Analysis of switching frequency variation in self-oscillating class-D audio amplifiers

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Abstract: In this letter, a mathematical model in self-oscillating class-D audio amplifiers is proposed to describe the switching frequency variation as a function of the modulation depth. It is alleged in a very simple form which can be applied to any general structure adopting phase-shifting in the feedback filter, time delay of the loop, and the hysteresis window at the comparator. The main focus is set on the analysis of the phase-shifting filter which shows the least dependency on the modulation depth, which is not yet revealed.

Keywords: class-D amplifier, self-oscillation, delta-sigma modulation, modulation depth

Classification: Integrated circuits

References


1 Introduction

The demand for near 1-W audio amplifier in mobile devices vitalizes the study on self-oscillating class-D amplifiers (SODA). Fig. 1(a) shows a typical structure of SODA. The oscillation frequency is determined by the total time delay of the loop ($t_D$), the hysteresis-based delay at the comparator ($h$), and the bandwidth of the feedback filter ($f_0$). A time delay can be effectively realized at the comparator, and possibly interpreted simply as $h\tau_{int}$ ($h$ is the hysteresis window, $\tau_{int}$ is the time...
constant of the integrator) if the signal is linearly approximated as shown in Fig. 1(c) [1].

The main feature of the self-oscillation scheme lies in a relatively lower switching (oscillating) frequency of several hundreds of kilo Hertz, tracking the input amplitude more or less. Thus, it generates less quantization noise and idle tones than a PWM or a delta-sigma modulator of the same order [1].

Fig. 1. Typical features of self-oscillating class-D amplifier (SODA) (a) basic structure (b) hysteric comparator (c) delay time generated in hysteresis window (d) proposed differential circuit structure with dual supply voltages (e) non-hysteric comparator circuit used in (b) (f) rail-to-rail folded-cacode 2 stage op-amp.
Fig. 1(d) shows the proposed circuit with dual supply voltages ($K>1$), the higher supply voltage at the output stage is used to transfer more power to the load. Fig. 1(b) shows an implementation of the hysteresis window of $h$ using two non-hysteric comparators which consist of a rail-to-rail pre-amplifier followed by a latch, and a differential-to-single converter as shown in Fig. 1(e). The op-amps used in the pre-amplifier (differential mode) and the integrator (single-ended mode) operate at rather low frequencies (less than 1 MHz), but require wide common-mode range, large output swing, and high dc gain. Accordingly, a rail-to-rail two stage scheme as shown in Fig. 1(f) is adopted where the 1st folded-cascode stage achieves the high gain, and the 2nd driver stage permits the wide output swing with extra gain.

Mobile devices generally employ a filter-less method in which the pulse-width modulated output signal is not reconstructed [2, 3], thereby, the electromagnetic interference (EMI) should be taken into consideration. In this regard, the self-oscillation gets some advantage as the self-oscillated frequency is discovered spread, distributing its energy over a range of switching frequencies.

On the other hand, the switching frequency variation is of a great concern for mobile applications. The differences in switching frequency may cause audible inter-modulation in multi-channel systems. Sometimes, the switching frequency may be degraded down into the signal band as the modulation depth increases. Therefore, it is important to analyze the effect of the modulation depth on the switching frequency variation.

Previous works disclose models for the delay time or hysteresis-based self-oscillation [3, 4], but any theoretical analysis for the phase-shifting self-oscillation has not yet been reported. This letter intends to reveal a very simple mathematical model of the oscillation frequency variation for a combined structure of the time delay, the hysteresis control, and the phase-shifting in feedback filter.

## 2 Analysis of switching frequency variation

When the loop gain in the oscillation range is large enough, the oscillation is always settled by the phase condition (Barkhausen’s criteria):

$$180\degree = \angle\text{integrator}(\theta_{\text{int}}) + \angle\text{feedback filter}(\theta_o)$$

$$+ \angle\text{hysteresis}(\theta_h) + \angle\text{time delay}(\theta_d)$$

(1)

It is possible that the phase shift at the 1st-order integrator $\theta_{\text{int}} = \pi/2$ [rad] at the oscillating frequency $f = f_{\text{osc}}$. With the adoption of a 1st-order passive filter (mostly probable as in Fig. 1(d)), the phase shift (in degree) at the feedback filter is

$$\theta_o = \frac{180}{\pi} \cdot \tan^{-1} \left( \frac{f_{\text{osc}}}{f_o} \right).$$

(2)

The proportionality between time and phase derives the phase shifts (in degree) of the hysteresis comparator and the delay stage ($\omega_u = 1/t_{\text{int}}, f_D = 1/t_D$):

$$\theta_h = 360 \cdot \frac{h f_{\text{osc}}}{\omega_u}, \quad \theta_d = 360 \cdot \frac{f_{\text{osc}}}{f_D}.$$  

(3)

A pulse-width modulated signal can be represented by the modulation depth (modulation index) $M$ which is defined between $+1$ and $-1$, and related to its duty
cycle $D$ as $M = 2 \cdot D - 1$. The time delay and the hysteresis delay in Fig. 1(a) are known to be dependent on the input amplitude (consequently, modulation depth $M$) [3, 4] given by:

$$
time \text{ delay} = \frac{t_D}{1 - M^2} \quad \text{hysteresis delay} = \frac{h \cdot \tau_{int}}{1 - M^2}
$$

(4)

where $t_D$, and $h \tau_{int}$ are measured when $M = 0$.

These simple equations are verified to be perfectly matched with the system-level simulations in the audio band [3, 4]. The delay (phase shift) caused by the feedback filter, on the other hand, is less dependent on $M$ as the input to the feedback filter is a digital pulse. Therefore, the oscillation criterion derived from (1) to (4) is given by:

$$
\pi = \frac{\pi}{2} + \tan^{-1}\left(\frac{f_{osc}}{f_0}\right) + \frac{2\pi f_{osc}}{(1 - M^2)} \left(\frac{h}{\omega_0} + \frac{1}{f_D}\right) \text{ [radian]}. \quad (5)
$$

Seemingly, the feedback filter term (arctangent) is not dependent on $M$ in (5).

3 Oscillating frequency model and simulation results

Arctangent approximation: To obtain the simplified expression for $f_{osc}$, the arctangent function in (5) is approximated. Three distinguishable polynomial functions can be utilized to approximate the arctangent function over the range of the argument (angle) [5]. Over the different ranges in $0^\circ$~$90^\circ$, the approximation has been obtained by:

$$
\tan^{-1}(x) \approx \varphi(x) \text{ for } 0^\circ \sim 50^\circ, \quad \tan^{-1}(x) \approx \pi/2 - \varphi(x) \text{ for } 50^\circ \sim 90^\circ \quad (x = f_{osc}/f_0).
$$

For the sake of the simplicity, the fitting function $\varphi(x)$ is proposed in terms of the following three different formula:

$$
\varphi(x) = \pi x \quad (6)
$$

$$
\varphi(x) = \frac{\pi}{4} x + 0.273 x (1 - x) \quad (7)
$$

$$
\varphi(x) = \frac{x}{1 + 0.28086x^2} \quad (8)
$$

The oscillation frequency is now approximated in each segmentation, e.g. in the range of $0^\circ$~$50^\circ$, the simplest expression is derived from (5) and (6) as

$$
f_{osc} \approx \frac{2\omega_0 f_0 f_D (1 - M^2)}{\omega_0 f_D (1 - M^2) + 8 f_0 (\omega_0 + h \cdot f_D)} \quad (9)
$$

However, the other expressions using (7) or (8) are too complicated to be expressed. The system-level simulations have been performed in Matlab based on Fig. 1(a). The results are shown in Fig. 2(a) by varying the phase shifting from $0^\circ$ (non-phase shifting) to $80^\circ$ at the loop filter. The design parameters are:

$f_0 = 384 \text{ kHz}$, $\omega_0 = 0.31 \cdot 10^6 \text{ [rad/s]}$, $f_D = 2.76 \cdot 10^7 \text{ [Hz]}$ for phase shifting $= 45^\circ$.

$f_0 = 222 \text{ kHz}$, $\omega_0 = 0.55 \cdot 10^6 \text{ [rad/s]}$, $f_D = 2.76 \cdot 10^7 \text{ [Hz]}$ for phase shifting $= 60^\circ$.

$f_0 = 33.6 \text{ kHz}$, $\omega_0 = 1.38 \cdot 10^6 \text{ [rad/s]}$, $f_D = 2.76 \cdot 10^7 \text{ [Hz]}$ for phase shifting $= 85^\circ$.

As expected, the normalized oscillation frequency with respect to $f_{osc} = 384 \text{ kHz}$ at $M = 0$ drops more rapidly as the filter phase decreases. The calculated errors deviated from the system-level simulations in Matlab are shown in Fig. 2(b)~(d).
The simplest model of Eq. (6) reveals the maximum errors, whereas the most complicated Eq. (8) shows the least errors for the most of cases except for the high phase shift $\phi_0 = 60^\circ$ at high modulation index $M > 0.9$.

Circuit simulation: The PSD of the simulation results of Fig. 1(d) in a 0.18 µm CMOS process is illustrated in Fig. 2(e). The design parameters are: $V_{DD} = 5$ V,
\[ K = 2.78 \ (V_{DD}/K = 1.8 \text{ V}), \ \tau_{int} = R_{int}C_{int} = 0.56 \mu s, \ \tau_{fb} = R_{fb}C_{fb} = 3.67 \mu s, \ h = 0.05 \] so that the phases are to be allocated as \( \theta_{int} = 90^\circ, \ \theta_{o} = 85^\circ, \ \theta_{h} = 5^\circ, \ \theta_{ld} = 0^\circ. \]

The dc gain of the op-amp is 120 dB.

A sinusoidal input of 0.2 V at 1.7 kHz and a sinusoidal supply noise of 0.5 V at 13 kHz are enforced. As the input signal gets a closed-loop dc gain of 2.8, the amplified value of \(-5 \text{ dB}\) (1 V amplitude is referenced to 0 dB) is seen at 1.7 kHz, and the oscillation frequency resides near 500 kHz. The input signal is reconstructed after the 4th-order Butterworth filter with the cutoff frequency of 40 kHz.

A feature to note is the suppression of the supply noise through the feedback, deriving the noise transfer function (NTF) as high-pass filtering. That explains why the supply noise even within the bandwidth (at 13 kHz) gets reduced by more than 30 dB. It should also be noted that the spread-spectrum characteristic of the oscillation is seen distributed around the center frequency as intended for the better EMI.

4 Conclusion

This article proposes a mathematical model used for self-oscillating class-D audio amplifiers including phase-shifting, time delay, and hysteresis window. The oscillating frequency variation in terms of the modulation depth is described; it is desirable that more portion of the phase shift in the feedback filter is to be allocated than the other two terms from the perspective of the less variation. The phase-shifting of the loop filter is assorted into different angular regions. That is, the proposed models adopting the arctangent approximation covering up \(0^\circ \sim 90^\circ\) were presented along with the behavioral-level simulations. The margin of less than 5% error was reported with the proper choice of a model. A test circuit was implemented, and proved the validity of the proposed scheme to a certain extent.

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