2 (e_t - p_t + p^*_t) + \gamma_2 y^*_t + \delta_2 y_t + u^*_t \\
(32) \quad i_t &= i^*_t + e_{t+1} - e_t
\end{align*}
\]

When the parameters in two countries are completely symmetric, the solutions and optimal monetary rules are equivalent to the case of \( N = 2 \) in the previous sections. This section considers how the solution under symmetric parameters will change when the parameters among two countries are different. For an arbitrary parameter \( k_i = a_i, b_i, c_i, a_i, \beta_i, \gamma_i \) and \( \delta_i \) when \( i = 1 \) and \( 2 \), define an average parameter by:

\[
(33a) \quad k \equiv (k_1 + k_2)/2,
\]

and the degree of asymmetry by:

\[
(33b) \quad \Delta k \equiv k_1 - k,
\]

where \( k_1 \) is a parameter of home country and \( k_2 \) a parameter of foreign country. By definition, it holds that \( k_2 - k = -\Delta k \).

We assume that for any \( k = a, b, c, \alpha, \beta, \gamma \) and \( \delta \), the absolute value of \( \Delta k \) is small enough to neglect the second-order effect of the asymmetry. That is, we assume that \( \Delta k \Delta h = 0 \) for any arbitrary parameters \( k \) and \( h \). For example, if \( k = a \) and \( h = b \) (where \( a_1 = a + \Delta a, a_2 = a - \Delta a, b_1 = b + \Delta b, b_2 = b - \Delta b \)), then it holds that \( \Delta a \Delta a = \Delta b \Delta b = \Delta a \Delta b = 0 \) by the assumption.

Under this assumption, the structural perturbation method proposed by Aoki (1981) can be applied to solve our model. That is, the solution under asymmetric parameters can be obtained as a deviation (or perturbation) from the solution under symmetric parameters.

For an arbitrary endogenous variable \( z \), define \( z^t \) as the variable \( z \) under symmetric parameters and \( \Delta z \) as a deviation of \( z \) from \( z^t \). For example, if \( y_t \) is real output under asymmetric parameters and \( y^*_t \) is real output under symmetric parameters, then the definition implies that \( \Delta y_t = y_t - y^*_t \).
Noting that \( t_{p_{t+1}} = t_{-1}p_t = e_{t+1} = 0 \), equations (29)-(32) approximately lead to:

\[
\begin{align*}
(34a) \quad \Delta m_t &- \Delta p_t = a \Delta y_t - b \Delta i_t + \Delta ay_t^* - \Delta bi_t^* \\
(34b) \quad \Delta m_t^* - \Delta p_t^* = a \Delta y_t^* - b \Delta i_t^* + \Delta ay_t^* + \Delta bi_t^*
\end{align*}
\]

\[
\begin{align*}
(35a) \quad \Delta y_t &- \Delta p_t = a \Delta y_t - b \Delta i_t + \Delta ay_t^* - \Delta bi_t^* \\
(35b) \quad \Delta y_t^* &- \Delta p_t^* = a \Delta y_t^* - b \Delta i_t^* + \Delta ay_t^* + \Delta bi_t^*
\end{align*}
\]

\[
\begin{align*}
(36a) \quad (1 - \delta) \Delta y_t &- (1 + \delta) \Delta i_t + \beta(\Delta e_t - \Delta p_t) + \delta \Delta y_t^* \\
&- \Delta a(i_t^* + p_t^*) + \Delta \beta(e_t - p_t^* + p_t^*) + \Delta \gamma y_t^* + \Delta \delta y_t
\end{align*}
\]

\[
\begin{align*}
(36b) \quad (1 - \delta) \Delta y_t^* &- (1 + \delta) \Delta i_t^* + \beta(\Delta e_t - \Delta p_t) + \delta \Delta y_t \\
&+ \Delta a(i_t^* + p_t^*) + \Delta \beta(e_t - p_t^* + p_t^*) - \Delta \gamma y_t + \Delta \delta y_t
\end{align*}
\]

\[
\begin{align*}
(37) \quad \Delta i_t &- \Delta i_t^* = \Delta e_t
\end{align*}
\]

where variables with superscript \( s \) respectively denote those under symmetric parameters, which are derived in the previous sections.

The structures of above equations are formally similar to those of equations under the symmetric structures. Thus, we again apply the decomposition method and solve the model.

### B. Decomposed Systems

We first consider the aggregate system. Define \( \Delta z^a = \Delta z + \Delta z^* \). Then, noting that \( e_t^* = -i_t^* \), equations (34)-(37) lead to:

\[
\begin{align*}
(38) \quad \Delta m_t^* - \Delta p_t^* &- a \Delta y_t^* - b \Delta i_t^* + \Delta ay_t^* - \Delta bi_t^* \\
(39) \quad \Delta y_t^* &- \Delta p_t^* = a \Delta y_t^* + \Delta ay_t^* - \Delta bi_t^* \\
(40) \quad (1 - \gamma + \delta) \Delta y_t^* &- a(\Delta i_t^* + \Delta p_t^*) - (\Delta a + 2 \Delta \beta)(i_t^* + p_t^*) + (\Delta y - \Delta \delta)y_t^*
\end{align*}
\]

where variables with superscript \( d \) are those of difference system under symmetric parameters (that is, the case that \( N = 2 \) in Section 3-B).

Hence, equations (38)-(40) lead to:

\[
\begin{align*}
(41) \quad \Delta y_t^* &- \frac{(1/\Omega)[\alpha \Delta m_t^* + \{\alpha(1 + b) \Delta c - bc(\Delta a + 2 \Delta \beta)\}p_t^* \\
&\quad - c(\Delta a + \beta(\Delta a - \Delta \gamma))y_t^* + c(\Delta a + b(\Delta a + 2 \Delta \beta)) \Delta y_t^* }] \\
(42) \quad \Delta i_t^* &- \frac{(1/\Omega)[-\{\alpha + (1 - \gamma + \delta)c\} \Delta m_t^* \\
&\quad + ((a - 1 + \gamma - \delta) \Delta c + (1 + ac)(\Delta a + 2 \Delta \beta))p_t^* \\
&\quad + ((a - 1 + \gamma + \delta)c \Delta a + (1 + ac)(\Delta a + 2 \Delta \beta)) \Delta y_t^* \\
&\quad + (-[\alpha + (1 - \gamma + \delta)c] \Delta b - (1 + ac)(\Delta a + 2 \Delta \beta)) i_t^* }] \end{align*}
\]

where \( \Omega = \alpha(1 + ac) + b(\alpha + (1 - \gamma + \delta)c) \).

It is quite noteworthy that except for the effect of exogenous money supply (i.e., \( \Delta m_t^* \)), each solution depends only on difference variables such as \( i_t^* \) and \( p_t^* \), but is independent of aggregate variables such as \( i_t \) and \( p_t \). Since the difference variables in Section 3-B depend only on difference shocks, this implies that the asymmetric parameters affect aggregate variables if and only if difference shocks such as \( u_t^* \), \( v_t^* \) and \( w_t^* \) are not negligible. This is in marked contrast with the solutions in Section 3-A because aggregate variables depend only on worldwide shocks under the symmetric parameters.

We next focus on the solution of the difference system. Define \( \Delta z^d = \Delta z + \Delta z^* \). Then,
noting that \( \Delta e_t = -\Delta i_t^d \), equations (34)-(37) lead to:

\[
(43) \quad \Delta m_t^d - \Delta p_t^d = a \Delta y_t^d - b \Delta i_t^d + \Delta a y_t^d - \Delta b i_t^d
\]

\[
(44) \quad \Delta y_t^d = c \Delta p_t^d + \Delta c p_t^d
\]

\[
(45) \quad (1 - \gamma + \delta) \Delta y_t^d = -\alpha(\Delta i_t^d + \Delta p_t^d) - \Delta \alpha(i_t^d + p_t^d) + (\Delta y_t^d + \Delta \delta) y_t^d
\]

Thus, equations (43)-(45) lead to:

\[
(46) \quad \Delta y_t^d = \frac{1}{\gamma}(ac \Delta m_t^d + \{\alpha(1 + b) \Delta c - bc \Delta a\} p_t^d - c[a \Delta a + b(\Delta y + \Delta \delta)] y_t^d + c(a \Delta b - b \Delta a) i_t^d]
\]

\[
(47) \quad \Delta i_t^d = \frac{1}{\gamma}[\{\alpha + (1 - \gamma + \delta)c\} \Delta m_t^d + \{(aa - 1 + \gamma - \delta) \Delta c + (1 + ac) \Delta a\} p_t^d + \{(a + (1 - \gamma + \delta)c \Delta a + (1 + ac)(\Delta y + \Delta \delta)) y_t^d + (-[\alpha + (1 - \gamma + \delta)c] \Delta b - (1 + ac) \Delta a) i_t^d]
\]

Contrary to the solution of aggregate variables, the above solutions depend only on aggregate variables such as \( i_t^d \) and \( p_t^d \) except for the effect of exogenous money supply \( \Delta m_t^d \).

That is, the asymmetric parameters affect difference variables if and only if worldwide shocks such as \( u_t^d, v_t^d \) and \( w_t^d \) are not negligible. This result is noteworthy because difference variables depend only on difference shocks under the symmetric parameters.

C. The Explicit Solution and Optimal Rules

Transforming one aggregate system and one difference system into the original two-country world, we can obtain the solutions in two-country world. That is, since \( \Delta z = (\Delta z^a + \Delta z^d)/2 \) and \( \Delta z^* = (\Delta z^a - \Delta z^d)/2 \), it holds that:

\[
(48) \quad \Delta y_t = (\Delta y_t^a + \Delta y_t^d)/2, \quad \Delta y_t^* = (\Delta y_t^a - \Delta y_t^d)/2,
\]

\[
(49) \quad \Delta i_t = (\Delta i_t^a + \Delta i_t^d)/2, \quad \Delta i_t^* = (\Delta i_t^a - \Delta i_t^d)/2,
\]

where \( \Delta y_t^a \) and \( \Delta i_t^a \) are given by (41)-(42), and \( \Delta y_t^d \) and \( \Delta i_t^d \) are by (46)-(47).

These equations state that asymmetric parameters change the solution under the symmetric parameters into two directions. First, when there exist some difference shocks, the asymmetric parameters will affect aggregate variables and change the solution of each country into the exactly same direction. Secondly, when there exist some worldwide shocks, the asymmetric parameters will affect difference variables and make the solution of each country diverge into two opposite directions.

In a case of the nominal exchange rate, it holds that \( \Delta e_t = -\Delta i_t^d \) where \( \Delta i_t^d \) is defined by (47). Hence, except for the effect of money supply (i.e., \( \Delta m_t^d \)), the asymmetric parameters will affect the exchange rate if and only if there exist some worldwide shocks in the economy. This result is noteworthy because under the symmetric parameters, exchange rate is independent of any worldwide shocks.

By using the above explicit solution under asymmetric parameters, we can obtain the optimal monetary rules under asymmetric parameters. An objective of the system is to choose the money supply so as to minimize the sum of the output variance of each country. If we define \( L = \text{Var}(y_t) + \text{Var}(y_t^*) \), the optimal rules are derived by setting \( \partial L/\partial m_t = \partial L/\partial m_t^* = 0 \).

Recall that \( y_t = y_t^a + \Delta y_t, y_t^* = y_t^{a*} + \Delta y_t^*, m_t = m_t^a + \Delta m_t, \) and \( m_t^* = m_t^{a*} + \Delta m_t^* \). Because \( E[(y_t^a)(\partial y_t^a/\partial m_t)] + (y_t^{a*})(\partial y_t^{a*}/\partial m_t^*) | I_t] = 0, \partial y_t^a/\partial m_t = \partial \Delta y_t/\partial \Delta m_t, \) and \( \partial y_t^{a*}/\partial m_t^{a*} = \partial \Delta y_t^* \),
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\[ \partial \Delta m^*, \text{ the optimal rules satisfy the following condition:} \]

\[ (50) \ E[(\partial Ay)^\prime(y)(\partial Ay)(\partial m)|I_t] = 0 \]

where \( I_t \) is the policymaker’s information set at time \( t \). Since \( Ay \) is independent of \( Ay^\prime \), this condition is equivalent to:

\[ (51) \ E[(\partial Ay)^\prime(y)(\partial Ay)(\partial m)|I_t] = 0 \text{ and } E[(\partial Ay)^\prime(y)(\partial m)|I_t] = 0. \]

Now assume that contemporaneous information in \( I_t \) is the exchange rate and nominal interest rates. Then, since \( e_t = -i_t \), the monetary rule can be written as \( \Delta m_t = A_i i_t \) and \( \Delta m_t = A_i d_t \).

Hence, the conditions in (51) lead to:

\[ (52a) \ \theta = -[1b - (b/a)(Aa + 2 A\beta)] + (1/\text{Var} (i_t))[-(((1 + b)/c) A - (b/a)(Aa + 2 A\beta)) \text{ Cov} (p_t, i_t) \]

\[ + \{Aa + (b/a)(A\delta - A\gamma)\} \text{ Cov} (y_t, i_t) \]

\[ (52b) \ \theta d = -[1b - (b/a) Aa] + (1/\text{Var} (i_t))[-(((1 + b)/c) A - (b/a) Aa) \text{ Cov} (p_t, i_t) \]

\[ + \{Aa + (b/a)(A\delta + A\gamma)\} \text{ Cov} (y_t, i_t) \]

Since \( \Delta m_t = (\Delta m_t + \Delta m_t)/2 \) and \( \Delta m_t = (\Delta m_t + \Delta m_t)/2 \), (52a) and (52b) lead to:

\[ (53a) \ \Delta m_t = (\theta ai_t^2 + \theta di_t^2)/2, \]

\[ (53b) \ \Delta m_t = (\theta ai_t^2 - \theta di_t^2)/2. \]

Equations (53a, b) describe how the asymmetry in parameters will change the optimal rules under the symmetric parameters. By definition, it holds that \( \Delta m_t = \Delta m_t = 0 \) when the structural parameters are symmetric between two countries. In general, the monetary rules in two countries will differ from those of Section 5 (i.e., (26)) under the asymmetric parameters.

In particular, the monetary rules in two countries tend to diverge from (26) into the same direction when \( \text{Cov} (p_t, i_t) \) and \( \text{Cov} (y_t, i_t) \) are large, while they tend to diverge from (26) into the opposite directions when \( \text{Cov} (p_t, i_t) \) and \( \text{Cov} (y_t, i_t) \) are large.

7. Conclusion

This paper analyzed desirable monetary rules in the N-country world. Decomposing the world system into \( N \) independent subsystems, we obtained an explicit form of solution among a large number of countries. The decomposition not only simplified the analysis but also made it possible to solve the model which is practically unsolvable without decomposition. Furthermore, the decomposition into one aggregate and difference systems itself had some meaningful economic implications.

The main results are summarized as follows; (i) The optimal monetary feedback rule for each country is composed of the feedback on the world average interest rate (i.e., the global money supply rule) and the feedback on the country specific interest rate or effective exchange rate (i.e., the intervention rule). (ii) Each type of rule depends on the relative magnitudes of exogenous variances and covariances. (iii) It also depends on the number of countries in general. (iv) When the weight of one country is quite small, the optimal feedback rule on the world average interest rate depends on the cross-country correlations of disturbances but is independent of the variances of disturbances, while the optimal feedback rule on the country specific interest rate
remains unchanged as long as the elasticity of aggregate demand for the domestic prices is bounded. (v) The asymmetry in parameters alters the solution of each country into the same direction when difference shocks exist, while it makes the solution of each country diverge into the opposite directions when worldwide shocks exist.

One noteworthy result in this paper is methodological. It turns out that if the decomposition method is used, the costs of computation to solve a $N$-country model are almost equivalent to the costs to solve a small open economy. Although the paper applied the decomposition method to decompose the specific world system with Mundell-Flemming feature, a similar decomposition is possible for various types of world economy model. Even in the closed economy model, the method may be applicable for decomposing the multiple sectors which interact with each other. Under some assumptions, the method can also be applied to the case that each country has a different structure. Hence, I believe that the gains from applying this decomposition method are not a few.

(Hitotsubashi University)

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REFERENCES

Hamada, Koichi, Jai-Won Ryou, and Yoshiro Tsutsui (1989) “Real and Monetary Disturbances in the G-7 Countries: Implications for the Choice of an International Monetary Regime,” mimeo.

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