Analytical inverter chain's delay and its variation model for sub-threshold circuits

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Abstract: Sub-threshold circuit is a promising circuit design style for IoT application. This paper concentrated on the delay model based on the transient current model in the sub-threshold region. In order to deduce the path delay model, two ways are adopted, which are the coupling capacitance equivalence and the output waveform equivalence. The distribution of path delays is rigidly proven to be lognormal distribution in the sub-threshold region. Considering different supply voltages, cell driven strengths and load capacitances, the proposed model is also validated by Monte Carlo Spice simulation under SMIC 40 nm CMOS process. Experiments show that proposed model agrees with MC simulation results with error 0.448% under the condition of 0.4 V and 99.7% probability, which proves the feasibility of the model.

Keywords: sub-threshold circuit, lognormal distribution, delay model, variation analysis

Classification: Integrated circuits

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1 Introduction

The Internet of things (IoT), which is also called machine to machine communication (M2M), has emerged as a new killer application in which billions of computers and sensor devices are interconnected with each other, enabling an autonomous exchange of information [1, 2, 3]. Low power and ultra-low power dissipation are important while enabling techniques for IoT devices. Due to the square relationship, the lower voltage is, the lower energy dissipation will be. Sub-threshold circuit [4] is a promising method for IoT, which operates at a supply voltage less than the transistor threshold voltage. Some specific applications such as wireless sensor devices that require extremely ultra-low power are all operated at sub-threshold voltage [5].

In view of transistor operation mechanisms, MOSFET changes from strong inversion to weak inversion with decreasing voltage, and its primary current also changes from drift to diffusion current [6]. Therefore, the process variation has different influences on the current and delay between super- and sub-threshold [7]. Global and local variations are the two kinds of process variations [8]. Global variation affects all devices in the same way and is the main source of variation in super-threshold. The delay distribution of super-threshold follows normal distribution, which is convenient to calculate. However, local variation influences in different ways even in the same chip and becomes increasingly significant in the sub-threshold. The delay distribution of sub-threshold follows lognormal distribution, which is much harder to analyze.

In order to understand the local variation more, many researches on path delay model under sub-threshold have been studied [9, 10, 11]. In [9], the Ion current

model is used, and the unit delay model is obtained according to a fitting parameter. Therefore, the unit delay model does not consider the real input waveform and different loads, which only give the result error of unit delay under FO4 load. Besides, the path delay is sum of unit delay with FO4 load, so it is not very reasonable for different loads. For the path delay distribution model, although lognormal distribution is proposed, the computation of the path delay variance should not be the sum of the every unit variation, and it should be obtained according to lognormal theory. The Ids current model is adopted in [10, 11], which considered the transient current with the changing of V_{gs} and V_{ds} . The delay model is accurate enough especially in [11]. However, there is still a lack of the path delay distribution modeling with process parameter variation.

In this paper, an analytical method involving the path delay distribution is proposed. The main contributions of this work are as followed. The exponential relationship between delay and threshold voltage is demonstrated in detail in Section 2, which has not been rigidly proved so far to our knowledge. Section 3 introduces the lognormal distribution theory, and path delay distribution is obtained under different voltages as well.

2 Delay model

Some analytical delay estimation techniques have been developed recently in [10, 11]. The transient current of NMOS and PMOS can be accurately approximated, and the unit and path delay are well estimated. However, none of path delay model take V_{th} variation into account.

2.1 Sub-threshold current

The sub-threshold drain-source current [11] of NMOS is given by

$$I_{dsn} = I_{0n} \frac{W_n}{L} \cdot \exp\left(\frac{V_{gs} - V_{thn0}}{n_n V_T}\right) \cdot \exp\left(\frac{\lambda_n V_{ds}}{n_n V_T}\right) \cdot \left(1 - e^{-\frac{V_{ds}}{V_T}}\right) \quad (1)$$

where

$$I_{0n} = \mu_n C_{ox} (n_n - 1) V_T^2 \quad (2)$$

$$V_T = kT/q. \quad (3)$$

λ_n is the DIBL (Drain Induced Barrier Lowering) coefficient, V_T is the thermal voltage, V_{thn0} is the threshold voltage at zero bias voltage and n_n is the sub-threshold slope factor. A variable M_n is defined and expressed by

$$M_n = I_{0n} \frac{W_n}{L} \exp\left(-\frac{V_{thn0}}{n_n V_T}\right). \quad (4)$$

Similarly, the drain-source current of PMOS can be obtained.

The sub-threshold current with changing input and output voltage is shown by the arrows in Fig. 1. S. Keller, et al. [9] build the delay model based on the on state current I_{on} , which is marked by red point. Obviously, I_{on} is greater than the actual current and cannot accurately represent the entire current, so the delay model is built based on a fitting parameter with FO4 load in [9]. However, in this paper, the transient current is studied based on (1), which reflect the relationship with V_{gs} and

Fig. 1. Transient current of NMOS.

V_{ds} respectively, and the ramp input is also considered, so the delay model is built without any fitting parameters.

Table I. Technology parameters of transistor in SMIC40 nm.

Parameters	NMOS	PMOS
I_0 (A)	7.74e-09	2.263e-09
λ	0.13	0.16
V_{th0} (V)	0.4895	-0.572
n	1.89	1.92

For SMIC 40 nm LVT technology at nominal temperature, transistor parameters are listed in Table I, where $L = 40$ nm and $W_P = W_N = 120$ nm. The comparisons of the transient current model with Spice simulation are shown in Fig. 2, where (a) and (b) are the results of NMOS and PMOS respectively.

Fig. 2. Comparisons of transistor transient current with Spice.

2.2 Output waveform

According to papers [10, 11], the output waveform can be expressed by piecewise function. In this paper, in order to simplify the computation and keep its accuracy, two ways are adopted in the modeling.

2.2.1 Gate-drain coupling capacitance equivalence

J. R. Tolbert et al. [10] do not consider coupling capacitance, and P. Corsonello et al. [11] have a complicated output waveform expression when taking coupling

capacitance into consideration, which results in complexity of the following delay model. Therefore, Miller equivalent method is adopted to simplify the complexity in this paper.

Fig. 3. Coupling capacitance equivalence.

Fig. 3 shows that coupling capacitance (C_M) exists between gate and drain of MOSFET. When the input and output waveform change in different directions, $2C_M$ is equal to the gate port, while C_M is equivalent capacitance with output unchanged. Moreover, in order to further simplify the capacitance, averaged $1.5C_M$ is used in the following delay computation.

Therefore, the equation of falling output waveform is expressed as

$$V_{out} = \begin{cases} \frac{-n_n V_T}{\lambda_n} \cdot \ln \left(\frac{M_n \lambda_n \tau}{V_{dd} C_L} (e^{\frac{V_{dd}}{n_n V_T \tau} t} - 1) + e^{-\frac{\lambda_n V_{dd}}{n_n V_T}} \right) & 0 < t < \tau \\ \frac{-n_n V_T}{\lambda_n} \cdot \ln \left(e^{-\frac{\lambda_n V_{out-\tau}}{n_n V_T}} + \frac{M_n \lambda_n e^{\frac{V_{dd}}{n_n V_T}}}{C_L n_n V_T} (t - \tau) \right) & \tau < t < t_{-3V_T} \\ V_T \cdot \ln(1 + (e^3 - 1) \cdot e^{-\frac{M_n e^{V_{dd}/n_n V_T}}{V_T C_L} (t - t_{-3V_T})}) & t > t_{-3V_T} \end{cases} \quad (5)$$

In a similar way, the result of the rising output waveform is given by

$$V_{out} = \begin{cases} \frac{n_p V_T}{\lambda_p} \cdot \ln \left(1 + \frac{M_p \lambda_p \tau}{C_L V_{dd}} (e^{\frac{V_{dd}-t}{n_p V_T}} - 1) \right) & 0 < t < \tau \\ \frac{n_p V_T}{\lambda_p} \cdot \ln \left(\frac{M_p \lambda_p \cdot e^{\frac{V_{dd}}{n_p V_T}}}{C_L n_p V_T} \cdot (t - \tau) + e^{\frac{\lambda_p V_{out-\tau}}{n_p V_T}} \right) & \tau < t < t_{-3V_T} \\ V_{dd} - V_T \cdot \ln \left(1 + (e^3 - 1) \cdot e^{\frac{(-\lambda_p + 1)V_{dd}}{V_T C_L} (t - t_{-3V_T})} \right) & t > t_{-3V_T} \end{cases} \quad (6)$$

2.2.2 Output waveform equivalence

There exist some papers introducing how to approximate output waveform. P. Corsonello et al. [11] divide output waveform into four linear parts and two of them are ramp, making the delay model complicated. N. Lotze et al. [12] deal a formula that fits well in the range of $V_{dd}/2$ to 0 for the fall transition and $V_{dd}/2$ to V_{dd} for the rise transition. The reason is that the last part for every transition has a much more significant effect on output voltage.

Combining the two minds, an approximate method is proposed and shown in Fig. 4. A line is used to describe the last part of every transition. Take the fall transition as an example, t_1 and t_2 are the corresponding points of output voltage

Fig. 4. Linear approximation of output waveform.

achieved $V_{dd}/2$ and V_T respectively. Then the line is extrapolated to V_{dd} and 0, and the corresponding points are t_{11} and t_{22} . The difference between t_{11} and t_{22} is considered as the transition time of the next stage. Between t_2 and t_{-3V_T} , the supply is $V_{dd} - 0.3V_T$, and the supply value is based on specific technology. In the last part the voltage is V_{dd} again. This way not only keeps it simple and convenient for next work but also fits the important part and keeps its accuracy.

2.2.3 Output waveform validation

Experiment is done to validate the proposed model with different sizes and loads. The circuit structure is shown in Fig. 5(a), where M represents the drive strength. The comparison of the model with Spice is shown in Fig. 5(b), which shows a good agreement between them.

2.3 Delay model

2.3.1 Derivation of delay model

Fig. 5(b) shows that the majority of second part in (5) are across the point $V_{dd}/2$. We consider the schematic in Fig. 6 and derive the delay formula, where t_{phl} represents the falling delay, τ_i is the transition time of the i_{th} stage, and C_{Li} is load capacitance.

(a) Circuit Structure

(b) Output Waveform

Fig. 5. The circuit structure and output waveform of arbitrary sizes.

Fig. 6. Schematic of inverter chains used for delay model

$$t_{phl} = \frac{n_n V_T}{I_{0n} \lambda_n} \frac{C_{Li}}{W_n/L} e^{\frac{-V_{dd}+0.3V_T}{n_n V_T}} \cdot \left(e^{\frac{-\lambda_n V_{dd}/2}{n_p V_T}} - e^{\frac{-\lambda_n V_{dd}}{n_p V_T}} \right) \cdot \exp\left(\frac{V_{thn}}{n_n V_T}\right) + \left(\frac{1}{2} + \frac{C_4}{V_{dd}}\right) \tau_i. \quad (7)$$

Equation (10) shows, the delay of the i_{th} stage inverter includes two parts. The first part is the step delay of i_{th} stage, and the second part is the influence of the $(i-1)_{th}$ stage. And the expression of τ_i is needed to derive.

According to Fig. 4, τ_i can be expressed by t_1 and t_2 :

$$\tau_i = (t_2 - t_1) \cdot \left(\frac{V_{dd}}{V_{dd}/2 - V_T} \right). \quad (8)$$

The difference between t_2 and t_1 is described as

$$t_2 - t_1 = \frac{C_{Li-1}}{W_p/L} \cdot C_3 \cdot \left(C_1 - C_2 \cdot e^{\frac{-\lambda_p V_{dd}}{2n_p V_T}} \right) \cdot e^{\frac{-V_{dd}}{2n_p V_T}} \cdot \exp\left(\frac{V_{thp}}{n_p V_T}\right) \quad (9)$$

where $C_1 = -\ln\left(\frac{e-1}{e^3-1}\right) + \frac{n_p}{\lambda_p} \exp\left(\frac{0.3-3\lambda_p}{n_p}\right)$, $C_2 = \frac{n_p}{\lambda_p} \exp\left(\frac{0.3}{n_p}\right)$, $C_3 = \frac{V_T}{I_{0p}}$, $C_4 = \left(-1 - n_n \exp\left(\frac{-0.7}{n_n}\right)\right) V_T$, and can be obtained by technology parameters.

So the expression of t_{phl} is

$$t_{phl} = f_1\left(V_{dd}, \frac{C_{Li}}{W_n/L}\right) \cdot \exp\left(\frac{V_{thn}}{n_n V_T}\right) + f_2\left(V_{dd}, \frac{C_{Li-1}}{W_p/L}\right) \cdot \exp\left(\frac{V_{thp}}{n_p V_T}\right) \quad (10)$$

where

$$f_1\left(V_{dd}, \frac{C_{Li}}{W_n/L}\right) = \frac{n_n V_T}{I_{0n} \lambda_n} \frac{C_{Li}}{W_n/L} e^{\frac{-V_{dd}+0.3V_T}{n_n V_T}} \cdot \left(e^{\frac{-\lambda_n V_{dd}/2}{n_p V_T}} - e^{\frac{-\lambda_n V_{dd}}{n_p V_T}} \right) \quad (11)$$

$$f_2\left(V_{dd}, \frac{C_{Li-1}}{W_p/L}\right) = \left(\frac{1}{2} + \frac{C_4}{V_{dd}}\right) \cdot \tau_i \cdot e^{\frac{-V_{thp}}{n_p V_T}}. \quad (12)$$

Similarly, the expression of t_{plh} can be obtained. And the delay of each stage has an exponential relationship with current and previous stage threshold voltage.

2.3.2 Delay model validation

The comparison of the delay model with Spice simulation is shown in Fig. 7. Besides, the result is also explored at 0.35 V and 0.3 V, which shows that they have a good agreement with each other, with errors are 1.42%, 4.03%, and 3.39%, respectively.

Fig. 7. Every stage delay of the arbitrary size inverter chains at different voltages.

3 Delay distribution model for sub-threshold

For discussion on the delay distribution, the concepts of probability density function (PDF) and cumulative density function (CDF) are introduced firstly.

3.1 Lognormal distribution

If a Random Variable (RV) X obeys a standard normal distribution, $Y = \exp(x)$ obeys lognormal distribution. Its PDF and CDF are

$$\ln N(y; \mu, \sigma^2) = \frac{1}{y\sqrt{2\pi}\sigma} \exp\left(-\frac{(\ln y - \mu)^2}{2\sigma^2}\right) \quad (13)$$

$$\int_0^y \ln N(y; \mu, \sigma^2) dy = \phi\left(\frac{\ln y - \mu}{\sigma}\right), \quad (14)$$

where ϕ is the CDF of standard normal distribution.

If Y_j is an independent lognormal distribution variable, $Z = \sum_{j=1}^N Y_j$ also obeys lognormal distribution. The variance and mean of Z are shown as following [13, 14]:

$$\sigma_z^2 = \ln\left(\frac{\sum e^{2\mu_j + \sigma_j^2} (e^{\sigma_j^2} - 1)}{(\sum e^{\mu_j + \sigma_j^2/2})^2} + 1\right) \quad (15)$$

$$\mu_z = \ln\left(\sum e^{\mu_j + \sigma_j^2/2}\right) - \frac{\sigma_z^2}{2}. \quad (16)$$

3.2 Path delay distribution in sub-threshold

Process variations include variations of threshold voltage, effective channel length, doping concentration, and more parameters. Since threshold voltage variation is a dominant factor affecting the delay in the subthreshold region [15], the following model only considers the variations of V_{th} .

The delay of the i_{th} stage is expressed as

$$t_{phi}(i) = f_1(i) \cdot \exp\left(\frac{V_{thn0}}{n_n V_T}\right) + f_2(i) \cdot \exp\left(\frac{V_{thp0}}{n_p V_T}\right). \quad (17)$$

And the delay of the $(i+1)_{th}$ stage is

$$t_{phi}(i+1) = f_3(i+1) \exp\left(\frac{V_{thp0}}{n_p V_T}\right) + f_4(i+1) \exp\left(\frac{V_{thn0}}{n_n V_T}\right). \quad (18)$$

So the path delay can be expressed as

$$t_p = \sum \left\{ \begin{array}{l} [f_1(i) + f_4(i+1)] \cdot \exp\left(\frac{V_{thn0}}{n_n V_T}\right) \\ + [f_3(i+1) + f_2(i+2)] \cdot \exp\left(\frac{V_{thp0}}{n_p V_T}\right) \end{array} \right\} \quad (i = 1, 3, 5 \dots). \quad (19)$$

Based on the transistor BSIM4 model, variation of V_{th} abides by standard normal distribution. We have the following expression:

$$V_{thn0} \sim N(\mu_n, \sigma_n^2) \quad (20)$$

where μ_n is the mean and σ_n^2 is the variance of minimal width transistor ($W_{N0} = 120$ nm and $L = 40$ nm)

$$\sigma_n^2 = \left(\frac{\sigma_{n0}}{\sqrt{W_N/W_{N0}}} \right)^2. \quad (21)$$

According to the property of standard normal distribution, we have

$$\frac{V_{thn0}}{n_n V_T} \sim N\left(\frac{\mu_n}{n_n V_T}, \left(\frac{\sigma_n}{n_n V_T}\right)^2\right). \quad (22)$$

Based on the definition of lognormal distribution, $\exp(V_{thn0}/n_n/V_T)$ follows lognormal distribution

$$\exp\left(\frac{V_{thn0}}{n_n V_T}\right) \sim \ln N\left(\frac{\mu_n}{n_n V_T}, \left(\frac{\sigma_n}{n_n V_T}\right)^2\right), \quad (23)$$

and

$$\begin{aligned} & [f_1(i) + f_4(i+1)] \cdot \exp\left(\frac{V_{thn0}}{n_n V_T}\right) \\ & \sim \ln N\left(\frac{\mu_n}{n_n V_T} + \ln[f_1(i) + f_4(i+1)], \left(\frac{\sigma_n}{n_n V_T}\right)^2\right) \end{aligned} \quad (24)$$

$$\begin{aligned} & [f_3(i+1) + f_2(i+2)] \cdot \exp\left(\frac{V_{thp0}}{n_p V_T}\right) \\ & \sim \ln N\left(\frac{\mu_p}{n_p V_T} + \ln[f_3(i+1) + f_2(i+2)], \left(\frac{\sigma_p}{n_p V_T}\right)^2\right), \end{aligned} \quad (25)$$

where f_1, f_2, f_3 and f_4 can be obtained based on the size and load of each stage. And according to (15) (16), the mean and variance of path delay can be obtained.

Fig. 8 shows the proposed model and spice match very well at 0.4 V. Furthermore, the delay distributions at 0.35 V and 0.3 V are explored. The error of path delay distribution is shown in Fig. 9, which is 0.906% (1.78%, 0.448%) when the probability is 0.3% (50%, 99.7%). And the error of other voltages are listed in Table II.

From the view of the principle and method of delay model, path delay error and distribution error, comparisons with previous works [9, 10, 11] are concluded and shown in Table II. [10] and [11] study the delay model in detail and not do the delay distribution model. [9] study the delay and delay distribution model, but the current model adopt on state current, so delay model based on fitting parameters,

Fig. 8. The comparison of path delay distribution of model with 10 K MC Spice simulation.

Fig. 9. Path delay distribution error.

and besides, the mean and variation of path delay is not based on lognormal theory. In this paper, the transient current model is used, so no fitting parameters such as K in [9] is incorporated in the delay model. For path delay model, Miller equivalence and output waveform simplicity are adopted, which have two sides. The advantage is making the model much simpler to compute the following delay distribution. And the disadvantage is bringing a side effect on the accuracy compared with [11], yet accurate enough for the following computation of path delay distribution. Besides, path delay distribution is proposed in this paper based on lognormal theory and the comparison results with [9] is shown in Table II, which is more accurate than [9].

Table II. Comparison with previous works

		[9]	[10]	[11]	This work		
Delay model Principle and method	Ids Model	$I_{ds} = k \cdot I_{on}$ k:fitting para.	$I_{ds} = f(V_{gs}, V_{ds})$ no fitting parameters				
	Supply	interval					
	Temp.	single					
	Input Slew	Step	Ramp input derived by previous stage				
	Output Slew	N/A	N/A	two ramp part	one ramp part		
	load	FO4	actual condition				
	Coupling Cap.	No	No	Yes: direct compute	Yes: miller equivalence		
	Path delay distri.	lognormal: u, σ not based on lognormal theory	No	No	lognormal: u, σ based on lognormal theory		
path delay error	Vdd Stage	0.3 V	N/A	0.3 V	0.4 V	0.35 V	0.3 V
	1		3.2%	1.75%	0.86%	1.23%	1.74%
	2	7.8%	13%	0.24%	1.06%	6.67%	3.05%
	3		10%	0.71%	1.45%	7.05%	5.33%
	4		13%	0.9%	1.01%	1.82%	1.87%
	5		13%	0.27%	1.37%	0.92%	4.23%
	6		13%		0.80%	2.08%	6.47%
	7		15%		0.34%	3.60%	4.74%
	8		14%		1.75%	5.07%	4.17%
	9		18%		4.62%	7.83%	1.78%
	10	8.9%	15%		0.90%	3.76%	0.49%
	avg error		12.72%	0.77%	1.42%	4.03%	3.39%
CDF error	P = 0.3%	43.75%			0.906%	2.95%	3.23%
	P = 50%	34.28%			1.78%	3.04%	6.65%
	P = 99.7%	8.5%			0.448%	4.64%	18.25%

4 Conclusions

In this paper, transient current is used to deduce unit the delay model. In order to simplify the computation process of obtaining the path delay model, two ways are adopted, which are coupling capacitance and output waveform equivalence. And they are explored under different voltages. According to the lognormal distribution theory, the path delay distribution model is obtained, which is consistent to spice MC simulation.

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