Improved Max-Log-MAP BICM-IDD receiver for MIMO systems

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Abstract: A novel bit-interleaved coded modulation with iterative detection and decoding (BICM-IDD) receiver for multiple-input and multiple-output (MIMO) systems using Max-Log-MAP algorithm is proposed. This receiver improves the detection and decoding performance by an improved turbo principle, in which pre-scaling the input information of the detector is performed at each iteration. From an information theory perspective, the proposed scheme is proved to outperform the traditional iterative architecture. Simulations results show that the proposed receiver significantly reduces the performance loss incurred by the suboptimal Max-Log-MAP detection and decoding algorithm with small additional complexity.

Keywords: BICM-IDD, MIMO, Max-Log-MAP

References


1 Introduction

MIMO systems based on bit-interleaved coded modulation with iterative detection and decoding (BICM-IDD) can approach the capacity of MIMO channels [1]. The optimal detection and decoding algorithm for BICM MIMO receiver are the log maximum a posteriori (Log-MAP) detection algorithm [1], and Log-MAP decoding algorithm [2], respectively. However, the Log-MAP algorithm is impractical for...
real systems due to its prohibitive complexity. Some suboptimal detection and decoding algorithms using the Max-Log approximation have been adopted, such as list sphere detection (LSD) [1], K-Best sphere detection [3], single tree search sphere detection (STS-SD) [4] and Max-Log-MAP turbo decoding algorithm [2]. The Max-Log-MAP algorithm significantly reduces the computational complexity of the optimal Log-MAP algorithm, and its performance is insensitive to a scaling of the input log-likelihood ratios (LLR), which implies the channel noise variance is not required. However, it introduces some performance loss due to the bias in the a priori information caused by the Max-Log approximation.

In this paper, we consider a BICM-IDD MIMO receiver employing the Max-Log-MAP algorithm. Based on the extrinsic scaling technique used in turbo decoder [5], a novel modification of the Max-Log-MAP BICM-IDD scheme is proposed. The approach compensates the performance loss by simply scaling the input extrinsic LLRs of the MIMO detector using an optimized scaling factor at each receiver iteration. From information theory perspective, we prove that the proposed method outperform the traditional architecture by providing additional mutual information between the detector and decoder. Simulation results indicate that this method can improve the performance of BICM-IDD MIMO receiver with Max-Log approximation.

2 System model

Conventionally, the receiver architecture employing the BICM-IDD scheme is given by Fig. 1 [1]. The MIMO detector generates extrinsic information \( L_{E1} \) using the received signal \( y \) and the a priori knowledge \( L_{A1} \) provided by the channel decoder. Then \( L_{E1} \) is de-interleaved and becomes the a priori input to the soft-input soft-output (SISO) channel decoder, which calculates extrinsic information \( L_{E2} \) of the coded bits. Then \( L_{E2} \) is re-interleaved and fed back as refined a priori knowledge to the detector. This process is known as an outer iteration, or turbo iteration, in contrast with the inner decoding iteration performed within the channel decoder. In the first iteration, \( L_{A1} \) is not available and is assumed to be 0.

Consider a MIMO system based on the Max-Log-MAP BICM-IDD scheme with \( N_T \) transmit antennas and \( N_R \) receive antennas. Assuming transmission over flat fading channel, the received symbol vector \( y \) can be written as

\[
y = Hs + n
\]  

(1)
where $\mathbf{H}$ is $N_T \times N_T$ channel matrix and $\mathbf{s}$ is a $N_T$ transmit symbol vector whose entries are taken from $M$-QAM Gray mapped constellation points with $M = 2^{M_c}$ and $M_c$ is modulation order. The vector $\mathbf{n}$ is zero mean independent and identically distributed Gaussian noise samples with variance $N_0$ per complex dimension.

Using Bayes theorem and exploiting the independence of each bit due to the BICM scheme, the extrinsic LLR value $L_{E1}(x_i)$ for each bit $x_i$ is computed as [1]:

$$L_{E1}(x_i) = \max_{x_i = \pm 1} \left\{ -\frac{1}{N_0} ||\mathbf{y} - \mathbf{Hs}||^2 + \frac{1}{2} \mathbf{x}^T[i]\mathbf{L}_{A1,i} \right\}$$

where $x_i = \pm 1$ stands for the $i$th bit of the block of $N_T \times M_c$ coded bits $\mathbf{x}$, $\mathbf{x}_{[i]}$ denotes the sub-vector of $\mathbf{x}$ obtained by omitting its $i$th bit $x_i$, and $\mathbf{L}_{A1,i}$ is the vector of priori LLRs without its $i$th element.

The extrinsic LLR of the decoder is

$$L_{E2}(x_i) = \max_{x_i = \pm 1} \left\{ \mathbf{1}_{x_i = \pm 1} \frac{1}{2} \mathbf{x}^T[i]\mathbf{L}_{A2,i} \right\} - \max_{x_i = \pm 1} \left\{ \mathbf{1}_{x_i = \pm 1} \frac{1}{2} \mathbf{x}^T[i]\mathbf{L}_{A2,i} \right\}$$

3 Proposed BICM-IDD receiver for MIMO systems

The proposed BICM-IDD receiver is to scale the $L_{A1}$ by the scaling factor $\alpha$, and thus we propose a modified BICM-IDD scheme for MIMO systems as shown in Fig. 2, where only one additional multiplication is required per bit compared to the traditional scheme. There are some other methods to compensate the performance degradation. A method called LLR correction is proposed in [4], which needs to choose a correction function depending on the side information such as received signal-to-noise ratio (SNR), the channel matrix $\mathbf{H}$, etc. That adds additional computation complexity.
To analyze the proposed scheme mathematically, we make two assumptions. The first assumption is the a priori and extrinsic information are statistically independent over many iterations. The second assumption is the extrinsic information can be modeled by a Gaussian random variable. The idea behind the proposed scheme is to provide some impurity into the a priori information of the detector. This impurity means that it slightly undermines the first assumption.

As shown in Fig. 2, in the $q^{th}$ iteration, the output LLR of the detector is modified to

$$L_{E1}^{(q)} = L_{E1}^{(q)} - a L_{E1}^{(q)} = L_{E1}^{(q)} + (1 - a) L_{E1}^{(q)}$$  \hspace{1cm} (4)

Then the a priori LLR of the decoder is computed as

$$L_{A2}^{(q)} = \Pi^{-1}(L_{E1}^{(q)}) = L_{E2}^{(q)} + (1 - a) L_{E2}^{(q-1)}$$  \hspace{1cm} (5)

Put this equation to (3) and the output LLR of the decoder becomes

$$L_{E2}^{(q)}(x_i) = \max_{x \in \mathcal{X}} \bigg\{ I_{(q+1)} 1/2 x^T [L_{A2}^{(q)} + (1 - a) L_{E2}^{(q-1)}] \bigg\}$$  \hspace{1cm} (6)

Let $\gamma_i = I_{(q+1)} 1/2 x^T [L_{A2}^{(q)} - L_{E2}^{(q-1)}]$, $\eta_i = (1 - a) I_{(q+1)} 1/2 x^T L_{E2}^{(q-1)}$, then it can be re-written as

$$\max_{x \in \mathcal{X}} \{ \gamma_i + \eta_i \} - \max_{x \in \mathcal{X}} \{ \gamma_i \} = \max_{x \in \mathcal{X}} \{ \gamma_i + \eta_i \} - \max_{x \in \mathcal{X}} \{ \eta_i \}$$  \hspace{1cm} (7)

where $\gamma_i$ with $b = \pm 1$ is a correction term between $x_{j,b}$ and $x_b$. If $x_{j,b} = x_b$, $\gamma_i = \gamma_i + \eta_i - (\gamma_i \neq \eta_i)$ and zero otherwise, in which the superscript $*$ denotes the optimum value of maximization. If the correction term is supposed to be zero, we can obtain the renewed output LLR of Max-Log-MAP decoder by

$$L_{E2}'(x_i) = L_{E2}(x_i) + (1 - a) \eta_i(x_i)$$  \hspace{1cm} (8)

For simplicity, we denotes $\eta_i(x_i)$ as $L_{E2}'$ and the equation becomes

$$L_{E2}'^{(q)} = L_{E2}^{(q)} + (1 - a) L_{E2}'^{(q-1)}$$  \hspace{1cm} (9)

According to the first assumption, $L_{E2}'$ is independent of $L_{E2}$. For the $(q + 1)^{th}$, the output LLR of the detector can be decomposed in a similar way above as $L_{E1}^{(q)} = L_{E1}^{(q)} + (1 - a) L_{E1}^{(q-1)}$. We now have

$$L_{A2}^{(q+1)} = L_{A2}^{(q+1)} + (1 - a) L_{E2}^{(q)} + (1 - a) L_{E2}^{(q-1)}$$  \hspace{1cm} (10)

where

$$L_{E2}'^{(q)} = L_{E2}^{(q)} + L_{E2}'^{(q-1)}$$  \hspace{1cm} (11)

Therefore, by applying (4) to (10), it can be proved that the output LLR of the detector and decoder can be decomposed as
\[ L'_E = L_E + (1 - \alpha)L_Z \] (12)

It is noted that \( L_{Z1} \) and \( L_{Z2} \) accumulate at each iteration. To prove the proposed scheme outperform the original Max-Log-MAP receiver, the mutual information between information bit and output LLR is used as a measure of performance. Let \( E \triangleq L_E, E'_T \triangleq L'_E, \) and \( Z \triangleq L_Z \). A lower bound on the mutual information between information bit \( X \) and impure extrinsic \( E_I \) is obtained by the convolution inequality for entropy powers [6]. The convolution inequality says that if \( x \) and \( y \) are independent random variable with sum \( z = x + y \), the \( 2H(Z) \geq 2H(X) + 2H(Y) \).

Let \( I(x; y) \) denote the mutual information between \( x \) and \( y \), and \( H(x) \) denote the differential entropy of \( x \). Using the convolution inequality for entropy powers in one dimension, we can obtain

\[
H(E'_T) \geq \frac{1}{2} \log_2 (2^{2H(E)} + 2^{H(1-\alpha)Z})
\]

(13)

\[
H(E'_T|X) = \frac{1}{2} \log_2 (2^{2H(E|X)} + 2^{H(1-\alpha)Z|X})
\]

(14)

When \( E \) and \( Z \) are conditionally Gaussian under \( X \), we get the equality equation. With (13), (14) and Jacobian logarithm, the mutual information between \( X \) and \( E'_T \) is bounded by

\[
I(X; E'_T) = H(E'_T) - H(E'_T|X)
\]

\[
\geq \frac{1}{2} \log_2 (2^{2H(E)} + 2^{H(1-\alpha)Z}) - \frac{1}{2} \log_2 (2^{2H(E|X)} + 2^{H(1-\alpha)Z|X})
\]

\[
\geq I(X; E) + \epsilon
\]

(15)

with equality only when \( \alpha = 1 \), where \( \epsilon = \frac{1}{2} \log_2 \left( \frac{1 + 2^{2H(E)-H(1-\alpha)Z)}}{1 + 2^{2H(E|X)-H(1-\alpha)Z|X}} \right) \). Since \( I(X; (1 - \alpha)Z) < I(X; E) \), we get \( \epsilon \geq 0 \). Hence the incremental information \( L_Z \) helps to compensate the performance loss due to Max-Log approximation.

\[ 4x4, \text{ turbo code rate}=1/2, 4 \text{ receiver iteration} \]

\[ \text{BER performance of the Log-MAP BICM-IDD, Max-Log-MAP BICM-IDD and the proposed improved Max-Log-MAP BICM-IDD for MIMO system with QPSK and 16-QAM modulation.} \]
4 Simulation results

We evaluate the performance of the proposed improved BICM-IDD receiver by simulation. We consider a coded 4 × 4 MIMO system utilizing QPSK and 16-QAM modulation over spatially uncorrelated Rayleigh MIMO channel with additive white Gaussian noise. The LTE turbo code is used, with constraint length = 4, polynomial: (feedback, redundancy) (13, 15) octal, block size = 1024 bits, code rate = 1/2 and 8 internal decoding iterations. Note that extrinsic scaling with a constant of 0.68 is used inside the turbo decoder when its component SISO decoders employed the Max-Log-MAP algorithm. At the receiver, the number of iteration between the detector and the turbo decoder is 4. Note that Max-Log-MAP detector employs the STS-SD algorithm to reduce computation complexity. In the case of the proposed Max-Log-MAP BICM-IDD receiver, the scaling factor $a$ is set to 0.45. Fig. 3 shows the BER performance of the Max-Log-MAP BICM-IDD and the proposed Max-Log-MAP BICM-IDD. The simulated performance of the optimal Log-MAP-IDD is also shown as a reference. As expected, the proposed Max-Log-MAP BICM-IDD has better BER performance than the traditional Max-Log-MAP BICM-IDD. At BER of $10^{-4}$, an improvement of 0.15 dB is obtain in QPSK case and the difference between Max-Log-MAP and Log-MAP is reduced to 0.45 dB. When 16-QAM modulation is used, the performance gain becomes more significant. The proposed Max-Log-MAP BICM-IDD scheme reduced the performance gap by 0.35 dB at BER of $10^{-4}$.

5 Conclusion

We proposed an improved turbo principle for Max-Log-MAP BICM-IDD receiver. The input extrinsic scaling method is used to correct the accumulated bias introduced by the Max-Log approximation. From an information theory perspective, the proposed method can provide incremental information for the detector and decoder at each iteration, which plays a significant role in compensating the loss due to Max-Log approximation. Through simulations, it is shown that this method improves the performance significantly even when the number of iterations is small (e.g. $N_I = 4$). As the scaling factor is a constant over all iterations, very small additional complexity is required. This optimized BICM-IDD receiver retains the low complexity and insensitivity to input LLR scaling which are the inherent advantages of the Max-Log-MAP algorithm.

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