Design methods for broadband 3 dB branch-line and rat-race hybrids

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Abstract: This paper considers band-broadening design methods for branch-line and rat-race hybrids such as 90- and 180-degree hybrids. For the 90-degree hybrid, utilizing external matching networks composed of coupled-transmission lines, broadband characteristics with a relative bandwidth of over 40% can be obtained for a 3 dB hybrid designed for operation at 3 GHz based on the equivalent admittance approach. For the 180-degree hybrid, a rat-race hybrid in which right-handed transmission lines are replaced with composite right-/left-handed transmission lines also realizes broadband characteristics. The validity of these design procedures has been demonstrated by electromagnetic simulations and experiments.

Keywords: branch-line, rat-race, band-broadening design, coupled-transmission line, equivalent admittance approach, CRLH-TL

Classification: Microwave and millimeter wave devices, circuits, and systems

References


1 Introduction

Directional couplers or hybrids play important parts in balanced amplifiers, mixers, phase shifters, and other devices in microwave/millimeter-wave systems. As a quadrature hybrid, a branch-line hybrid consisting of four quarter-wavelength lines based on a distributed circuit theory is usually used. Applications of this hybrid for microwave systems, however, are limited by its performance because of its relatively narrow bandwidth of about 10%. To broaden the bandwidth, some band-broadening design methods have been reported [1, 2, 3, 4, 5, 6]. In [1, 2], although relatively broadband characteristics of return loss and isolation were achieved using multisection branch-line structures, flat coupling could not be obtained. Moreover, flat coupling over relative bandwidths of about 26% and over 50% has been realized by utilizing external matching networks consisting of an impedance step and an open-circuited stub [3, 4]. These couplers, however, occupy a large space in an MIC or MMIC. The authors have also reported broadband/tri-band branch-line couplers utilizing open-circuited series stubs [7, 8] and coupled-transmission lines [9]. Wide band 3 dB branch-line hybrids with relative bandwidths of about 50% and 42% have been reported by using a $\lambda/4$ open-circuited coupled line with a suspended microstrip structure [10] and a triplate structure [11], respectively. In addition, some design techniques for the dual-band operation of branch-line couplers have also been reported such as utilizing a dual-stage branch-line coupler with impedance transformers [12], a cross-coupled branch-line structure [13], rectangular patches [14], and shunt stubs at the input/output ports [15] or at the branches [16, 17].

As a while, a rat-race configuration which consists of three quarter-
wavelength lines (with an electrical length of 90 degrees at the design frequency) and a 3/4-wavelength line (270 degrees) has usually been used as an 180-degree hybrid. In the same manner as the branch-line hybrid, the circuit size tends to become large in an MIC or MMIC, particularly at low frequencies. To solve this problem, design techniques utilizing composite right-/left-handed transmission lines (CRLH-TLs) [18, 19] have also been reported [20, 21]. Other techniques using capacitor loading [22], coupled lines [23], dual coupled lines [24] have also been reported. In these papers, hybrid rings in which a 270-degree line is replaced with a −90-degree CRLH-TL are proposed. This, however, cannot sufficiently reduce the implementation size. Although the authors have also reported compact 180-degree hybrids utilizing CRLH-TLs, optimization of their circuit parameters is necessary [25, 26].

In this paper, we propose broadband branch-line couplers with coupled-transmission lines as external matching networks. The proposed couplers occupy a small area, because the coupled-transmission lines are integrated in input/output transmission lines, and accomplish a broadband characteristic of over 40% based on the equivalent admittance approach [3, 6]. We also propose a 180-degree hybrid composed of three −90-degree CRLH-TLs and a 90-degree right-handed transmission line (RH-TL) [27]. As a result of designing the hybrid for operation at 1 GHz, a broadband and very compact rat-race hybrid can be obtained. The validity of these design procedures is illustrated by electromagnetic simulations and experiments.

2 90-degree hybrid

2.1 Circuit construction and design method

Figure 1 shows the circuit construction of a branch-line coupler with coupled-transmission lines as external matching networks (MNs). In this figure, the characteristic impedance of each transmission line is normalized by that of the input/output port. A coupled-transmission line can be equivalently expressed as two open-circuited stubs in series and an impedance step as shown in Fig. 2 (a) [28]. In Fig. 2 (a), \( Z_{0e} \) and \( Z_{0o} \) are the even/odd-mode impedances.

Fig. 1. Circuit construction of branch-line hybrid with coupled-transmission lines as matching networks.
Fig. 2. (a) Equivalent circuit of coupled-transmission line and (b) quarter one-port circuit of proposed branch-line hybrid.

of a coupled-transmission line, respectively, and the parameters of the equivalent circuit are given by the following equations through even/odd-mode analysis.

\[
Z_s = \frac{Z_{0e} - Z_{0o}}{2} \tag{1}
\]

\[
Z_{\text{stub}} = Z_{0o} \tag{2}
\]

The circuit shown in Fig. 1 can be designed basis of the equivalent admittance approach because it possesses twofold symmetry about the planes AA’ and BB’. Figure 2 (b) shows the quarter one-port circuit of the proposed branch-line coupler including port 1 (#1). The intrinsic power split ratio and the equivalent admittance of the original branch-line coupler composed of only four quarter-wavelength transmission lines are given by [6]

\[
R = \frac{Y_2^2 - Y_2^2}{Y_2^2} \tag{3a}
\]

\[
Y_{eq} = \sqrt{RY_2} \left| \frac{1}{\sin \theta} \right| - j(1 + \sqrt{R + 1})Y_2 \cot \theta. \tag{3b}
\]

From the above equations, by connecting the same matching network, which can match a one-port circuit with equivalent admittance \(Y_{eq}\) to each input/output port, the circuit operates as a quadrature hybrid with a power split ratio of \(R\). Furthermore, since the intrinsic power split ratio of \(R\) is not dependent on frequency, flat coupling can be obtained in order to achieve the broadband matching of circuits.

Then, we analyze the branch-line coupler with coupled-transmission lines to perfectly match the circuits at three frequencies, which are two arbitrary frequencies \(f_\pm (f_+ > f_-)\) and their average center frequency \(f_0 (= (f_+ + f_-)/2)\). The equivalent impedance \((Z_{eq} = 1/Y_{eq})\) through two open-circuited stubs in series and an impedance step can be written as

\[
Z_{eq\text{all}} = Z_s + jZ_s \tan \theta + jZ_{\text{stub}} \cot \theta = Z_s + jZ_{eq} \tan \theta, \tag{4a}
\]

where

\[
Z_{eq} = \frac{1}{Y_{eq}} = \frac{1}{Y_{eq}} + jZ_{\text{stub}} \cot \theta. \tag{4b}
\]
In the above equations, $Z_{\text{stub}}$ and $Z_s$, respectively denote the characteristic impedances of the open-circuited stubs and the impedance step as shown in Fig. 2. Assuming that the lengths of the open-circuited stubs and the impedance step are quarter-wavelengths, the matching condition at the center frequency $f_0$ is given by

$$Z_{eq} = Z_s^2. \quad (5)$$

**Fig. 3.** Locus of equivalent admittances with some reference planes.

**Fig. 4.** Scattering parameters of proposed branch-line hybrid with coupled-transmission line. The circuit parameters are $Z_1 = 0.56$, $Z_2 = 0.79$, $Z_s = 0.89$, $Z_{\text{stub}} = 1.86$ ($Z_{0e} = 3.64$, $Z_{0o} = 1.86$). (a) Magnitude and (b) phase difference between output ports.
Then, the matching conditions at the two frequencies $f_{\pm}$ can be written

$$\text{Re}\left\{Z''_{eq\pm}\right\} = 1 \quad (6a)$$

$$\text{Im}\left\{Z''_{eq\pm}\right\} = Z_{stub} \cot\left(\frac{\pi f_{\pm}}{2 f_0}\right). \quad (6b)$$

Using these three matching conditions, a triband matching circuit can be successfully obtained.

### 2.2 Circuit design and scattering matrix

In this section, we design a broadband 3 dB branch-line coupler by moving

![Diagram of a 3 dB branch-line coupler](image)

**Fig. 5.** Simulation results for proposed 3 dB branch-line hybrid with coupled-transmission lines. (a) Simulation pattern, (b) magnitude, and (c) phase difference between output ports.
the two frequencies $f_\pm$ to the center frequency $f_0$. Here, assuming the two frequencies to satisfy $f_\pm/f_0 = 0.83$ and $1.17$, the circuit parameters can be obtained from the design procedure in the previous section. Figure 3 shows the locus of the equivalent admittance with some reference planes. The locus of the equivalent admittance through the matching network passes through a matching point at three frequencies as shown in Fig. 3 (b). Figure 4 shows the frequency characteristics of the scattering parameters of the proposed branch-line coupler with coupled-transmission lines. A flat power balance and a 90-degree phase difference can be obtained over a wide frequency range. The relative bandwidth is about 40.0% for 20 dB return loss. This value is about four times larger than that of a conventional branch-line coupler without matching networks.

2.3 Simulation and experimental results

To validate the proposed design procedure, we simulate the designed 3 dB branch-line coupler with coupled-transmission lines using a commercial elec-

![Fig. 6. Experimental results for fabricated 3 dB branch-line hybrid. (a) Photograph, (b) magnitude, and (c) phase difference between output ports.](image)
Electromagnetic simulator (Sonnet EM). We assume a substrate with a relative permittivity of 4.5 and a thickness of 0.508 mm. Figure 5 (a) shows the circuit pattern of the 3 dB coupler designed with a center frequency of 3 GHz. In this figure, the width and gap of the coupled-transmission lines are decided basis of a design chart [29]. Figures 5 (b) and (c) show the simulation results of the scattering parameters of the 3 dB coupler. These results are in close agreement with the theoretical results.

For demonstration, the designed broadband 3 dB branch-line coupler is fabricated on a Rogers TMM4 substrate with a relative permittivity of 4.5 and a thickness of 0.508 mm. Figure 6 (a) shows a photograph of the fabricated 3 dB coupler. The measured scattering parameters of the 3 dB branch-line coupler are illustrated in Figs. 6 (b) and (c). Although the power balance is slightly deteriorated, close agreement with the simulation results is obtained as shown in these figures.

### 3 180-degree hybrid

Figure 7 shows the design concept of the proposed CRLH-TL rat-race hybrid. Figure 7 (a) shows a conventional rat-race hybrid composed of four sections of an RH-TL, which have 90-degree and 270-degree phase delays. On the other hand, Fig. 7 (b) shows the phase delay of each transmission line for the proposed CRLH-TL rat-race hybrid, which has three sections of CRLH-TLs with a −90-degree phase delay and an RH-TL with a 90-degree phase delay at the design frequency.

#### 3.1 Design of CRLH-TL

Figure 8 shows the CRLH-TL model used in this paper. Here, an LH-TL unit cell consists of in series two capacitances ($C_L$) and a shunt inductance ($L_L$), and RH-TLs are connected on both sides of the LH-TL section in the form of distributed constant circuits. Here, we consider the LH-TL section as a cascade connection of T-circuits, that is, an LH-TL unit cell, and denote

![Fig. 7. Phase delay of each transmission line for rat-race hybrids. (a) Conventional hybrid and (b) proposed hybrid at design frequency.](image-url)
$N$ as the number of LH-TL unit cells. We give the design equation of the LH-TL unit cell below. The phase delay of the LH-TL unit cell is given by

$$\theta_L = \frac{\theta_C - \theta}{N}, \quad (7)$$

where $\theta_L$, $\theta_C$, and $\theta$ denote the phase delays of the LH-TL unit cell, the whole CRLH-TL model, and the RH-TL, respectively.

Next, we consider an RH-TL with characteristic impedance $Z$ and negative electric length $\theta_L$ ($< 0$) as shown in Fig. 9. To obtain a T-circuit with the equivalent of the negative line, the following equations are obtained by equating the $F$-matrices of both circuits.

$$\begin{bmatrix}
\cos \theta_L & jZ \sin \theta_L \\
\frac{1}{2} Z \sin \theta_L & \cos \theta_L
\end{bmatrix} = \begin{bmatrix}
1 + \frac{X_{T2}}{X_{T1}} & j \left(2X_{T2} + \frac{X_{T2}^2}{X_{T1}}\right) \\
-j \frac{1}{X_{T1}} & 1 + \frac{X_{T2}}{X_{T1}}
\end{bmatrix} \quad \quad (8)$$

Here, $Z$ is normalized by the characteristic impedance of the input/output port. Then, by replacing $X_{T1}$ and $X_{T2}$ for the T-circuit with $L_L$ and $C_L$, $L_L$ and $C_L$ can be obtained

$$L_L = -\frac{Z}{\sin \theta_L}, \quad \quad (9a)$$

$$C_L = \frac{\sin \theta_L}{Z (\cos \theta_L - 1)}, \quad \quad (9b)$$

where $L_L$ and $C_L$ are normalized by the characteristic impedance of the input/output port and the center angular frequency.

### 3.2 Circuit construction of 3 dB rat-race hybrid

In this section, we propose the circuit construction of three $-90$-degree CRLH-TLs and a $90$-degree RH-TL as shown in Fig. 7 (b). Here, we assume that the number of LH-TL unit cells is $N = 2$ and derive the parameters of the CRLH-TL model using the procedure described in Sec. 3.1. In this design, because we have a degree of freedom of the electrical length of the RH-TL, $\theta = 4 \, [\mathrm{deg.}]$ is decided upon consideration of the circuit size.
Fig. 10. Circuit construction of rat-race hybrid using three 
−90-degree CRLH-TLs and a 90-degree RH-TL.

Fig. 11. Scattering characteristics of designed rat-race hy-
brid with $Z = 1.4$, $L_{L1} = L_{L2} = 1.9$, $C_{L1} = 
C_{L2} = 1.6$, and $\theta = 4$ [deg.]. (a) Magnitude and
(b) phase difference between output ports.

The circuit construction of the proposed CRLH-TL 3 dB rat-race hybrid
is shown in Fig. 10 with its design parameters. The frequency characteristics
of the scattering parameters are shown in Fig. 11. As shown in this graph, an
equal power split ratio with the in/reverse phase can be obtained over a wide
operation band. For $-20$ dB reflection and isolation and a $\pm 0.5$ dB power
unbalance, a relative bandwidth of about 46.7% can be obtained. Compared with the original hybrid, which has a relative bandwidth of 27.8%, the proposed hybrid has an extremely wide operation band.

### 3.3 Simulation and experimental results

In this section, we simulate the designed CRLH-TL 3 dB rat-race hybrid to validate the design procedure. Figure 12 (a) shows the circuit pattern of CRLH-TL rat-race hybrid designed with a center frequency of 1 GHz, a dielectric constant of 4.5, and a dielectric substrate thickness of 0.508 mm. For the lumped elements of the LH-TL, a rectangular spiral inductor (whose minimum linewidth and line gap are 0.1 mm) and an ideal chip capacitor are implemented. Furthermore, by meandering the 90-degree RH-TL, the

![Simulation results of designed CRLH-TL rat-race hybrid with $Z = 70.7 \, \Omega$, $L_{L1} = L_{L2} = 15.4 \, \text{nH}$, $C_{L1} = C_{L2} = 5.2 \, \text{pF}$, and $\theta = 4 \, \text{deg.}$. (a) Simulation pattern, (b) magnitude, and (c) phase difference between output ports.](image-url)
size can also be reduced. The area occupied by the circuit of the proposed hybrid is about 1/25 that of the original hybrid. Figures 12 (b) and (c) show the circuit parameters of the designed hybrid at 1 GHz and the simulation results of the $S$-parameters. These results are in good agreement with the theoretical results shown in Fig. 11.

Finally, we fabricate a prototype circuit using the above simulation pattern. In our experiment, a Rogers TMM4 dielectric substrate ($\epsilon_r = 4.5$, $h = 0.508$ mm) is used. Figure 13 shows a photograph of the trial circuit and the $S$-parameters measured by a vector network analyzer. Although the reflection coefficients are slightly deteriorated, broadband characteristics and very flat coupling can be obtained.

**4 Conclusion**

In this paper, we have demonstrated a band-broadening design method for
3 dB branch-line and rat-race hybrids. A 90-degree branch-line hybrid with coupled-transmission lines as matching networks was successfully designed with broadband characteristics having a relative bandwidth of over 40%. Furthermore, a 180-degree rat-race hybrid composed of CRLH-TLs also obtained broadband characteristics of over 40%. The validity of this design procedure has been demonstrated by electromagnetic simulations and experiments.

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