Using a version of two-asset optimal lifetime consumption model with human wealth, we analyze the effect of income tax on interest on portfolio selection and savings rate as compared to that of capital gains tax and the combined effect of the two taxes. We suggest a method of measuring the welfare cost of capital income taxes in this life-cycle setting and apply it to Japanese workers households classified by cohort. We calculate the ratio of compensation to lifetime expenditures considered as a measurement of the welfare cost. Households who bear these taxes would pay this ratio of lifetime consumption expenditures before taxation so as to avoid remaining the post-tax level of utility. The estimation results show that the welfare cost of both income tax on interest and capital gains tax is between 3% and 7% of the sum of lifetime expenditures at each period after taxation.

1. Introduction

The welfare cost of capital income taxes has hitherto been measured using life-cycle models that do not treat households’ portfolio selection. As a result, little attention has been paid to the comparison of welfare cost of interest income tax with that of capital gains tax, as well as the imposition of both taxes. However, concerning the current tax reforms in Japan, it has been argued that increasing the rate of income tax on interest due to the abolition of the tax-free interest and the imposition of capital gains tax may influence households’ portfolio selection and savings rate. The change in savings rate distort the optimal path of consumption in comparison to pre-tax situation and affects the welfare cost of these taxes.

The purpose of this paper is twofold. We first analyze the effect of income tax on interest as well as capital gains tax on portfolio selection and savings rate in the case where households accumulate their assets by selecting their portfolio so as to maximize their lifetime utility.

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1) The difference in these tax rates has been pointed out in Ihori (1990).
Secondly, we suggest a method of measuring the welfare cost of these capital income taxes in this life-cycle setting and applies it to Japanese workers households classified by cohort.

The effects of capital income taxes on asset demand and savings have been studied using different models. One-period asset demand functions have been used to study the effects of the taxes on portfolio selection (Sandmo (1977), Honma and Atoda (1990)). The life-cycle models have been employed to study the effect of capital income tax on savings. Since the effect on savings distort the path of optimal consumption in a life-cycle setting, Feldstein (1978), Green and Shesinski (1979), Summers (1981), and Keen (1990) expanded the method of measuring this distortion to include the welfare cost. Furthermore, Hatta and Nishioka (1992) estimated the welfare cost of capital income taxes in Japan by using a version of the neoclassical growth model with these taxes as in Chamley (1981), and Nishioka (1989).

The decomposition of the analyses of the effect of capital income tax on portfolio selection and that on savings depends on the basic characteristic feature of the models proposed by Atkinson and Stiglitz (1980) and Sandmo (1985). That is, asset demand functions are similar in form between one-period setting and life-cycle setting under the following conditions: (a) additive separability in time of utility function with constant relative risk aversion, and (b) lack of labor income in the budget constraint on the maximization of households' lifetime expected utility.2)

However, in the literature of optimal lifetime consumption models (Merton (1971), Breeden (1979), Grossman and Shiller (1982), Ingersoll (1987), He and Pages (1990)), labor income is included in the budget constraint. In this case, the explicit solutions for asset demand derived by lifetime expected utility maximization include human wealth. The similarity in form between asset demand functions under life-cycle setting and one-period setting do not hold. Hence in order to extend the analysis we eliminate precondition (b). Consequently, the effect of income tax on interest on portfolio include human wealth effect, that is, the effect on asset demand of an increase in the value of human wealth caused by an increase in the rate of the tax.

In order to analyse this effect and calculate the welfare cost of the taxes, we use a version of Merton's optimal consumption model with human wealth (Merton (1971)). Following Bover (1989) and Blundell, Browning and Meghir (1989), we shall replace the discounted sum of utility functions by that of indirect utility functions as the objective function in Merton's model. The reason for combining these models is twofold: (1) we can derive the explicit solutions for consumption function and asset demand functions. (2) we can estimate Arrow and Pratts' measure of relative risk aversion of this model using the method proposed by Blundell et. al. (1989). For tractability of the model, we shall focus on the two-asset case where income tax on interest is regarded as tax on rate of return on safety asset, while capital gains tax is tax on rate of return on risky asset.

The welfare cost of these capital income taxes is caused by intertemporal substitution for consumption in comparison to the pre-tax situation. This substitution appears as the change in savings rate. In our model, it is related to the change in the conditional mean and variance of the rate of return on non human wealth composed by optimal portfolio.

---

2) The precondition (b) is assumed in Hamilton (1987). Hence he did not analyse the human wealth effect of income tax on interest.
Hence, we shall first examine the effects of the taxes on the share of risky asset to nonhuman wealth (this share is called social risk taking), the conditional mean and variance. Here we shall take into account the human wealth effect of income tax on interest. This has not been explored in one-period models. When the effects of the taxes are ambiguous, we use simulation method based on estimated parameters for Japanese workers households classified by cohort. Secondly, we shall explain the change in savings rate in relation to intertemporal substitution of consumption. Thirdly, we shall estimate the welfare cost of these taxes. The method that we propose here is to calculate the ratio of compensation to lifetime expenditures before taxation so that households who bear these taxes would be prepared to pay this compensation to avoid remaining the post-tax level of lifetime utility.

This paper is organized as follows. The next section presents our model. In the third section we consider the effects of income tax on interest and capital gains tax on social risk taking, the conditional mean and variance of rate of return on the optimally composed non-human wealth, and savings rate. The fourth section describes the estimation method of the welfare cost of these taxes as well as Arrow and Pratts' measure of relative risk aversion and explains the data used for the analysis. In the fifth section, we summarize the estimation results. The quantitative effects on portfolio, the conditional mean, savings rate and the welfare cost of the taxes are also calculated using the statutory tax rates. The final section presents concluding remarks.

2. The Model

Households obtain income from labor income and capital income accruing from the returns on safety asset and risky asset and save the income less consumption at each period of their lifetime. Under this budget constraint, households determine the path of shares of the two assets to non-human wealth and that of consumption so as to maximize their expected utility. We assume that households' initial wealth is given and that households' lifetime utility function is additively separable in time as well as in consumption of goods and leisure. Under these assumptions, the behavior of households can be considered as a form of two stage budgeting: 1) for a given labor income and prices of the goods, households determine the optimal path of consumption and portfolio under the wealth accumulation equation; 2) for given prices of goods and consumption expenditure chosen by the first stage at each period, the demand for goods is determined so as to maximize the within period utility.

In this two stage budgeting, the objective function is expressed by the discounted sum of within period indirect utility \( v(t) \). The preference ordering of consumption expenditures among different periods depends on the transformation of \( v(t) \), for the transformation \( F(v(t)) \) may determine the magnitude of risk aversion, and its reciprocal relates to the elasticity of intertemporal substitution. We adopt the specification of \( F(v(t)) \) suggested by Blundell et al. (1989):

\[
F(v(t)) = \frac{1}{1 + \rho} v(t)^{1+\rho}, \quad 1 + \rho < 1 \text{ and } \rho \neq 0
\]

The transformation \( F(v(t)) \) exhibits constant relative risk aversion and \( -\rho \) is equal to Arrow and Pratts' measure of relative risk aversion.

The within period indirect utility \( v(t) \) is specified as:

\[
v(t) = v(p_t, C_t) = [C_t - a(p_t)]/b(p_t),
\]

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where \( a(p_t) = \sum_{i=1}^{n} a_i p_{it} \), \( b(p_t) = \prod_{i=1}^{n} b_{it} \). For given path of consumption goods prices, indirect utility \( v(t) \) implies consumption expenditure less committed consumption divided by \( b(p_t) \), the cost of marginal utility of income. Within period consumption expenditure \( C(t) \) is obtained by inverting Eq. (2) to the within period expenditure function:

\[
(3) \quad C(t) = C(p_t, v(t)) = a(p_t) + b(p_t) v(t)
\]

Applying Roy’s identity to Eq. (2) leads to Linear Expenditure System of within period demand functions:

\[
(4) \quad p_i q_{it} = a_i p_{it} + b_i \left( C(t) - \sum_{i=1}^{n} a_i p_{it} \right).
\]

For given prices of goods and labor income at each period, households maximize their expected utility by choosing the path of indirect utility \( v(t) \) and share of risky assets to non-human wealth \( x(t) \) under the budget constraint (see Appendix 1). This maximization problem is expressed by:

\[
(5) \quad I[W(t_1), t_1] = \max_{v(t), x(t)} E_t \left\{ \int_{t_1}^{t_2} e^{-\delta t} \frac{1}{1+\rho} v(t)^{1+\rho} dt \right\} + I[W(t_2), t_2],
\]

s.t. \( dW(t) = \{[x(t)(\mu_i - i_t) + i_t]W(t) + i_tY(t) - v(t)] dt + x(t)\sigma_i n(t) W(t) \sqrt{dt} \),

where \( t_1 \) is the period when the taxes are introduced, \( t_2 = t_1 + \Delta t \) is some period after taxation, \( \mu_i \) is the post-tax expected rate of return on risky asset, \( i_1 \) is the post-tax rate of return on safety asset. By denoting nonhuman wealth and human wealth at \( t \)-th period by \( A(t) \) and \( H(t) \), \( W(t) \) and \( Y(t) \) are respectively nonhuman wealth and human wealth divided by \( b(p_i) \), i.e., \( W(t) = A(t)/b(p_i) \), \( Y(t) = H(t)/b(p_i) \). \( I[W(t), t] \) is the value function which is assumed to be twice continuously differentiable:

\[
(7) \quad I[W(t), t] = \max_{v(t), x(t)} E_t \left\{ \int_{t}^{T} e^{-\delta s} \frac{1}{1+\rho} v(s)^{1+\rho} ds \right\} + I[W(T), T],
\]

where \( T \) is the date of death.

The post-tax rate of return on safety asset, \( i_1 \), is given by \( i(1-\tau_i) \) where \( i \) is the pre-tax interest rate assumed to be constant and \( \tau_i \) is the rate of income tax on interest. Since labor income is not stochastic, the sum of labor income is discounted by post-tax rate of interest rate to get the human wealth at the \( t \)-th period:

\[
(8) \quad H(t) = y_1(1-\exp(i_1(t-T)))/i_1,
\]

where \( y_1 \) is labor income at the \( t \)-th period, \( y_i \), less committed consumption, that is, \( y_i = y_i - a(p_i) \), and \( y_i \) is assumed to be constant.\(^3\)

On the other hand, the post-tax rate of return on risky asset in the limit of unit interval \( dt \) at the \( t \)-th period is given by a random variable, \( r_1(t) = r(t)(1-\tau_r) \), where \( r(t) \) is pre-tax rate of return and \( \tau_r \) is the capital gains tax rate. We assume the stochastic process which generates the variety of \( r_1(t) dt \) as follows:

\[
3) \quad \text{This assumption implies that some kind of exemption from wage income tax is necessary to maintain } y_1 \text{ constant.}
\]

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(9) \[ r_1(t) \, dt = \left[ \mu(1 - \tau_r) - (\sigma_1^2/2) \right] + dN, \]
where \( dN = \sigma_1 n(t) \sqrt{dt} \) is the Wiener process and \( n(t) \) is a random variable with independently and identically distributed standard normal distribution. Following Atkinson and Stiglitz (1980), we make two assumptions. The first one is that the government compensates for the loss in capital gains tax.\(^4\) Hence the variance of the stochastic process of (9) is expressed in post-tax terms, that is, \( \sigma_1^2 = (\sigma (1 - \tau_r))^2 \), where \( \sigma \) is the variance under pre-tax situation. This assumption enables us to solve Eq. (5) under the constraint of Eq. (6) by using a version of the trial solution proposed by Merton (1969). The second assumption is that the post-tax rate of return on risky asset is larger than that of safety asset to assure interior solutions, that is \( \mu_1 - i_1 > 0 \).

By applying the Bellman and Dryfess equation to the maximization problem (5) and by using the definitions of \( W(t) \) and \( Y(t) \) lead to the optimal solutions for the indirect utility and the share for risky asset:\(^5\)

(10) \[ v(t) = g(t)^{1/\beta}[A(t) + H(t)], \]
(11) \[ x(t) = \left[ (\mu_1 - i_1) / \sigma_1^2 (1 - \rho) \right] [1 + H(t) / A(t)]. \]

where

(12) \[ g(t) = \left[ 1 + (\nu \xi - 1) \exp(\nu(t - T)) \right] / \nu - \rho, \quad \nu \neq 0, \quad 6 \]
(13) \[ \nu = 1/(-\rho) [\delta - (1 + \rho)] \left[ (\mu_1 - i_1) \xi / (2\sigma_1^2 (1 + \rho)) + i_1 \right], \quad \text{and} \quad (14) \xi^{1-\rho} = g(T). \]

The consumption function is obtained by substituting Eq. (10) into Eq. (3) as follows:

(15) \[ C(t) = a(p_0) + b(p_0) / g(t)^{1/\beta}[A(t) + H(t)]. \]

From Eq. (15), the savings rate \( s \), according to Atkinson and Stiglitz (1980) is the ratio of non-human wealth less consumption and minimum expenditure \( a(p_0) \) to total wealth, \( i.e., s = [A(t) + H(t) - (C(t) - a(p_0))] / [A(t) + H(t)]. \) Setting \( b(p_0) \) equal to one, \( s \) becomes equal to \( 1 - g(t)^{1/\beta} \). Using Eq. (12) and denoting \( H(t) / A(t) \) by \( h,t \), we obtain the conditional mean \( (z_t) \) and variance \( (\sigma_t^2) \) of rate of return on optimally composed non-human wealth respectively:

(16) \[ z_t = E[x(\mu_1 + dN) + (1 - x) i_1] = (\mu_1 - i_1)^2 (1 + h_t) / \sigma_1^2 (1 - \rho) + i_1, \]
(17) \[ \sigma_t^2 = \text{Var}[x(\mu_1 + dN) + (1 - x) i_1] = (\mu_1 - i_1)^2 (1 + h_t)^2 / \sigma_1^2 (1 - \rho). \]

3. The Effects of Capital Income Taxes on Social Risk Taking and Savings Rate

The welfare cost of capital income taxes is induced by the intertemporal substitution of consumption in comparison to the pre-tax situation. In our model, this substitution appears as the change in savings rate between successive two periods \( s_t \). And if we take the limit \( T \to \infty \), \( s_t \) becomes related to \( z_t \) and \( \sigma_t^2 \). The relations among \( z_t, \sigma_t^2 \), and \( s_t \) can be seen as follows:

\(^4\) This assumption implies that there is no asymmetry concerning tax systems in our model. According to Mackie-Mason (1990), this assumption is the typical approach to modeling tax systems. However he employs asymmetries of tax system to study its effects on firms’ investment in the presence of uncertainty.

\(^5\) \( C(t) > 0, \, W(t) > 0, \, W(0) > 0 \), and the sufficient conditions are assumed to be satisfied.

\(^6\) If \( \nu = 0 \), Eq. (12) becomes \( g(t) = [1 / (T - t + \xi)] \). According to the data we used, \( \nu \) is not equal to zero.
Taking the limit \( T \to \infty \) in \( s_t = 1 - g(t)^{1/\rho} \) yields:

\[
(18) \quad s_t = 1 - \frac{1}{(1 + \rho)} \left[ \delta - \frac{1}{1 + \rho} \left( \frac{i_t h_t}{1 + h_t} + \frac{z_t}{1 + h_t} - \frac{\sigma_t^* \rho}{2(1 + h_t)} \right) \right].
\]

Combining Eqs. (16) and (17) with Eq. (18), we obtain (see Appendix 2):

\[
(19) \quad s_t = 1 - \left[ \frac{\delta}{-\rho} - \frac{1}{-\rho} \left( \frac{i_t h_t}{1 + h_t} + \frac{z_t}{1 + h_t} - \frac{\sigma_t^* \rho}{2(1 + h_t)} \right) \right].
\]

Income tax on interest influences \( z_t \) through the effect on \( x(t) \) and \( h_t \), because \( H(t) \) depends on this tax, while capital gains tax affects \( z_t \) and \( \sigma_t^* \) because of the assumption of full loss offset. The change in savings rate occurs from the combined effect of \( h_t \) and \( z_t \) when income tax on interest is imposed and from that of \( z_t \) and \( \sigma_t^* \) in the case of capital gains tax.

3.1 The Effects on Social Risk Taking, the Conditional Mean and Variance of the Rate of Return on Optimally Composed Nonhuman Wealth

We first examine the effects of capital income taxes on social risk taking, \( x(t) \). The examination of the effects of these taxes when human wealth is included is very important, since this case has not been explored in one-period model. Differentiating Eq. (11) with respect to \( \tau_t \) and \( \tau_r \), and considering \( H(t)/\partial \tau_t > 0 \) lead us to the following expressions:

\[
(20) \quad \frac{\partial x(t)}{\partial \tau_t} = i_t/\left[ \sigma_t^* (1 - \rho) \right] \left( 1 + h_t \right) + \left( \mu_t - i_t \right) \left( \partial H(t)/\partial \tau_t \right) \left[ \sigma_t^* (1 - \rho) A(t) \right] > 0,
\]

\[
(21) \quad \frac{\partial x(t)}{\partial \tau_r} = x(t)/(1 - \tau_r) - \left( 1 + h_t \right) i_t/\left[ \sigma_t^* (1 - \rho) (1 - \tau_r) \right].
\]

By rearranging Eq. (21), we can obtain the following proposition:

**Proposition 1** (i) An increase in the rate of income tax on interest increases social risk taking \( x(t) \).

(ii) If \( x(t)^* > (\prec) i_t(1 + h_t)/\sigma_t^* (1 - \rho) \), an increase in the rate of capital gains tax decreases (increases)\(^8\) social risk taking \( x(t) \), where \( x(t)^* \) is the optimal social risk taking before the change in the tax rate.

Eq. (20) implies that the substitution for portfolios which is induced by increase in the rate of income tax on interest is larger than that which would be caused without human wealth due to the ratio of \( 1 + h_t \) and the magnitude of the second term in the equation. The second term of Eq. (20) can be called the human wealth effect of income tax on interest on social risk taking, because it accrues from the increase in the value of human wealth after the decrease in the post-tax rate of return on safety assets. On the other hand, since the first term of Eq. (21) is positive while the second term is negative as long as \( (1 - \rho) \) is positive, the change in \( x(t) \) induced by capital gains tax is ambiguous. The magnitude of the second term is larger by \( h_t \) than that without human wealth (in the case where \( h_t = 0 \)). Hence the inclusion of human wealth lowers the change in social risk taking by \( h_t \) than that without it. Thus, since the effect of income tax on interest is positive and that of capital gains tax on \( x(t) \) is ambiguous, the combined effect of both

---

7) In the case where \( T \to \infty \), \( H(t) = y_t i_t = y_t/\left[ i_t (1 - \tau_t) \right] \) and \( \partial H(t)/\partial \tau_t > 0 \).

8) In each Proposition, the signs in the parentheses correspond to each other. The proof of this proposition is available upon request.

---
taxes will also be ambiguous and it will be examined by using simulation method in Section 5.

The changes in the conditional mean of rate of return on optimally composed nonhuman wealth, \( z_t \), caused by increases in the rate of income tax on interest and capital gains tax are given by:

\[
\frac{\partial z_t}{\partial \tau_i} = (2i)x(t) + x(t)(\mu_1 - i_1)(\frac{\partial H(i)}{\partial \tau_i})\left[\frac{A(i) + H(i)}{i} - 1\right],
\]

\[
\frac{\partial z_t}{\partial \tau_r} = -2i_1(\mu_1 - i_1)(1 + h_t)\left[\frac{\sigma^2(-\rho)(1 - \tau_r)}{1}\right] < 0,
\]

respectively. The last inequality follows from the assumption that \(*1 - i_1 > 0\). By comparing the optimal value of \(x(t)^*\) with the ratio of excess return multiplied by human wealth effect over the total wealth, that is, \(k = \frac{[\partial H(i)/\partial \tau_r](\mu_1 - i_1)}{(A(i) + H(i))}\), the effect of capital income taxes on \(z_t\) are summarized as follows (See Appendix 3):

**Proposition 2**

(i) If \(x(t)^* > (\leq) \frac{1}{2 + k}\), then \(\frac{\partial z_t}{\partial \tau_i} > (\leq) 0\), that is, an increase in the rate of income tax on interest increases (decreases) \(z_t\).

(ii) When human wealth is excluded, if \(x(t)^{**n} > (\leq) \frac{1}{2}\), then \(\frac{\partial z_t}{\partial \tau_i} > (\leq) 0\), where \(x(t)^{**n}\) is the optimal value of \(x(t)\) without human wealth.

(iii) an increase in the rate of capital gains tax decreases \(z_t\).

Since \(k\) is positive from the assumption: \(\mu_1 - i_1 > 0\), we have: \(1/(2 + k) < 1/2\). Hence, when the asset demand function includes human wealth, the level of optimal social risk taking by which the effect of income tax on interest on \(z_t\) becomes positive or negative \(x(t)^*\) is smaller than that without human wealth \(x(t)^{**n}\).

Finally we examine the effect of the taxes on the conditional variance \(\sigma_t^2\). Using Eqs. (20) and (17), we obtain the following results:

\[
\frac{\partial \sigma_t^2}{\partial \tau_i} = 2\sigma_t^2 x(t)[\partial x(t)/\partial \tau_i] > 0,
\]

\[
\frac{\partial \sigma_t^2}{\partial \tau_r} = -\left[\frac{2(\mu_1 - i_1)h_t(1 + h_t)}{\sigma^2(-\rho)(1 - \tau_r)}\right] < 0.
\]

**Proposition 3**

(i) an increase in the rate of income tax on interest increases \(\sigma_t^2\).

(ii) an increase in the rate of capital gains tax decreases \(\sigma_t^2\).

The reason for (i) is that an increase in \(\tau_i\) increases social risk taking, while the reason for (ii) is that because of loss offset, the variance of rate of return on risky asset becomes \(\sigma_t^2 = (\sigma(1 - \tau_r))^2\).

### 3.2 The Effects on Savings Rate

We shall examine the effect of income tax on interest and capital gains tax on savings rate in relation to its effect on intertemporal substitution of consumption. Differentiating \(s_t\) in Eq. (19) with respect to \(\tau_i\) and \(\tau_r\), we obtain:

\[
\frac{\partial s_t}{\partial \tau_i} = [(1 + 1/\rho)](1 - (\mu_1 - i_1))/\sigma_t^2,
\]

\[
\frac{\partial s_t}{\partial \tau_r} = (1 + 1/\rho)(\mu_1 - i_1)/\sigma_t^2(1 - \tau_r),
\]

respectively. From Eq. (26) we can find that the change in savings rate caused by income tax on interest depends on the magnitude of relative risk aversion \((-\rho)\) and the ratio of excess return...
over post-tax variance \((\mu_1 - i_1)/\sigma^2_t\). Using these two factors leads us to:

**Proposition 4**  
(i) If \(-\rho\) is larger than one, and if \((\mu_1 - i_1)/\sigma^2_t > (\cdot <)\) 1, then \(\partial s_i/\partial \tau_i < (\geq 0)\), that is, an increase in the rate of income tax on interest decreases (increases)\(^7\) savings rate \(s_i\).

(ii) if \(-\rho\) is smaller than one, and if \((\mu_1 - i_1)/\sigma^2_t > (\cdot <)\) 1, then \(\partial s_i/\partial \tau_i < (\geq 0)\), that is, an increase in the rate of income tax on interest increases (decreases)\(^8\) savings rate \(s_i\).

On the one hand, since the term in brackets is positive under the assumption that \(\mu_1 - i_1 > 0\), the effects of capital gains tax on savings rate are given by:

**Proposition 5** If \(-\rho\) is larger (smaller) than one, \(\partial s_i/\partial \tau_r > (\cdot <)\) 0, that is, an increase in the rate of capital gains tax increases (decreases)\(^9\) savings rate \(s_i\).

In order to explain the changes in savings rate summarized in Propositions 4 and 5, we decompose the gross substitution effect induced by change in the rate of each tax into the compensated substitution effect and the wealth effect in a similar way proposed by Merton (1969). These three effects of income tax on interest are given by:

\[
\begin{align*}
\frac{\partial C_i}{\partial \tau_i} &= \frac{(1 + \rho)i}{-\rho} \left(1 - \frac{\mu_1 - i_1}{\sigma^2_t}\right)(W_{t_i} + Y_{t_i}) + g(t_i)^{1/\rho} \frac{\partial Y_{t_i}}{\partial \tau_i}, \\
\frac{\partial C_i}{\partial \tau_i} \bigg|_{t_i} &= \frac{i}{-\rho} \left(1 - \frac{\mu_1 - i_1}{\sigma^2_t}\right)(W_{t_i} + Y_{t_i}), \\
\frac{\partial C_i}{\partial A_{t_i}} &= -i \left(1 - \frac{\mu_1 - i_1}{\sigma^2_t}\right)(W_{t_i} + Y_{t_i}) + g(t_i)^{1/\rho} \frac{\partial Y_{t_i}}{\partial \tau_i},
\end{align*}
\]

respectively, where \(\partial C_i/\partial \tau_i \big|_{t_i}\) is the partial derivative of consumption with respect to \(\tau_i\) with \(I_{t_i} = I[W(t_i), t_i]\) being held fixed, and the wealth effect is given by \(\partial C_i/\partial A_{t_i} = \partial C_i/\partial \tau_i - \partial C_i/\partial \tau_i \big|_{t_i}\).

| \(-\rho\) \(\sigma_1\) | \(\frac{\mu_1 - i_1}{\sigma^2_t}\) | \(\frac{\partial C_i}{\partial \tau_i} \big|_{t_i}\) | \(\frac{\partial C_i}{\partial A_{t_i}}\) first term | \(\frac{\partial C_i}{\partial A_{t_i}}\) second term | \(\frac{\partial C_i}{\partial \tau_i}\) | savings rate \(s_i\) |
|---|---|---|---|---|---|---|
| ii-a | >1 | >1 | + | + | + | - |
| ii-b | >1 | <1 | + | - | + | - |
| iii-a | <1 | >1 | - | + | + | - |
| iii-b | <1 | <1 | + | - | + | - |

1) In the column of \((-\rho), \ (>1\) \ and \ (<1\) indicate that \(-\rho\) is larger than one and that it is smaller than one respectively.

The sign of the compensated substitution effect and the two terms of wealth effect and resulting change in savings rate are summarized in Table 3.1. From Table 3.1, we can see that for high risk
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aversers (−ρ > 1) if (μ_1 − i_1)/σ_1^2 > 1, then the wealth effect dominates the compensated substitution effect, while if (μ_1 − i_1)/σ_1^2 < 1 then the first term of the wealth effect dominates the compensated substitution effect and the second term of the wealth effect (human wealth effect). For low risk averters (−ρ < 1), if (μ_1 − i_1)/σ_1^2 > 1 then the compensated substitution effect dominates the wealth effect, while if (μ_1 − i_1)/σ_1^2 < 1 then the compensated substitution effect and the second term of wealth effect dominate the first term of wealth effect.

Next, the gross and compensated substitution effects and wealth effect concerning capital gains tax are expressed respectively as:

\[
\begin{align*}
\partial C_n/\partial \tau_r &\equiv \frac{\partial}{\partial \tau_r} \left[ \frac{(1 + \rho) i_1 (\mu_1 - i_1)}{(-\rho) \sigma_1^2(1 - \tau_r)} \right] (W_{t_1} + Y_{t_1}), \\
\partial C_n/\partial A_{t_1} &\equiv \frac{\partial}{\partial A_{t_1}} \left[ \frac{\mu_1 - i_1}{\sigma_1^2(1 - \tau_r)} \right] (W_{t_1} + Y_{t_1}) > 0, \\
\partial C_n/\partial A_{t_1} &\equiv -\frac{\partial}{\partial A_{t_1}} \left[ \frac{\mu_1 - i_1}{\sigma_1^2(1 - \tau_r)} \right] (W_{t_1} + Y_{t_1}) < 0,
\end{align*}
\]

From Eqs. (31), (32) and Eq. (33) we get Table 3.2. It shows that for high risk averter the wealth effect dominates the compensated substitution effect, while for low risk averters the relationship is reversed.

| Table 3.2 | The substitution effects, wealth effect, and the change in savings rate according to the value of −ρ for capital gains tax |
|-----------------|-------------------------------------------------|-------------------------------------------------|-----------------|-----------------|-----------------|-----------------|
| −ρ^1) | \(\frac{\partial C_n}{\partial \tau_r}\) | \(\frac{\partial C_n}{\partial A_{t_1}}\) | \(\frac{\partial C_n}{\partial \tau_r}\) | savings | rate | \(s_t\) |
| ii′ | >1 | + | − | − | − | + |
| iii′ | <1 | + | − | + | − |

1) In the column of (−ρ), ‘>1’ and ‘<1’ indicate that −ρ is larger than one and that it is smaller than one respectively.

While the effect of capital gains tax on savings rate is determined according to the magnitude of relative risk aversion, the effect of income tax on interest is ambiguous because the latter is related to the ratio of excess return over variance. Hence in order to elucidate the combined effect of these taxes on savings rate and to calculate resulting welfare cost of these taxes, we will use simulation method in Section 5.

4. Estimating Equations and Data

4.1 The Measurement of the Welfare Cost

We now propose a measurement of the welfare cost of capital income taxes on those households who accumulate their assets by selecting their portfolio in a life-cycle setting. We denote the measurement of the loss caused by income tax on interest, capital income tax, and both taxes by \(L(\tau), L(\tau),\) and \(L(\tau),\) respectively. Furthermore let us define this measurement by:

\[
\begin{align*}
V[\{A_{t_1} + H_{t_1}\}(1 + L(\tau)), \ldots, (A_T + H_T)(1 + L(\tau)), i, \mu] \\
= V[A_{t_1}^1 + H_{t_1}^1, \ldots, A_T^1 + H_T^1, i^1, \mu^1],
\end{align*}
\]

where \(L(\tau)\) stands for \(L(\tau), L(\tau),\) or \(L(\tau),\) the suffix 1 represent post-tax variables, and \(t (= t_1, \ldots, T)\) indicates the period after imposition of the tax. \(V[\cdot\cdot\cdot]\) is the discounted sum of
within period indirect utility on the optimal solution under the assumption that prices of goods are equal to one. This measurement \( L(\tau) \) is the ratio of compensation on pre-tax total asset holdings by which households' total asset is multiplied to equalize their pre- and post-tax levels of utility. For the purpose of calculation, we adopt the discrete time form of \( V[\cdot] \):

\[
(35) \quad V[A_{t_1} + H_{t_1}, \ldots, A_T + H_T, i, \mu] = \sum_{t=t_1}^{T} \frac{1}{(1 + \delta)^{t-t_1}} \left[ \frac{1}{1 + \rho} v(t)^{1+\rho} \right],
\]

where \( v(t) = g(t)\delta^{1/\rho} [A(t) + H(t)] \) from Eqs. (17) and (18) with \( b(p_t) = 1 \).

Ex post, according to the method proposed by Keen (1990), we can show that \( L(\tau) \) is related to the lifetime intertemporal equivalent variation.\(^9\) This intertemporal variation implies the maximum amount of income that the household would be prepared to pay at the budget level before taxation to avoid the change in the path of relative prices for consumption between periods. This intertemporal variation \( \pi_i \) is given by: \( \pi_i = E_i(p_0^i, u_0^i) - E_i(p_0^i, u_a^i) \), where \( E_i(p, u) \) is a within period expenditure function, \( p_0^i \) is a price vector at the \( t \)-th period before price change, \( u_0^i \) and \( u_a^i \) are within period utility before and after the change respectively. We note that the path of \( u_a^i \) is chosen so as to maintain the original lifetime utility level.

Under the assumption that \( b(p_t) \) is set equal to one, \( p_0^i \) is regarded as the relative price of consumption between successive two periods before the imposition of capital income taxes \( P_{t+1}^1 \), for \( z_1^i \) corresponds to post-tax relative price of consumption due to the expression: \( P_{t+1}^1 = 1/(1 + z_1^i) \). Since \( v_1^i \) indicates the post-tax value of within period indirect utility, while \( v_0^i \) is its pre-tax value which is chosen so as to maintain post-tax level of lifetime utility by Eq. (34), \( \pi_i \) is rewritten as: \( \pi_i = C_i(z_0^i, v_1^i) - C_i(z_0^i, v_0^i) \).

Denoting the sum of \( \pi_i \) for the lifetime after imposition of the taxes by \( \prod \), we obtain:

\[
(36) \quad \prod = \sum_{t=t_1}^{T} [C(z_0^i, v_1^i) - C(z_0^i, v_0^i)]
= \sum_{t=t_1}^{T} [(1 + L(\tau))g(t)\delta^{1/\rho}(A_t + H_t) - g(t)\delta^{1/\rho}(A_t + H_t)]
= L(\tau) \sum_{t=t_1}^{T} g(t)\delta^{1/\rho}(A_t + H_t).
\]

It follows from the last line in Eq. (36) that:

\[
(37) \quad L(\tau) = \prod / \left[ \sum_{t=t_1}^{T} g(t)\delta^{1/\rho}(A_t + H_t) \right].
\]

Thus, ex post, \( L(\tau) \) becomes the ratio of the intertemporal equivalent variation on pre-tax lifetime consumption expenditure less committed consumption for the periods after imposition of the taxes. If the level of utility after taxation is decreased by the taxes, this intertemporal variation could be negative. Hence we shall examine the absolute values of the ratio \( L(\tau) \) in the simulation analysis.

By using Eq. (35) and solving Eq. (34) for \( L(\tau) \), and setting \( \xi = 0 \) in Eq. (16), we obtain:

---

\(^9\) By following Keen's suggestion and by rewriting his measurement by using equivalent variation, we can derive the equation for \( \pi_i \). This \( \pi_i \) is expressed as a value at the \( t \)-th period.
Y. Kaneko: Asset Demand and Welfare Cost of Capital Income Taxes in a Life-Cycle Setting

where $v_1$ is the estimated value of $v$ after the taxes are imposed. We calculate the values of $L(t_i)$, $L(t_r)$, and $L(t_{ir})$ by simulating the time path of $A(t)$, $H(t)$, and $v$ for pre- and post-tax cases by each cohort.

4.2 The Estimation Method and Data

To calculate the effects of the taxes and the welfare cost, we need the parameters which affect indirect utility function, asset demand functions and savings rate. The important parameter which is usually included in these functions is the Arrow and Pratts' measure of relative risk aversion ($\rho$). Although several attempts have been made to estimate this parameter for Japan, we still do not have any conclusive values. Hence we adopt the estimation method proposed by Blundell, et al. (1989). The features of their method is as follows: first calculate the values of indirect utilities by estimating within period demand systems; secondly, using the data, estimate the Euler equation which includes the parameter, $\rho$. Here we note that the Euler equation used for estimation is the equation obtained from the first order conditions for the discrete time model, which could be approximated to our continuous time model according to the method proposed by Ingersoll (1987, ch. 13). By denoting the rate of return on safety asset at unit interval by $i_t$, the Euler equation is given by:

$$F'(v(t)) = [(1 + i_t)/(1 + \delta)]E_t[F'(v(t + 1))].$$

where $F'(v(t)) = \partial F(v(t))/\partial v(t)$. Substituting Eq. (1) into Eq. (39) and differentiating both sides of this outcome with respect to $C_t$, we obtain:

$$\nu'_t \nu'_t = [(1 + i_t)/(1 + \delta)]E_t[\nu^p_{t+1} \nu^p_{t+1}].$$

where $\nu'_t = \partial v(t)/\partial C_t$. Following the assumptions by Blundell, Browning and Meghir (1989) about the stochastic term associated with the values of $v(t)$, we eliminate the conditional expectation operator in Eq. (40). And taking the logarithm of both sides, Eq. (40) is rewritten as a linear regression equation for the parameter $\rho$:

$$\Delta \log \nu_{t+1} + \Delta i_{t+1} = c + \rho \Delta \log \nu_{t+1} + e^*_t + 1,$$

where $\Delta$ expresses the difference in each variable between $t$ and $t + 1$ periods, $c$ is a constant term, and $e^*_t + 1$ is a disturbance term.

In order to estimate Eq. (41), we need the values of indirect utility, and these are obtained by estimating the within period demand system. Furthermore, since Eq. (41) expresses the optimal condition for maximization of households' lifetime utility, it may be desirable to estimate Eq. (41) by cohort data. In order to construct cohort data of Japanese workers household, we use the monthly income and expenditure per household classified by the age of the household head and by

$10) e^*_t$ is assumed to be a random variable such that $E_t[e^*_t] = 0$ and that $\log e^*_t + 1 = -d + e^*_t + 1$ where $e^*_t + 1$ is log normally distributed and $d$ is a conditional variance.
prefectures ('Setai Nushi no Nenrei Kaikyu Betsu Ichi Setai Atari Ikkagetsu Kan no Shunyu to Shishutsu (Kinrousha Setai), (To-Dou-Fu-Ken Betsu)), reported in The National Survey of Family Income and Expenditure, Vol. 4.

The data was obtained from the surveys published in 1974, 1979, and 1984. Since the table contains 5 age-class data of expenditures for goods, we reconstructed them into 10 age-class data by combining the columns of the 5 age classes. We therefore obtained data on expenditures for 4 cohorts; first is the cohort born before 1925, which we refer to as Cohort-0, second is the cohort born between 1925 and 1934, Cohort-1, third is the cohort born between 1935 and 1944, Cohort-3, and fourth is the cohort born between 1945 and 1954, Cohort-4. The reason for separating the Cohort-0 is that this cohort was over 60 years old in 1988 when the interest income tax rate was increased and they may have been retired, while the other cohorts should be working.

We used the price index reported in 'Subgroup Index (Chuu Bunrui Hyou)' of The Consumer Price Index. By using the weights in the table, we reconstructed the price indexes according to the division of the goods for the within period demand system in 1974, 1979, and 1984, and we set the indexes equal to one in 1980. The within period demand system (Linear Expenditure System) that we estimated consisted of six kinds of goods; 1) foods, 2) drinks and beverages, 3) housing, fuel and water, 4) clothing, 5) durable goods, and 6) other living expenditure. The expenditure on durable goods are the sum of expenditures on furniture and automobile. In order to satisfy the adding up restraint for the demand system, we exclude the demand function for other living expenditure. We applied Seemingly Unrelated Regression to the demand system because we used the pooling cross section data described above. For this estimation, we imposed the symmetry and homogeneity conditions. The reason is that if we do not impose these restraints, the estimated parameters included in the indirect utility function will depend on which one of the demand functions is deleted.

We estimated the Euler equation by applying the two stage least squares regression. We selected as instrumental variables the one period lagged data of the difference of $v_{i,t+1}$ and $v_{i,t+1}$, and the number of the dependent of households at the observed period.

In order to calculate the asset demands, we estimated the values of the expected rate of return on safety and risky assets, the variance of the rate of return, and human wealth. The expected rate of return on safety asset, $i$, is calculated by taking the weighted average of interest rates on time and demand deposits. We used the data of each interest rate between 1960 and 1988 reported in Economic Statistics Annual. The weight is the share of the average value of deposit for all cohorts in the total of the two deposits reported in Family Savings Survey. Thus the value of $i$ was set equal to 4.64%. The expected rate of return on risky asset, $\mu$, is calculated by the weighted average of rate of returns on bonds and securities between 1960 and 1988. The weight is the share of the average value of securities for all cohorts in the total of these two assets. We use the data of rate of return on bonds reported in Economic Statistics Annual and that of securities in Rates of Return on Common Stocks. Thus we set the value of $\mu$ equal to 10.65%.

11) The estimated parameters on the within period demand system are available upon request. In estimation of the Euler equation we assumed that households' expectation is rational. Hence following Mankiw, Rotemberg and Summers (1985), we estimated the Euler equation by using the two stage least squares regression in SAS/ETS as an instrumental variable method.
Since there are no effective tax rates after the tax reforms in 1988 and 1989 classified by cohort, we used the statutory tax rates. The rate of income tax on interest is 20%, while the rate of capital gains tax is 26%. In this analysis, we shall focus the effect of differential taxation on the rate of returns on safety and risky assets. Hence, according to Ihori (1990), we regard the rate of capital gains tax as the sum of the rate of separate taxation on capital gains at 20% through the filling of final returns and the rate of individual inhabitants tax on capital gains at 6%. As a result, post-tax rate of returns on safety and risky assets become 3.71% and 7.88%, respectively.

We calculated the initial value of human wealth by substituting the value of labor income classified by age classes which was reported in Annual Report on the Family Income and Expenditure Survey into Eq. (8). Secondly, we adjusted this initial value by multiplying the parameter with which the estimated value of savings rate \( s_t \) and the share of risky asset \( x_t \) could approximate the actual values.

Since we could not estimate the discount factor \( \delta \) from the estimation of the Euler equation, we selected the set of values of \( \delta \) by each cohort which was larger than interest rate on safety asset. Then we took some possible sets of values of \( \delta \) and an adjusted multiplier for human wealth and narrowed them down so that the absolute values of \( L(\tau) \) obtained by using these values converge between positive values. The values of \( \delta \) we chose were 6% for Cohort-1, 15% for Cohort-2, and 8% for Cohort-3 respectively.

Thus for the given initial value of human wealth and the given value of discount factor, we obtained the initial value of net consumption \( (C_t - a(r_t)) \) from Eq. (3). On the one hand, under the assumption that \( b(r_t) \) is set equal to one, changing the parameter for time \( t \) yields the path of \( g(t) \), since \( g(t) \) becomes independent of non-human wealth. By combining the initial values of net consumption and adjusted human wealth with the path of \( s_t \), and by retaining the estimated values of change in non-human wealth period by period, we calculated the paths of non-human wealth and resulting net consumption.

### 5. The Estimation Result

The parameter estimates of the Euler equation by cohort are presented in Table 5.1. The results show that Arrow and Pratts’ measure of relative risk aversion \(-\rho \) is larger than one for every cohort. The estimated value of \(-\rho \) of Cohort-0 is much larger than those of the other cohorts, which may correspond to the different patterns of savings behavior of Cohort-0. Since

---

12) The effective tax rates of income tax on interest and capital gains tax for the periods before and after the current tax reform have been estimated by Tajika and Yui (1990) and Iwamoto (1991), respectively. However, their tax rates were not classified by cohort.

13) The other taxation on capital gains which a taxpayer can choose in place of the taxation mentioned here is separate taxation withheld at source at 20% on the deemed gain of 5% of the proceeds from the sales of listed stocks and other publicly traded securities.

14) In comparison to the values of \( s_t \) and \( x_t \) in Table 5.2(A) which were estimated by the simulation method, we calculated the savings rate \( s_t \) and the share for risky asset \( x_t \) by using the data in 1988 quoted from “Family Savings Survey” and “Annual Report on the Family Income and Expenditure” classified by cohort. According to this calculation, \( s_t \) for Cohort-1, for Cohort-2, and for Cohort-3 is 0.75, 0.59, and 0.51 respectively, while \( x_t \) for each cohort is 0.38, 0.34 and 0.24 respectively.
they have retired by 1988, they seem to be more cautious than the other cohorts in their savings decision. Hence we eliminate Cohort-0 in the analysis below.

Table 5.1 The parameter estimates of The Euler Equation and the elasticity of intertemporal substitution

<table>
<thead>
<tr>
<th>cohort</th>
<th>constant</th>
<th>$\rho$</th>
<th>$R^2$</th>
<th>the elasticity of inter-temporal substitution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.743**</td>
<td>-24.05**</td>
<td>0.09</td>
<td>0.159</td>
</tr>
<tr>
<td>(before 1925)</td>
<td>(1.813)</td>
<td>(8.846)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.217</td>
<td>-1.224</td>
<td>0.01</td>
<td>0.667</td>
</tr>
<tr>
<td>(1925-1934)</td>
<td>(1.300)</td>
<td>(3.616)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.834**</td>
<td>-3.618**</td>
<td>0.06</td>
<td>0.300</td>
</tr>
<tr>
<td>(1935-1944)</td>
<td>(0.305)</td>
<td>(1.493)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.369**</td>
<td>-1.622**</td>
<td>0.03</td>
<td>0.697</td>
</tr>
<tr>
<td>(1945-1954)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: **shows that the value is significant at 5%. The values of year in the first column indicate the period when the sample of the cohort was born. Source: author's estimation. The value in parentheses is standard error.

Since the intertemporal elasticity of consumption, $\varepsilon$, is reciprocal to Arrow and Pratts' measure, it is inelastic as can be seen from the right column of Table 5.1. Kaneko (1991) calculated this elasticity by using estimation results of Engel curves. The data used in his analysis is the same cohort data in this paper, but the classification of consumption expenditures consists of five kinds of goods. The elasticity based on the estimation of the Engel curves is between 0.4 and 0.9. On the one hand, Ogawa (1986) and Honma, Atoda and Iwamoto (1987) estimated the elasticity through the method developed by Mankiw, Rotemberg and Summers (1985) based on consumption expenditure data in Annual Report on the Family Income and Expenditure. Ogawa's elasticity estimates under the assumption of separability between consumption and leisure are between 0.3 and 1.1. Honma et al. did not adopt separability assumption and the range of estimated values was very wide, that is, between 0.3 and 23.7. Our classification of the data by cohort has therefore confirmed that the intertemporal elasticity of substitution of Japanese workers households classified by age classes is inelastic.

By using the parameter value of $\rho$ in Table 5.1, we can calculate the share of risky asset before and after taxation from Eq. (11). The results are presented in Table 5.2.14,15 As in Eq. (22), the share of risky assets increases after income tax on interest is imposed ((B) of Table 5.2). The percentage change in share of risky asset induced by human wealth is the ratio of difference

15) Since the coefficient of determinants are low for all cohorts, it may be better to improve the estimation of the Euler equations. We should note that the simulation result in this analysis be regarded as one of the plausible results and that the sensitivity analyses of $s_t$, $x_t$, and $L(t)$ with respect to the pair of $\rho$ and $\delta$ is needed to fulfill further research.
Table 5.2  The effects of interest income tax and (or) capital gains tax on share of risky asset, savings rate, and expected rate of return on composed nonhuman wealth

<table>
<thead>
<tr>
<th></th>
<th>cohort</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(A) pre-tax</strong></td>
<td>* $x(t)$</td>
<td>0.444</td>
<td>0.151</td>
<td>0.336</td>
</tr>
<tr>
<td></td>
<td>* $s_i$</td>
<td>0.697</td>
<td>0.604</td>
<td>0.559</td>
</tr>
<tr>
<td></td>
<td>* $z_i$ (%)</td>
<td>7.32</td>
<td>5.54</td>
<td>6.66</td>
</tr>
<tr>
<td><strong>(B) interest income tax</strong></td>
<td>$x(t)$</td>
<td>0.513</td>
<td>0.174</td>
<td>0.338</td>
</tr>
<tr>
<td></td>
<td><strong>% change in $s_i$ (%)</strong></td>
<td>-10.92</td>
<td>-16.84</td>
<td>-22.34</td>
</tr>
<tr>
<td></td>
<td>$z_i$ (%)</td>
<td>7.27</td>
<td>4.92</td>
<td>6.06</td>
</tr>
<tr>
<td></td>
<td><strong>% change in $x(t)$ caused by human wealth (%)</strong></td>
<td>0.43</td>
<td>1.31</td>
<td>1.54</td>
</tr>
<tr>
<td><strong>(C) capital gains tax</strong></td>
<td>$x(t)$</td>
<td>0.438</td>
<td>0.148</td>
<td>0.331</td>
</tr>
<tr>
<td></td>
<td><strong>% change in $s_i$ (%)</strong></td>
<td>14.77</td>
<td>24.22</td>
<td>31.69</td>
</tr>
<tr>
<td></td>
<td>$z_i$ (%)</td>
<td>6.06</td>
<td>5.12</td>
<td>5.71</td>
</tr>
<tr>
<td><strong>(D) interest income tax and capital gains tax</strong></td>
<td>$x(t)$</td>
<td>0.308</td>
<td>0.104</td>
<td>0.233</td>
</tr>
<tr>
<td></td>
<td><strong>% change in $s_i$ (%)</strong></td>
<td>4.17</td>
<td>-7.38</td>
<td>8.92</td>
</tr>
<tr>
<td></td>
<td>$z_i$ (%)</td>
<td>4.99</td>
<td>4.15</td>
<td>4.68</td>
</tr>
</tbody>
</table>

Notations:  * $x(t)$: share for risky asset, that is, social risk taking; $s_i$: savings rate; $z_i$: expected rate of return on composed nonhuman wealth.

Source: author's estimation. **The values of $x(t)$, $s_i$ and $z_i$ are calculated by using Eq. (11) and the data on $\mu_i$, $i_1$ and $h_i$ in 1988. **This value is the average value of the percentage change of $s_i$ through the lifetime after imposing the tax in each case.

The percentage change in savings rate is calculated by the average values of savings rate through the lifetime after imposing the tax in each case. As we see from (B) of Table 5.2, income between the post-tax share of risky asset with human wealth ($x_h^i$) and that of risky asset which would hold without it ($x_{no}^i$) to total change in that with human wealth ($x_h^i - x_h$). This is given by $(x_h^i - x_{no}^i)/(x_h^i - x_h)$. These values are less than 2%. Therefore it can be concluded that the magnitude of increase in the share of risky asset induced by human wealth effect is small for all cohorts.

On the other hand, capital gains tax decreases the share of risky asset. The combined effect of income tax on interest and capital gains tax is a decrease in the share of risky asset as shown in (C) of Table 5.2. Interest income tax and capital gains tax decrease the conditional mean of rate of return on composed nonhuman wealth. The combined effect of both taxes is a large decrease in the conditional mean.

The percentage change in savings rate is calculated by the average values of savings rate through the lifetime after imposing the tax in each case. As we see from (B) of Table 5.2, income...
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tax on interest decreases the savings rate, and it is less than minus 20% for all cohorts. Since the estimated value of Arrow and Pratts' measure of relative risk aversion \((-\rho)\) is larger than one, this outcome is consistent with the analysis in §3.3. On the other hand, as the analysis in §3.3 implies, capital gains tax increases savings rate for all cohorts. Hence, looking at the combined effect of income tax on interest and capital gains tax by using predicted values of \(s_t\), this effect decreases savings rate for Cohort-2, while it increases this savings rate for Cohort-1 and Cohort-3 ((D) of Table 5.2).

Table 5.3  The welfare cost of income tax on interest, capital gains tax, and both of the taxes

<table>
<thead>
<tr>
<th>cohort</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>interest income tax only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L(\tau_i)); the ratio of intertemporal compensation to total asset holdings</td>
<td>0.051</td>
<td>0.042</td>
<td>0.032</td>
</tr>
<tr>
<td>capital gains tax only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L(\tau_c)); the ratio of intertemporal compensation to total asset holdings</td>
<td>0.036</td>
<td>0.029</td>
<td>0.026</td>
</tr>
<tr>
<td>combined effect of both taxes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L(\tau_{ic})); the ratio of intertemporal compensation to total asset holdings</td>
<td>0.071</td>
<td>0.031</td>
<td>0.042</td>
</tr>
</tbody>
</table>

Source: author's estimation. The notation of each cohort is explained in §4.2. The values of \(L(\tau)\) are expressed in terms of absolute values.

The estimated value of the welfare cost of capital income taxes are shown in Table 5.3. \(^{15}\) \(L(\tau_i)\), \(L(\tau_c)\), and \(L(\tau_{ic})\) indicate the cost of income tax on interest, capital gains tax, and both taxes respectively. The ratio of intertemporal compensation of interest income tax \(L(\tau_i)\) is larger than that of capital gains tax \(L(\tau_c)\). The range of the ratio \(L(\tau_i)\) of income tax on interest is between 3.2% and 5.1%, while that of capital gains tax \(L(\tau_c)\) is between 2.6% and 3.6%. The ratio of intertemporal compensation for both taxes \(L(\tau_{ic})\) tends to be larger than that of each tax. The range of the ratio \(L(\tau_{ic})\) is between 3.1% and 7.1%. The only exceptional case where \(L(\tau_i)\) is larger than \(L(\tau_{ic})\) is that of Cohort-2.

If we ignore committed consumption, \(L(\tau_i)\) would be compared with that calculated by Keen (1989) and the ratio of equivalent variation as a percentage of the initial consumption wealth in Hatta and Nishioka (1992). Since they do not consider uncertainty in the rate of return, capital income tax corresponds to income tax on interest in our model. Keen shows that when the value of intertemporal elasticity of substitution \(\varepsilon\) is 0.5, the discount rate is 4%, the interest rate is 3%, and \(\tau_i\) is 25%, the ratio of intertemporal compensating variation to lifetime consumption expenditures becomes 1.6%. On the other hand, according to Hatta and Nishioka (1992), the range of the rate of compensation falls between 1.31% and 2.15%. This is due to the fact that before the abolition of the exemption, so called 'the Maruyu system', the initial marginal rate of

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15: This notation was used in the previous discussions.
income tax on interest was estimated to be between 44.65% and 54.34%.

Hence, as for income tax on interest, the ratio of compensation that we calculated for Japanese workers households seems to be larger than the ratio obtained by Keen for British households and the ratio analysed by Hatta and Nishioka for Japanese households in the period before the abolition of the exemption system.

6. Concluding Remarks

Using a version of two-asset optimal lifetime consumption model with human wealth, we have analysed the effect of income tax on interest on portfolio selection and savings rate as compared to that of capital gains tax and that of both taxes. Then we have suggested a method of measuring the welfare cost of capital income taxes in this life-cycle setting.

Since we used an optimal consumption model where asset demand functions include human wealth, the effect of income tax on interest on the share of risky asset to nonhuman wealth, $x_t$, includes human wealth effect. The increase in $x_t$ is greater in this case and makes the effect of the tax on the conditional mean of optimally composed nonhuman wealth, $z_t$, ambiguous. On the other hand, capital gains tax decreases $x_t$ and $z_t$. The change in savings rate caused by income tax on interest is determined by the combination of the magnitude of relative risk aversion and excess return over variance, while that of capital gains tax depends only on relative risk aversion.

The empirical part of this paper follows from these theoretical discussions. First we estimated the Arrow and Pratts' measure of relative risk aversion using the data of Japanese workers households classified by cohort. The outcome implies that the elasticity of intertemporal substitution of consumption is inelastic for each cohort. The simulation results suggest that the magnitude of increase in the share of risky asset induced by human wealth effect is small among all cohorts and that the decrease in conditional mean $z_t$ occurs evidently after the taxes are imposed.

The combined effect of income tax on interest and capital gains tax decreases the average savings rate through the lifetime after imposing the taxes for Cohort-2, while it increases this savings rate for Cohort-1 and Cohort-3. The welfare cost where both of income tax on interest and capital gains tax are imposed is found to be between 3.1% and 7.1% of total consumption expenditure less committed consumption for the whole period after taxation. Hence, the welfare cost of both capital income taxes imposed are likely to appear substantial even if the intertemporal elasticity of substitution is inelastic for each cohort. In this respect, the possibility of improvement in the welfare cost which could be accomplished by replacing these capital income taxes with other kinds of taxes seems to be very important topics which needs to be considered in this life-cycle setting.

Appendix 1: The Derivation of Budget Constraint

The budget constraint Eq. (1.1) means that nonhuman wealth plus human wealth at the $t$-th period is equal to total wealth and labor income at $t_1$-th period less consumption expenditure plus capital income accruing from investment of safety asset and risky asset with optimal share of each asset at the $t_1$-th period plus the value of human wealth evaluated at the $t$-th period.

\[
A(t) + H(t) = x(t_1) \frac{X_1(t_1) - X_1(t)}{X_1(t_1)} (1 - \tau_v) + (1 - x(t_1)) \frac{X_2(t_1) - X_2(t)}{X_2(t_1)} (1 - \tau_v)
\]
where $X_1(t)$ and $X_2(t)$ are the prices of risky asset and safety asset at the $t$-th period, respectively, and $y_1$ is an exogenous labor income $y_t$ less committed consumption $a(p_t)$. $y_1$ is assumed to be constant. Subtracting $H(t)$ from both sides and using the equations: $y_1 \Delta t = H(t) - H(t_1)$ and $y_1 \Delta t - y_1 \Delta t = a(p_t)$, we obtain:

$$A(t) = \left[ X(t) \frac{X_1(t) - X_1(t_1)}{X_1(t_1)} (1 - \tau_1) + (1 - x(t)) \frac{X_2(t) - X_2(t_1)}{X_2(t_1)} (1 - \tau_1) \right]$$

$$\times \left[ A(t_1) + y_1 \Delta t - C(t_1) \Delta t \right]$$

$$+ i_1 \Delta t H(t) + A(t_1) + a_1(p_t) - C(t_1) \Delta t,$$

Subtracting $A(t_1)$ from both sides, substituting the expenditure function Eq. (5), and dividing both sides by $b(p_t)$, Eq. (1.2) is rewritten as:

$$(1.3) \quad W(t) - W(t_1) = \left[ x(t) \frac{X_1(t) - X_1(t_1)}{X_1(t_1)} (1 - \tau_1) + (1 - x(t)) \frac{X_2(t) - X_2(t_1)}{X_2(t_1)} (1 - \tau_1) \right]$$

$$\times \left[ W(t_1) + y_2 \Delta t - (C(t_1)/b_1(p_t)) \Delta t \right] + i_1 \Delta t Y(t) - \nu(t) \Delta t,$$

where $y_2 = y_1/b(p_t)$. Using the assumptions underlying the rate of returns, it can be expressed as:

$$(1.4) \quad W(t) - W(t_1) = [x(t_1) \exp \left( (\mu_1 - \frac{n_1}{t}) \Delta t + \Delta N \right) + (1 - x(t_1)) \exp (i_1 \Delta t - 1)]$$

$$\times \left[ W(t_1) + y_2 \Delta t - (C(t_1)/b_1(p_t)) \Delta t \right] + i_1 \Delta t Y(t) - \nu(t) \Delta t.$$

Linearly approximating $(\mu_1 - \frac{n_1}{t}) \Delta t + \Delta N$ and $i_1 \Delta t$, substituting these outcomes into Eq. (1.4) leads us to:

$$(1.5) \quad W(t) - W(t_1) = [x(t_1)\mu_1 + (1 - x(t_1))\frac{1}{t}] W(t_1) \Delta t + y_1 \Delta t^2 - (C(t_1)/b_1(p_t)) \Delta t^2$$

$$+ i_1 \Delta t Y(t) - \nu(t) \Delta t + x(t_1) \sigma_1 n(t) W(t_1) \Delta t \sqrt{\Delta t}$$

$$x(t_1) \sigma_1 n(t) \left[ \frac{y_2 \Delta t^2 - (C(t_1)/b_1(p_t)) \Delta t^2}{\Delta t} \right]$$

Hence by the linear approximation in Eq. (1.5) we obtain:

$$(1.6) \quad W(t) - W(t_1) = [x(t_1)\mu_1 + (1 - x(t_1))\frac{1}{t}] W(t_1) \Delta t + i_1 \Delta t Y(t) - \nu(t) \Delta t$$

$$+ x(t_1) \sigma_1 n(t) W(t_1) \Delta t \sqrt{\Delta t} + o(\Delta t).$$

Dividing both sides by $\Delta t$ and taking the limit in Eq. (1.6) yields Eq. (6).

Appendix 2: Derivation of Eq. (19)

We note that:

$$i_1 + \frac{(\mu_1 - i_1)^2}{2\sigma_1^2(-\rho)} = \frac{i_1 (1 + h_t)}{1 + h_t} + \frac{(\mu_1 - i_1)^2}{\sigma_1^2(-\rho)} = \frac{(\mu_1 - i_1)^2(-\rho)}{2\sigma_1^2(-\rho)}$$

$$= \frac{i_1 h_t}{1 + h_t} + \frac{i_1}{1 + h_t} + \frac{(\mu_1 - i_1)^2}{\sigma_1^2(-\rho)}$$

$$= \frac{(\mu_1 - i_1)^2(-\rho)}{2\sigma_1^2(-\rho)}.$$

On the other hand, from Eqs. (16) and (17) we have:

$$\frac{z_t}{1 + h_t} = \frac{i_1}{1 + h_t} + \frac{(\mu_1 - i_1)^2}{\sigma_1^2(-\rho)},$$

$$\frac{\sigma_1^2}{(1 + h_t)^2} = \frac{(\mu_1 - i_1)^2(-\rho)}{\sigma_1^2(-\rho)^2},$$

respectively. Substituting Eqs. (2.2) and (2.3) into Eq. (2.1) yields:

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Combining this with Eq. (18), we get Eq. (19) as follows:

\[
(2.5) \quad s_t = 1 - \left[ 1 + \delta - \frac{1 + \rho}{\rho} \frac{1 + \rho}{\rho} \left[ \frac{1}{1 + h_t} + \frac{z_t}{1 + h_t} - \frac{\sigma_i^2}{2(1 + h_t)^2} \right] \right].
\]

Appendix 3: A Proof of Assertion (i) in Section 3.1

From Eq. (24) we have:

\[
\frac{\partial x}{\partial \tau} \begin{cases} > (\cdot) \Rightarrow (2i) x(t) + \frac{x(t)(\mu_1 - i_0)(\partial H_i/\partial \tau)}{A_i + H_i} - i > (\cdot) 0. 
\end{cases}
\]

Then we can rewrite this inequality as follows:

\[
(2i) \frac{x(t)(\mu_1 - i_0)(\partial H_i/\partial \tau)}{A_i + H_i} - i > (\cdot) 0.
\]

Thus we obtain:

\[
x(t) > (\cdot) \frac{1}{2i + [(\mu_1 - i_0)(\partial H_i/\partial \tau)]/[i(A_i + H_i)]}.
\]

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