MARKET STRUCTURE AND CONDUCT:
GENERALIZED INDUSTRY PERFORMANCE
GRADIENT INDEXES
FOR OLIGOPOLY MARKETS*

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This paper generalizes Dansby-Willig’s industry performance gradient index so as to explicitly relate the maximal feasible welfare improvement rate in imperfectly competitive markets with the number of firms perceiving market power and the industry-wide distribution of marginal costs. Two specific examples of the proposed generalized index are fully explored to obtain two results: (i) the greater number of oligopolistically behaving firms lowers but (ii) the greater variance of the marginal-cost distribution over the industry increases the maximal feasible rate of welfare improvement.

1. Introduction

In imperfectly competitive markets there is persistent market distortion caused by market power perceived by firms. Industrial organization theorists often try to capture this distortion by using the concentration ratio or the Herfindahl index. However their use as a distortion measurement index often lacks theoretical validity. Against this shortcoming Dansby and Willig (1979) proposed the so-called “industry performance gradient index” to evaluate the maximum feasible rate of welfare improvement given the market distortion arising from imperfect competition. However fruitful insights their index brings for us, it is somewhat weak in distinguishing among several market conduct and structure factors in regard to the maximum feasible welfare improvement rate, such as the distribution of market power perception by firms, the concentration ratio or the Herfindahl index and the distribution of cost efficiency among firms.

This paper, properly extending Dansby-Willig’s industry performance gradient index (hereafter the DW index), constructs much more general industry performance gradient indexes. They are shown to succeed in making up for the above shortcomings in the original index. In section 2 we propose a generalized industry performance gradient index based on the original construction procedure of the DW index, and in sections 3 and 4 we examine the properties of two specific

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examples of the generalized industry performance gradient index.

2. Market Conduct and Generalized Industry Gradient Performance Index

Consider an industry producing \( n \) close substitutes, each of which is produced by a single firm. We let \( N = \{1, \ldots, N\} \) denote this set of active firms. Let \( x = \{x_1, \ldots, x_n\} \) denote the output configuration of the industry and let a twice-continuously differentiable concave function \( S(x) \) express the gross benefits of consuming the output bundle \( x \). The inverse demand function facing firm \( i \) (i.e., a firm producing good \( i \)) is given by \( P_i(x) = S_i(x) = \partial S(x)/\partial x_i \) for \( \forall \ i \in N \). The consumer surplus is thus equal to \( S(x) - \sum_{i \in N} P_i(x)x_i \). Firm \( i \) has a twice-continuously differentiable cost function \( C_i = C_i(x_i) \), and earns profits \( \pi_i(x) = P_i(x)x_i - C_i(x_i) \). Thus the social welfare \( W(x) \) is given by:

\[
W(x) = S(x) - \sum_{i \in N} P_i(x)x_i + \sum_{i \in N} \pi_i(x) = S(x) - \sum_{i \in N} C_i(x_i).
\]

The following are the basic assumptions underlying the present Industry:

\[ \langle A-1 \rangle \]

\( P_{ij}(x) = \partial P_i(x)/\partial x_j \leq 0 \) for \( \forall i, j \in N \) with a strict inequality for \( j = i \).

\( |P_{ii}(x)| \geq \sum_{j \in N, j \neq i} |P_{ij}(x)| \) for \( \forall x > 0 \) and \( \forall i, j \in N \) (\( j \neq i \)).

\[ \langle A-2 \rangle \]

\( C_i'(x_i) > 0 \) for \( \forall x_i > 0 \) and \( \forall i \in N \).

\[ \langle A-3 \rangle \]

\( W(x) \) is strictly concave in \( x \).

\( \langle A-1 \rangle \) (i) implies that the demand function facing each firm is downward sloping and that the products in question are mutually weak substitutes. \( \langle A-1 \rangle \) (ii) implies that the own effect of output increase on the price outweighs the cross effect when all the firms simultaneously increase their outputs by one unit. \( \langle A-2 \rangle \) means that the marginal cost of each firm is strictly positive. And \( \langle A-3 \rangle \) ensures that there exists a unique social optimum.

Given the market conduct (i.e., the mode of competition among the firms which will be later captured by their conjectural variations), there arises a certain equilibrium output configuration \( x = (x_1, \ldots, x_n) \). Hereafter we impose:

\[ \langle A-4 \rangle \]

There exists some \( i \in N \) such that \( W_i(x) = \partial W(x)/\partial x_i \neq 0 \).

Otherwise, \( \langle A-3 \rangle \) implies that the status quo is the unique social welfare optimum, so that there is no problem of market distortion. Then our question is how much social welfare can be improved by suitably adjusting the industry output configuration given this equilibrium \( x \) as the status quo involving some market imperfection. Since this rate of potential welfare improvement hinges on the scale of output adjustments, Dansby and Willig (1979) first constrains the adjustment scale measured by the Euclidean norm (called the DW norm below) up to a certain level and next obtain a marginal welfare gain by computing a limit of the rate in welfare improvement with regard to the adjustment scale. More specifically, the DW norm is given by:

\[
\rho_{DW}(Ax, p) = \sqrt{\sum_{i \in N} (p_i Ax_i)^2}
\]
where $\Delta x_i = x'_i - x_i$ for $\forall i \in N$, $\Delta x = (x'_1 - x_1, \ldots, x'_N - x_N)$, $p_i = P'(x)$, and $p = (p_1, \ldots, p_N)$ is the market price vector corresponding to $x$.

As can be easily understood, the derived maximal rate of welfare improvement depends on the norm employed. The characteristic of the DW norm (2) is that Dansby and Willig treat all the firms equally, which implicitly assumes that each firm's output adjustment incurs the same social or administrative costs. This disregards the cause of market imperfection, i.e., presence of firms perceiving market power.

For example, it is well known in the literature of public economics and industrial organization that public utility regulation incurs extra social costs called X-inefficiency. In fact, when a firm perceives market power, it means that the firm decides on its output by taking an account of its influence on the market as a whole. Thus, unless the government can directly control each firm (which is impossible in reality), its indirect policy to affect the firms' output decision tends to fail in achieving the expected result, for the firms may predict the government response to their behavior and thus behaves strategically.

This game-like situation between the government and the industry implies that regulation of the firms perceiving market power incurs additional social costs not captured by the welfare measurement above. A short-cut way to overcome this shortcoming is to amend the norm (2) in such a way that it is more difficult to change the oligopolistic firms' outputs than the competitive ones. More specifically, we may employ the following more generalized norm $p$:

$$
(3) \quad p(\Delta x, p) = \sqrt{\sum_{i \in N} a_i(p_i \Delta x_i)^2},
$$

where $a_i (> 0$ for $\forall i \in N)$ measures the weight associated with firm $i$. If firm $i$'s market power is greater than firm $j$'s, then we may let $a_i > a_j$. The DW norm is a special case of (3) given by $a_i = 1$ for $\forall i \in N$.

Given (3) replacing the DW norm (2), the constrained welfare maximization problem posed by Dansby and Willig is expressed by:

$$
(4) \quad \text{Max } W(x') \text{ subject to } p(\Delta x, p) \leq t,
$$

where $t$ is the allowed adjustment scale of the industry output configuration. This problem can be solved as below. Let $L$ denote the Lagrangian function for the problem (4) and $\lambda$ denote the Lagrangian multiplier associated with the constraint, i.e.,

$$
L(x', \lambda) = W(x') + \lambda[t - p(\Delta x, p)].
$$

In view of $\langle A-3 \rangle$ and $\langle A-4 \rangle$, the solution expressed by $\{x(t), \lambda(t)\}$ must satisfy the following first-order condition:

$$
(5) \quad \partial L(x(t), \lambda(t)) / \partial x_i = P'(x(t)) - C'(x_i(t)) - \lambda(t) a_i \Delta x_i(t) \frac{p_i}{\rho(\Delta x_i(t), p)} \leq 0, \quad x_i(t) \geq 0, \text{ for } \forall i \in N,
$$

$$
(6) \quad \partial L(x(t), \lambda(t)) / \partial \lambda = t - p(\Delta x, p) \geq 0, \lambda(t) \geq 0,
$$

where $\Delta x_i(t) = x_i(t) - x_i, \Delta x(t) = (\Delta x_1(t), \ldots, \Delta x_N(t))$ and use was made of:

$$
W_i(x) = P'(x) - C'(x_i) \text{ for } \forall i \in N,
$$

by virtue of (1). (5) is the optimization condition with respect to each firm's output, and (6) requires that the constraint in question be satisfied.
Without loss of generality, we assume that the solution is interior. Then (5) can be rewritten as:

\[
\frac{P'(x_i(t)) - C'(x_i(t))}{P'(x(t))} \cdot \frac{1}{\sqrt{\alpha_i}} = \lambda(t) \frac{\sqrt{\alpha_i P_i \Delta x_i(t)}}{\rho(\Delta x(t), p)} \quad \text{for } \forall i \in N.
\]

Square and sum this over \( i \in N \), and obtain:

\[
\lambda(t) = \sqrt{\sum_{i \in N} \frac{1}{\alpha_i} \left[ \frac{P'(x_i(t)) - C'(x_i(t))}{P'(x(t))} \right]^2},
\]

where use was made of (6). Note that \( \lambda(t) > 0 \) holds by virtue of \( \langle A-4 \rangle \).

As is well known, the equilibrium Lagrangian multiplier \( \lambda(t) \) measures the marginal gain associated with an infinitesimal relaxation of the constraint, i.e., \( \lambda(t) = dW(x(t))/dt \). The industry gradient performance index proposed by Dansby and Willig is nothing but this value when we let \( t \) approach zero. Similarly, given our general norm (3), the index \( \phi(x) \) which we propose is expressed by:

\[
\phi(x) = \lambda(0) = \sqrt{\sum_{i \in N} \frac{1}{\alpha_i} \left[ \frac{P'(x_i) - C'(x_i)}{P'(x)} \right]^2},
\]

where use was made of (8) and \( x(0) = x \).

Now we rewrite (9) by using the parameters expressing the market conduct and structure. For this purpose, we note that the first-order condition for profit maximization for firm \( i \) in the present quasi-Cournot oligopoly market is described as:

\[
\pi_i'(x) = \frac{\partial \pi'(x)}{\partial x_i} = P'(x) + x_i \sum_{j \in N} P_j'(x) \mu_{ij} - C'(x_i) = 0,
\]

where \( \mu_{ij} = \partial x_j/\partial x_i \) is firm \( i \)'s conjectural variation with respect to firm \( j \) for \( \forall i, j \in N \). As for each firm's conjectural variations, we impose the following condition:

\[
\langle A-5 \rangle \quad \mu_{ij} \text{ is a non-negative constant for } \forall i, j \in N.
\]

Generally, the firm’s conjectural variation may depend on the extent of its output increase as well as the output configuration at the time of expectation formation. But taking them into consideration would make the succeeding analysis too messy to trace. \( \langle A-5 \rangle \) is assumed for the purpose of simplifying the analysis.

Let \( \mu_i = \sum_{j \in N} P_i'(x) \mu_{ij} P_j'(x) \). From \( \langle A-1 \rangle \), \( \mu_i \leq \sum_{j \in N} \mu_{ij} \). In the case of a homogeneous industry, \( \mu_i = \sum_{j \in N} \mu_{ij} \) holds, for \( P_i'(x) = P_j'(x) = P'(X) \) where \( X = \sum_{j \in N} x_j \) denotes the total industry output and \( p = p(X) \) ( = \( P'(X) \)) describes the market demand function in the homogeneous oligopoly. Under \( \langle A-1 \rangle \) and \( \langle A-5 \rangle \), each \( \mu_i \) is non-negative. Then (10) can be rewritten as:

\[
\pi_i'(x) = P'(x) + x_i P_j'(x) \mu_{ij} + C'(x_i) = 0 \quad \text{for } \forall i \in N,
\]

Note that \( F_s = \{ i \in N | \mu_i > 0 \} \), the set of firms perceiving market power, is non-empty in view of \( \langle A-1 \rangle \) (i) and \( \langle A-4 \rangle \). And (11) can be further rewritten as:

\[
\frac{P'(x) - C'(x_i)}{P'(x)} = \frac{\mu_i}{e_i'(x)} \quad \text{for } \forall i \in N;
\]
where $\varepsilon_i(x) = -\left[ \partial \ln P^i(x)/\partial \ln x_i \right]^{-1}$ is the own price elasticity of firm $i$'s demand. Put (12) into (9), and obtain:

$$\phi(x) = \sqrt{\sum_{i \in N} \frac{1}{\alpha_i} \left[ \frac{\mu_i}{\varepsilon_i(x)} \right]^2},$$

In the case of a homogeneous product industry, i.e., $p(X) = P^i(x)$ for $i \in N$, $\phi(x)$ is expressed in the following form:

$$\phi(x) = \frac{1}{\varepsilon(X)} \sqrt{\sum_{i \in N} \frac{(s_i \mu_i)^2}{\alpha_i}},$$

where $\varepsilon(X) = -[d \ln p(X)/d \ln X]^{-1}$ is the price elasticity of the market demand and use was made of $p(X) = P^i(x)$, $p'(X) = P'_j(x)$ for $i, j \in N$, $s_i = x_i/X$ firm $i$'s market share for $i \in N$, and $\varepsilon_i(x) = s_i/\varepsilon(X)$ for $i \in N$.

As Dansby and Willig proved for their original industry performance gradient index, one can prove in a straightforward fashion that the above generalized industry performance gradient index $\phi(x)$ satisfies the following properties:

**Property 1:** $\phi(x) = 0$ if and if $P^i(x) = C_i'(x_i)$ for $i \in N$. Furthermore, zero is the minimum value of $\phi(x)$.

**Property 2:** If output adjustments are continuously made in the best local direction, then while social welfare $W(x)$ increases, $\phi(x)$ will fall monotonically.

**Property 3:** $W(x') - W(x) \leq \phi(x) \rho(dx, \mu)$.

**Property 4:** In a homogeneous Cournot industry with an equal weight for each firm, i.e., $\alpha_i = \alpha$ for $i \in N$,

$$\phi(x) = \frac{\sqrt{H(x)}}{\alpha \varepsilon(X)},$$

where $H(x) = \sum_{i \in N} s_i^2$ is the Herfindahl index of market concentration.\(^2\)

However, when we look at the DW index, it is not so clear how each firm's behavior pattern represented by $\mu_i$, differences in each firm's cost conditions, and the concentration ratio such as the Herfindahl index affect the possible magnitude of further welfare improvement. To improve the index in this respect, I propose several specific examples of the above generalized industry performance gradient index by suitably choosing the weight profile $\alpha = (\alpha_1, \ldots, \alpha_N)$.

### 3. Harmonic Mean of Conjectural Variations and Market Conduct

The first example of the above generalized industry performance gradient index, denoted by

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2) The welfare implication of the Herfindahl index is also explored by Kiyono (1988b) and Farrell and Shapiro (1990) in a different context. They are concerned with the direct welfare-improving effect of a rise in the Herfindahl index, while in the present paper we are concerned with its effect on the maximal feasible rate of welfare improvement rate.
\(\phi_i(x)\), which I propose, is of the following form:

\[
\phi_1(x) = \sqrt{\sum_{i \in F_+} \left[ \frac{P_i'(x) - C_i'(x)}{P_i'(x)} \cdot \frac{\bar{\mu}}{N_+ + \mu_i} \right]^2},
\]

where \(N_+\) is the cardinality of \(F_+\), \(\bar{\mu} = \frac{1}{\sum_{i \in F_+} \mu_i}\) the harmonic mean of the industry’s conjectural variations, and (15) is obtained from (9) by letting \(\alpha_i = \begin{cases} \left( \frac{N_+ \mu_i}{\bar{\mu}} \right)^2 & \text{for } i \in F_+ \\ \frac{\bar{\mu}}{N_+ \mu_i} & \text{otherwise} \end{cases}\).

\(0, 1)\) for \(\forall i \in F_+, \alpha_i \geq \alpha_j\) for \(\forall i, j \in N\) if and only if \(\mu_i \geq \mu_j\) with an equality only when \(\mu_i = \mu_j\). This satisfies our requirement on the weight profile \(\alpha = (\alpha_1, \ldots, \alpha_N)\).

\(\bar{\mu}\) measures the harmonic mean of the conjectural variations among the firms perceiving market power, i.e., \(\mu_i > 0\). It should be noted that the harmonic mean is always smaller than the arithmetic mean unless the components are all identical. Thus for a fixed arithmetic mean the harmonic mean is the largest (and equals the arithmetic mean) when the firms hold the same conjectural variations, i.e., \(\mu_i = \bar{\mu}(>0)\) for \(\forall i \in N\).

Put (12) into (15), and obtain:

\[
\phi_1(x) = \frac{\bar{\mu}}{N_+} \sqrt{\sum_{i \in F_+} \frac{1}{[\epsilon_i'(x)]^2}}.
\]

In view of the present norm, Property 4 can be expressed in the more specific fashion as below:

**Property 4*: In a homogeneous quasi-Cournot oligopoly, industry,**

\[
\phi_1(x) = \frac{\bar{\mu}/H_+(x)}{N_+ \epsilon(X)},
\]

where \(H_+(x) = \sum_{i \in F_+} s_i^2\) is the Herfindahl index of market concentration for the firms perceiving market power, i.e., \(\mu_i > 0\).

One feature not captured by the DW index is the role of the number of firms perceiving market power \(N_+\). As shown in the equation listed in Property 4*, given the Herfindahl index for the firms perceiving market power \(H_+(x)\), the greater value of \(N_+\) lowers the maxim rate. This is because an increase in \(N_+\) implies the smaller difference of price-marginal costs among the active firms in the industry as a whole, implying the less room for welfare improvement in view of the second-best theory.

Furthermore, with this generalized industry performance gradient index, we can distinguish the factors of market concentration or structure from those of market conduct in evaluating the present room for welfare improvement. Concretely, we have the following assertion on the market conduct which is not made clear by the DW index:
Assertion 1: Given the industry’s Herfindahl index $H(\mu)$, the larger harmonic mean of the industry’s conjectural variations $\tilde{\mu}$ implies the larger room for further welfare improvement by suitable adjustment of the industry output configuration.

By suitably choosing the weight profile $a$, we can construct another measure which is very useful for evaluating the maximum feasible rate of further welfare improvement with respect to the market conduct and the distribution of each firm’s cost conditions, which we discuss in the next section.

4. Industry-wide Cost Conditions and Market Structure

The second generalized industry performance gradient index, denoted by $\phi_2(x)$, when $\mu_i > 0$ for $\forall i \in N$, is described as:

$$
\phi_2(x) = \sqrt{\sum_{i \in F_+} \left( \frac{p_i - C_i'(x_i)}{p_i} \right)^2 \frac{\tilde{\mu}}{N_+ \mu_i}}
$$

which is obtained by letting $a_i = \begin{cases} \frac{N_+ \mu_i}{\tilde{\mu}} & \text{for } i \in F_+ \\ 1 & \text{otherwise} \end{cases}$

Let $b_i = \begin{cases} \frac{\tilde{\mu}}{N_+ \mu_i} & \text{for } i \in F_+ \\ 0 & \text{otherwise} \end{cases}$, and $b = (b_1, \ldots, b_N)$. Then since $b_i \in [0, 1)$ for $\forall i \in N$, $b_i = 0$ only for $V_\infty \notin F_+$, and $\sum_i b_i = 1$ hold, the profile $b$ can be used as a probability distribution.

When we notice it, the above second index (17) is very useful to evaluate the maximum feasible rate of welfare improvement in relation to the industry-wide distribution of each firm’s marginal costs in a homogeneous quasi-Cournot oligopoly industry as follows.

First, from the first-order condition for profit maximization (11), we have:

$$
\frac{\mu_i}{\mu} x_i = x_j + \frac{\varepsilon(x)X}{\mu_i \varepsilon(X)} [C'_j(x_j) - C'_i(x_i)]
$$

Summing (18) over $j \in N$, we obtain:

$$
\frac{\mu x_i}{\mu X} = \frac{1}{N_+} + \frac{\varepsilon(x)X}{\tilde{\mu} \varepsilon(X)} [C'(x) - C'_i(x_i)],
$$

where $C'(x) = \sum_{j \in N} \beta_j C'_j(x_j)$, a kind of the industry mean marginal cost.

Again using (11), we can rewrite (19) as:

$$
\frac{p(X) - C'_i(x_i)}{p(X)} = \frac{\tilde{\mu} x_i}{\varepsilon(x)} = \frac{\tilde{\mu} x_i}{\varepsilon(X)} \frac{C'(x) - C'_i(x_i)}{\mu X} + \frac{C'(x) - C'_i(x_i)}{p(X)},
$$

for $\forall i \in N$. Squaring (20) and taking the mean over $i \in N$ with the probability distribution $b$, we obtain:

$$
[\phi_2(x)]^2 = \left[ \frac{\tilde{\mu}}{N_+ \varepsilon(X)} \right]^2 + \text{Var} \left[ \frac{C'(x)}{\varepsilon(X)} \right] \left[ \frac{\tilde{\mu}}{p(X)} \right]^2,
$$

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where $\text{Var}[C'(\theta)] = \sum_{i \in N} \beta_i [C'(x_i) - C'(\theta)]^2$ is the variance of the industry marginal-cost distribution. Thus we obtain:

$\phi_2(x) = \sqrt{\frac{\mu}{N} \sigma(x)} + \frac{\text{Var}[C'(\theta)]}{\{p(X)\}^2}$.

Note that this second index also has the same implication as the first one as regards the number of firms perceiving market power as well as the market conduct. A new insight is the following concerning the implication of the industry-wide distribution of cost efficiency:

**Assertion 2:** Other things being equal, in a homogeneous quasi-Cournot oligopoly industry the larger variance of the marginal costs among the firms implies the larger room for further welfare improvement by suitable adjustment of the industry output configuration.

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