An improved figure-of-merit equation for op-amp evaluation

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Abstract: An improved figure-of-merit (FoM) equation is proposed for op-amp evaluation. This equation has a modification factor to include the effect of the phase margin, which is not considered in a conventional FoM equation. Simulation results confirm that the proposed FoM equation drastically suppresses the effect of load capacitance variations. By using the proposed FoM equation, evaluations of op-amps become more reasonable as compared with the comparisons using the conventional FoM equation.

Keywords: op-amp, figure of merit, gain-bandwidth product, phase margin

Classification: Integrated circuits

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1 Introduction

When designing an operational amplifier (op-amp) and comparing its performance with those of other op-amps, a figure-of-merit (FoM) equation is typically used [1, 2, 3, 4, 5, 6, 7]. This equation is defined such that the FoM is proportional to the gain-bandwidth product (GBW) and the load capacitance, and is reciprocally proportional to the total current [1]. The FoM equation is useful in judging the

efficiency of op-amp operation in terms of the GBW and the drivability of the load capacitance per unit current.

However, the shortcoming of this FoM equation is that the phase margin of the op-amp is not considered whatsoever. This issue is explained as follows.

First, an op-amp is assumed to consist of two stages (a so-called two-stage op-amp). In the two-stage op-amp, a large load capacitance decreases the frequency of the second pole and hence decreases the phase margin. Therefore, the larger the load capacitance, the lower the phase margin. However, the phase margin is not considered in the FoM equation. A large FoM value is no longer meaningful if the phase margin is too small, such that the op-amp oscillates. Even though op-amps can have phase margins larger than 45°, it is difficult to fairly compare several op-amps if their phase margins are not equal.

To resolve this shortcoming of the FoM equation, a modified FoM equation, in which the phase margin is considered, is proposed.

2 Conventional FoM equation

A conventional FoM equation for GBW is defined as follows:

$$\text{FoM} = \text{GBW} \times C_L / I_{\text{total}}, \quad (1)$$

where C_L is the load capacitance and I_{total} is the total current of the op-amp [1].

Since the FoM is proportional to the load capacitance C_L , a large load capacitance leads to a large FoM value. However, a large load capacitance decreases the phase margin. The problem is that the phase margin is not incorporated into this FoM equation. Therefore, it is possible to obtain a large FoM value by using a very large load capacitance if the phase margin is sacrificed.

3 Proposed FoM equation

The proposed FoM equation is described as follows:

$$\text{FoM} = \frac{\text{GBW} \times C_L}{I_{\text{total}}} \times f(\phi_M), \quad (2)$$

where ϕ_M is the phase margin and $f(\phi_M)$ is a modification factor for considering the effect of the phase margin. The modification factor $f(\phi_M)$ is defined as follows:

$$f(\phi_M) = \frac{\tan \phi_M}{\tan \phi_{\text{Mref}}}, \quad (3)$$

where ϕ_{Mref} is a phase margin used as a reference value.

In the following, the op-amp is assumed to be a typical two-stage op-amp as shown in Fig. 1. Fig. 2 shows a Bode plot of the two-stage op-amp. Suppose that this op-amp has a unity-gain frequency of f_u , the second pole f_{p2} , and a phase margin of ϕ_{M1} with a load capacitance C_{L1} . Let us assume that ϕ_{M1} is a reference phase margin ϕ_{Mref} . Then the phase margin ϕ_{M1} ($= \phi_{\text{Mref}}$) is calculated as follows:

$$\phi_{M1} = 90^\circ - \tan^{-1} \frac{f_u}{|f_{p2}|} = \tan^{-1} \frac{|f_{p2}|}{f_u}. \quad (4)$$

When the load capacitance C_{L1} increases to C_{L2} (i.e., $C_{L2} > C_{L1}$), the second pole moves from f_{p2} to f_{p2}' . Here $|f_{p2}'|$ is lower than $|f_{p2}|$. Then the phase margin

Fig. 1. CMOS two-stage op-amp.

Fig. 2. Bode plot of a two-stage op-amp.

reduces from ϕ_{M1} to ϕ_{M2} ($\phi_{M2} < \phi_{M1}$) and the new phase margin ϕ_{M2} is described as follows:

$$\phi_{M2} = 90^\circ - \tan^{-1} \frac{f_u}{|f_{p2}'|} = \tan^{-1} \frac{|f_{p2}'|}{f_u}. \quad (5)$$

From Eqs. (4) and (5), the following is obtained:

$$\tan \phi_{M1} = \frac{|f_{p2}|}{f_u}, \text{ and} \quad (6)$$

$$\tan \phi_{M2} = \frac{|f_{p2}'|}{f_u}. \quad (7)$$

$$\begin{aligned} \therefore f(\phi_{M2}) &= \frac{\tan \phi_{M2}}{\tan \phi_{Mref}} = \frac{\tan \phi_{M2}}{\tan \phi_{M1}} = \frac{|f_{p2}'|}{|f_{p2}|} \\ &= \left(\frac{g_{m6}}{2\pi C_{L2}} \right) / \left(\frac{g_{m6}}{2\pi C_{L1}} \right) = \frac{C_{L1}}{C_{L2}}, \end{aligned} \quad (8)$$

where g_{m6} is the transconductance of an NMOS transistor M6 in Fig. 1.

Then the proposed FoM with C_{L2} is as follows:

$$\begin{aligned}\text{FoM}(\text{with } C_{L2}) &= \frac{GBW \cdot C_{L2}}{I_{\text{total}}} \cdot f(\phi_{M2}) \\ &= \frac{GBW \cdot C_{L2}}{I_{\text{total}}} \cdot \frac{C_{L1}}{C_{L2}} = \frac{GBW \cdot C_{L1}}{I_{\text{total}}}.\end{aligned}\quad (9)$$

On the other hand, when $C_L = C_{L1}$ and the phase margin is $\phi_{M1} (= \phi_{Mref})$,

$$\begin{aligned}\text{FoM}(\text{with } C_{L1}) &= \frac{GBW \cdot C_{L1}}{I_{\text{total}}} \cdot f(\phi_{M1}) \\ &= \frac{GBW \cdot C_{L1}}{I_{\text{total}}} \cdot \frac{\tan \phi_{M1}}{\tan \phi_{Mref}} = \frac{GBW \cdot C_{L1}}{I_{\text{total}}} \cdot \frac{\tan \phi_{M1}}{\tan \phi_{M1}} \\ &= \frac{GBW \cdot C_{L1}}{I_{\text{total}}}.\end{aligned}\quad (10)$$

$$\therefore \text{FoM}(\text{with } C_{L2}) = \text{FoM}(\text{with } C_{L1}). \quad (11)$$

As shown above, FoM values are equal for the load capacitances of C_{L1} and C_{L2} .

It is well known that a phase margin of 60° is considered to be the optimum value because of little ringing, fast settling, and robust stability [8]. By setting $\phi_{Mref} = 60^\circ$, the general modification factor becomes

$$f(\phi_M) = \frac{\tan \phi_M}{\tan 60^\circ}. \quad (12)$$

Hence, the proposed FoM is as follows:

$$\text{FoM} = \frac{GBW \cdot C_L}{I_{\text{total}}} \cdot \left(\frac{\tan \phi_M}{\tan 60^\circ} \right). \quad (13)$$

4 Simulations

To verify the proposed FoM equation, a two-stage CMOS op-amp was simulated, as shown in Fig. 1, using SPICE parameters for a 0.18- μm CMOS process. Table I shows the op-amp device sizes used in HSPICE simulations. The power supply voltage was 1.8 V, and the total current was 178 μA . With a capacitive load of 10 pF, a DC gain of 76 dB, a unity gain frequency of 6.1 MHz, and a phase margin of 64° were obtained.

Fig. 3 shows the load capacitance and phase margin of the op-amp. As expected, the phase margin decreases when the load capacitance increases.

Table I. Op-amp device sizes

device	W [μm]	L [μm]
M1, M2, M8	12	1.2
M3, M4	3.6	1.2
M5	12×2	1.2
M6	3.6×8	1.2
M7	12×8	1.2
$I_{\text{REF}} = 16 \mu\text{A}, C_C = 3 \text{ pF}$

Fig. 3. Phase margin and load capacitance.

(a)

(b)

Fig. 4. FoM and load capacitance. (a) Before normalization. (b) After normalization.

Fig. 4(a) shows load capacitance and FoM values calculated from the simulation results and Eq. (13). As mentioned above, a phase margin of 60° is considered optimal. Therefore, Fig. 4(a) was normalized using the FoM value at a phase margin of 60° . The normalized FoMs are shown in Fig. 4(b).

Table II shows comparison of the phase margin and FoMs for load capacitances of 5, 10, and 30 pF. The proposed FoM equation drastically suppresses the effect of load capacitance variations for a wide range of phase margins.

Table II. Comparison of the phase margin and FoMs for load capacitances of 5, 10, and 30 pF

Load capacitance [pF]	5	10	30
Phase margin [°]	72	64	45
Conventional FoM [MHz pF/mA]	181	348	850
Proposed FoM [MHz pF/mA]	332	419	501
Normalized conventional FoM	0.41	0.79	1.93
Normalized proposed FoM	0.75	0.94	1.13

5 Conclusion

In this paper, an improved FoM equation for op-amp evaluation was proposed. This equation includes a modification factor which is related to the phase margin of the op-amp.

Simulation results confirm that the proposed FoM equation drastically suppresses the effect of load capacitance variations, thereby making the comparison and evaluation of op-amps more reasonable, as compared with the comparisons using the conventional FoM equation.

Acknowledgments

This work is supported by VLSI Design and Education Center (VDEC), the University of Tokyo in collaboration with Synopsys, Inc., and Rohm Corporation.