Advanced polarization estimation method using the spatial polarimetric characteristic of antenna

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Abstract: The polarization of radar antenna vary with scan direction. This phenomenon is called Spatial Polarization Characteristic (SPC) of antenna. Based on the SPC of single polarized antenna, a novel polarization estimation method is established. Comparing with the traditional estimation method, this one has the following advantages: (i) It does not need dual-polarization channels to estimate the signal polarization; (ii) The polarization and angle of arriving (AOA) of a periodic signal can be estimated simultaneously; (iii) The full-polarization pattern of the antenna can be matched automatically during the estimation. The feasibility of this method is proved by simulation.

Keywords: spatial polarization characteristic (SPC), angle of arriving (AOA), polarization estimation, multiple signal classification (MUSIC)

Classification: Microwave and millimeter wave devices, circuits, and systems

References

1 Introduction

The usefulness of polarization information has already been established in radar applications [1, 2]. Usually the signal polarization can only be estimated using multiple polarization antennas and multiple polarization channels [2]. The single polarization radar doesn’t own the ability of polarization information processing because a single polarized antenna is unable to measure polarization. Therefore the performance of single polarization radar is directly restricted.

This restriction is broken in [3]. According to antenna theory, for a given frequency and space point in the far-field area, antenna has a predefined polarization direction. At constant frequency, the antenna polarization of radiation field varies with different observe directions, suggesting that the antenna polarization is a function of the spatial direction. The antenna polarization changing in different positions and oriented directions is known as spatial polarization characteristic (SPC) of antenna [3]. Based on the analysis of spatial polarization characteristic (SPC) of single polarization antenna, the signal polarization is estimated by using least square method. Although huge ground break is made in [3], the feasibility of this method is limited because AOA of the signal can not be estimated and the full-polarization (full-polar) pattern of the antenna can not be matched automatically with the received signals.

These problems are resolved in this paper. Firstly, the model of antenna SPC is introduced based on the full-polar pattern of an offset parabolic reflector antenna measured in outfield. Then the polarization estimation method in [3] and its limitation are summarized. An improved polarization estimation method based on antenna SPC is proposed. Not only polarization but AOA of the signal can be estimated by this new method. Finally, the feasibility of this method is testified by simulation experiment. The working performance of single polarization radar, such as traditional weather radar, can be improved remarkably, after gaining the ability of polarization information processing by using this method.

2 SPC model of antenna

In order to obtain the prior knowledge of SPC of a typical parabolic reflector antenna, the co-polarization (co-polar) and cross-polarization (cross-polar) gain of an offset parabolic reflector antenna is measured in outfield. The polarization characteristic of the antenna with 0° elevation angle is shown in Fig. 1. It is shown in Fig. 1 that the antenna polarization characteristic changes significantly and regularly in azimuth direction. Similar results and more details can be found in [4].

Suppose the peak gain of the radar antenna is $G$, the normalized co-polar pattern is $g_{//}(\varphi, \theta)$ and the normalized cross-polar pattern is $g_{\perp}(\varphi, \theta)$, where $\varphi$ and $\theta$ denote the elevation and the azimuth angle respectively. On the polarization basis, the antenna SPC can be described by

$$G(\varphi, \theta) = G \begin{bmatrix} g_{//}(\varphi, \theta) & g_{\perp}(\varphi, \theta) \end{bmatrix} = Gg(\varphi, \theta)$$
where $g(\varphi, \theta) = [g_{//}(\varphi, \theta) \quad g_{\perp}(\varphi, \theta)]^T$ is the antenna spatial polarization vector.

In addition, when the antenna SPC is only concerned within a particular elevation $\varphi$, the two-dimensional function can be simplified into a one-dimensional function denoted by $G(\theta)$. When the radar antenna scans in azimuth direction, the antenna full-polar pattern can be characterized by time as

$$G(t) = G \cdot g(\omega t + \theta_0)$$

here $\omega$ is the antenna scanning speed, $\theta_0$ is the initial angle.

### 3 Polarization estimation using SPC

#### 3.1 Signal model

Suppose the pulse width of the signal is, the signal can be described as

$$s(t) = \text{rect}(t, \tau)e(t) \left[ 1 \quad \tan(\gamma)e^{j\phi} \right]$$

where $e(t)$ is the complex envelope, $\gamma$ and $\phi$ are the angle descriptors of signal polarization, $\text{rect}(t, \tau) = 1$, if $0 < t < \tau$.

The received signal of the radar antenna in the pulse width can be described as

$$s_r(t) = (G)^T(t)s(t) = G\text{rect}(t, \tau)e(t)\mathbf{g}^T(\omega t + \theta_0)\mathbf{j}(\gamma, \phi)$$

here $\mathbf{j}(\gamma, \phi) = \left[ 1 \quad \tan(\gamma)e^{j\phi} \right]^T$ is the signal polarization and is considered as a constant during antenna scan period.

Suppose the signal is periodic with the period $T_s$, the scan period of antenna is $T_a$. Usually $T_a \gg T_s$, the antenna gain can be considered as a constant during the signal period, therefore the received signal can be expressed as

$$s_r = \begin{bmatrix}
    g_{//}(\theta_1) & g_{\perp}(\theta_1) \\
    g_{//}(\theta_2) & g_{\perp}(\theta_2) \\
    \cdot & \cdot & \cdot \\
    g_{//}(\theta_N) & g_{\perp}(\theta_N)
\end{bmatrix}
\begin{bmatrix}
    s_{//}(\theta_1) \\
    s_{//}(\theta_2) \\
    \cdot \\
    s_{//}(\theta_M)
\end{bmatrix}$$

here, $N$ is the number of samples of antenna SPC during $T_a$ with the sampling interval $\omega T_s$; $M$ is the number of signal samples in $T_s$. The spatial relation is shown in Fig. 2.
3.2 Limitation of polarization estimation using SPC

In [3], a novel polarization estimation method is proposed. The method is summarized as following:

The received signal in Eq. (5) can be written in the form of linear equation as

\[ s_r(θ, t) = G^T(θ)s(t) + n(θ, t) \]  

(6)

Here, \( n(θ, t) \) is the additional white Gaussian noise (AWGN).

Apparently, \( s(t) \) can be estimated by using least square method

\[ s(t) = (G^H(θ)G(θ))^{-1}G^H(θ)s_r(θ, t) \]  

(7)

It is obvious that the estimation result will be wrong if the SPC \( G(θ) \) used in Eq. (7) is not matched with the one in Eq. (6). Since AOA of the signal is unknown before estimation, it is hard to achieve the matching. Furthermore, AOA of the signal can not be measured after processing of Eq. (7), since the affect of the antenna pattern is eliminated. These problems seriously restrict the feasibility of this method.

3.3 Improved polarization and AOA estimation method

To resolve the problems, an improved estimation method is shown as following:

The covariance matrix of the receiving signal \( R \) is given by

\[ R = E[s_s s_r^H] = G^T(θ)j(γ, φ)E[e_s e_r^H]j^H(γ, φ)G^*(θ) + σ^2 I \]

\[ = A(θ, γ, φ)R_i A^H(θ, γ, φ) + σ^2 I \]  

(8)

where \( e_s = [e(t_1), e(t_2), \cdots, e(t_M)]^T \) is the samples of \( e(t) \) in \( T_s \); \( σ^2 \) is the variance of noise; \( I \) is unit matrix; \( A(θ, γ, φ) = G^T(θ)j(γ, φ) \).

Although the antenna SPC \( G(θ) \) can be measured at all directions previously, the segment \( G(θ_k), k = 1, 2, \cdots, N \) which matches the received signal is unknown. \( γ \) and \( φ \) are the polarization state need to be estimated.

For MUSIC algorithm, the eigendecomposition of \( R \) is

\[ R = U_i \Sigma_i U_i^H + U_n \Sigma_n U_n^H \]  

(9)
where $\Sigma_i$ is the diagonal matrix contains the largest eigenvalue, $U_i$ denotes the corresponding eigenvector; $\Sigma_n$ is the diagonal matrix contains the rest eigenvalues, and $U_n$ is the corresponding orthogonal eigenvectors.

Then the polarization and AOA of the signal can be estimated from the location of the peak of the following MUSIC spectrum function

$$P_{\text{MUSIC}}(\gamma, \phi) = \frac{1}{A^H(\theta, \gamma, \phi)U_nU_n^HA(\theta, \gamma, \phi)}$$

(10)

It should be noticed that the full-polar pattern of the antenna $G(\theta_k), k = 1, 2, \cdots, N$ can be matched automatically by searching the peak of $P_{\text{MUSIC}}(\gamma, \phi)$ in the azimuth direction $\theta$ using the SPC samples of the antenna measured previously.

### 4 Simulation results

In the simulation experiment, a radar adopted reflector antenna is mechanical scanning in azimuth direction. Its antenna pattern is shown in Fig. 1. The scan speed of the antenna is 6 rpm. The signal is a pulse with frequency 300 MHz, pulse-width 10 $\mu$s, PRI 25 $\mu$s, and polarization state $\gamma$ 35°, $\phi$ 45°. The AOA of the signal is 3°. The waves received between the azimuth angle window 2° ~ 4° is sampled with frequency 1 GHz. Signal to noise ratio is 20 dB. The spatial and time relation is shown in Fig. 2. The MUSIC spectrum is calculated based on the received waves.

The simulation results are shown in Fig. 3. In Fig. 3 (a), the MUSIC spectrum is given with different $\theta$ when $\gamma$ = 35°, $\phi$ = 45°. It is obvious that the peak of MUSIC spectrum is located at the AOA of the signal. In Fig. 3 (b), the MUSIC spectrum with different $\gamma$, $\phi$ is shown, when $\theta$ = 3°. The peak of the MUSIC spectrum is located at the polarization state of the signal with $\gamma$ = 35°, $\phi$ = 45°. Therefore, the polarization and AOA of the signal can be estimated correctly at the peak location of the MUSIC spectrum with the automatically matched full-polar pattern of the antenna.

![Fig. 3. MUSIC spectrum with different $\gamma$, $\phi$ and $\theta$](image)

### 5 Conclusion

An improved polarization estimation method using SPC of antenna is proposed in this paper. Comparing with the traditional estimation method, this
method has the following advantages: (i) It does not need dual-polarization channels to estimate the signal polarization; (ii) The polarization and AOA of periodic signals can be estimated simultaneously; (iii) The full-polarization pattern of the antenna can be matched automatically. The feasibility of this method is testified by simulation experiment. The working performance of single polarization radar can be enhanced after owning the ability of polarization information processing by using this method.

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