A simple current-reversible chaotic jerk circuit using inherent $\tanh(x)$ of an opamp

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Abstract: A simple current-reversible chaotic jerk circuit is proposed and particularly demonstrates the use of inherently $\tanh(x)$ nonlinearity of a single opamp for a chaotic jerk circuit. No additionally nonlinear devices are required. Bifurcation is electronically tunable through a current source in a reversible direction. The largest Lyapunov exponent of either direction forms mirror images, whereas the chaotic attractor of either direction forms anti-symmetric images.

Keywords: chaotic jerk oscillator, current tunable

Classification: Electron devices, circuits and modules

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1 Introduction

Autonomous chaotic oscillators have recently been of much interest in order to a variety of applications in science and engineering such as chaos-based secure communications [1], cryptography [2] or wide-band communication systems [3].

A category of chaotic circuits based on an operational amplifier (opamp), or opamps, has been of particular interest owing to its simple realization. For instance, the chaotic circuits that were early proposed based on a positive-feedback loop [4] and a biquadratic filter [5], a chaotic circuit based on a single opamp has employed a diode [6], whereas a chaotic jerk circuit based on two opamps has employed $\text{sgn}(x)$ nonlinearity [7]. Recently, current-tunable chaotic circuits have been demonstrated for such a category using either exponential nonlinearity of a diode with a single opamp [8, 9], or $\text{sgn}(x)$ nonlinearity with two opamps [10]. Although the former has exploited a current source I_0 for a dynamical model [8] and a jerk model [9], I_0 has encountered difficulty in a reverse direction due to the diode direction. Although the later [10] has exploited current-reversible I_0 without such difficulty, the two required opamps have resulted in a relatively complicated chaotic jerk circuit. It is natural to wonder whether a simple current-reversible chaotic jerk circuit is possible based on a single opamp without a diode. In this paper, a simple current-reversible chaotic jerk circuit is presented using neither a diode nor a two-opamp approach. The paper particularly demonstrates the use of inherently $\tanh(x)$ nonlinearity of a single opamp for a chaotic jerk circuit. The direction of I_0 is reversible and no longer depend on the diode direction when a single opamp is employed. The largest Lyapunov exponent of either direction shows mirror images, whereas the chaotic attractor of either direction shows anti-symmetric images.

2 Circuit realisations

Fig. 1 shows a simple current-reversible chaotic jerk oscillator based on inherent $\tanh(x)$ of a single opamp. A resistor $R_L = R_1 + R_2$ where R_1 is an internal resistance of an inductor L and R_2 is an external resistor. Voltages v_{C1} and v_{C2} are across capacitors C_1 and C_2 , respectively. Currents i_L and i_R flow through L and a resistor R , respectively. A DC current source I_0 is connected to a DC voltage source $-V_{SS}$. Let V_S be a saturation voltage of the opamp, and V_r be an arbitrary reference voltage = 1 V. Variables (X, Y, Z), derivatives ($\dot{X}, \dot{Y}, \dot{Z}$), constants (A, B, C) and time $\tau = t/\tau_L$ are normalized parameters as summarised in (1) where time constants $\tau_L = L/R_L$, $\tau_1 = R_L C_1$, $\tau_2 = R C_2$.

Fig. 1. A simple current-reversible chaotic jerk circuit based on inherent $\tanh(x)$ of a single opamp.

$$\begin{bmatrix} \dot{X} & X & A \\ \dot{Y} & Y & B \\ \dot{Z} & Z & C \end{bmatrix} = \begin{bmatrix} dX/d\tau & v_{C1}/V_r & \tau_L/\tau_1 \\ dY/d\tau & v_{C2}/V_r & \tau_L/\tau_2 \\ dZ/d\tau & i_L R_L/V_r & (V_S/V_r)\tau_L/\tau_2 \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} 0 & 0 & A \\ 0 & -B & A \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} 0 \\ C \tanh(kX) + D \\ 0 \end{bmatrix} \quad (2)$$

As the $\text{sgn}(X)$ of the opamp can be closely approximated by $\tanh(x) = \tanh(kX)$ for larger scaling values of k , the normalized dynamic presentation of the circuit is shown in (2) where $D = \tau_L I_0 / (C_2 V_r)$. Equation (2) can be transformed into a jerk form as

$$\ddot{X} = -a_1 \ddot{X} - a_2 \dot{X} - a_3 X - a_4 \tanh(kX) + a_5 \quad (3)$$

where coefficients $a_1 = (B + 1)$, $a_2 = (2A + B)$, $a_3 = AB$, $a_4 = AC$, and $a_5 = AD$. The model in (2) possesses two equilibrium points depending on the direction of I_0 . For a positive value of I_0 and D is positive, the equilibrium point $P_1 = (X_1, Y_1, Z_1) = (-Y_1, Y_1, 0)$, where the two equations $X_1 = -Y_1$, and $Y_1 = [C \tanh(kX_1) + D]/B$ can be solved for the two unknown (X_1, Y_1) . For a negative value of I_0 where D is negative, the equilibrium point $P_2 = (X_2, Y_2, Z_2) = (Y_1, -Y_1, 0)$.

3 Simulations and experiments

Table I. Two examples of (P_1, P_2) and eigenvalues at $I_0 = 5.1$ mA

Example 1 $C_1 = 9.87$ nF, $R_L = 100.05$ Ω	Example 2 $C_1 = 12.8$ nF, $R_L = 190.25$ Ω
$P_1 = (0.0333, -0.0333, 0)$ $P_2 = (-0.0333, 0.0333, 0)$	$P_1 = (0.0333, -0.0333, 0)$ $P_2 = (-0.0333, 0.0333, 0)$
$\lambda_1 = -4.3239$ $\lambda_{2,3} = 1.5370 \pm j4.6021$	$\lambda_1 = -2.0885$ $\lambda_2 = 0.4929 \pm j2.0192$

Table II. Comparisons of current-tunable chaotic jerk circuits

References	Opamp	Diode	I_0	Passive	Total	$-I_0$
This paper	1	0	1	5	7	yes
[4]	1	1	1	4	7	no
[5]	2	0	1	5	8	yes

Solutions of (2) are numerically simulated using a fourth-order Runge-Kutta integrator with a step size of 0.001. Initial conditions are at $(X_0, Y_0, Z_0) = (0.1, 0, 0)$. Parameters $C_1 = C_2 = 10 \text{ nF}$, $L = 500 \text{ } \mu\text{H}$, $R = 2 \text{ k}\Omega$, $R_1 = 0.5 \text{ }\Omega$, $R_2 = 100 \text{ }\Omega$ and $k = 50$. The opamp is TL082 with a dual supply of $\pm 12 \text{ V}$, i.e. the saturation voltage $V_S \cong 9 \text{ V}$, and I_0 is an adjustable current source LM334. Figs. 2a–c show numerical trajectories on an $[X + Y, -Z]$ plane, whereas Figs. 2d–f show oscilloscope traces on a $[v_{C1} + v_{C2}, i_L R_L]$ plane, using $I_0 = (5.20, 5.40, 5.70) \text{ mA}$, for period-2, period-4, and chaos respectively. Note that period-1 is omitted for simplicity. In addition, Fig. 3 shows two numerical trajectories of chaotic attractors with anti-symmetric images on a $[Y, Z]$ plane, using $I_0 = \pm 5.6 \text{ mA}$. Fig. 4a shows a bifurcation diagram of the maximum of X ($X_{\max}$), and the largest Lyapunov exponent (LLE) versus I_0 from 5.4 to 5.9 mA. By contrast, Fig. 4b shows another bifurcation diagram of $X_{\max}$ and the LLE versus I_0 from -5.9 to -5.4 mA . A period doubling route to chaos can be observed, whereas the LLEs of Figs. 4a and b for either I_0 direction show mirror images.

Table I summarizes Examples 1 and 2 of equilibrium points (P_1, P_2) and their eigenvalues, using two different sets of C_1 and R_L at $I_0 = 5.1 \text{ mA}$. Each example comprises a negative real eigenvalue (λ_1) and a complex conjugate pair of eigenvalues ($\lambda_{2,3} = \alpha \pm j\beta$) where $\alpha > 0$. Therefore, the equilibrium points are spiral saddle points with an index 2. Example 1 corresponds to a general case where the equilibrium point is located off the attractor, as depicted in Fig. 2c and f. On the other hand, Example 2 corresponds a particular case where the equilibrium point intersects the attractor, as illustrated by a homoclinic orbit on an $[X, Y, Z]$ plane in Fig. 5. In addition, $|\lambda_1| > |\alpha|$ follows the Shil'nikov condition [5] for a proof of chaos. Table II compares the proposed circuit with two existing current-tunable chaotic jerk circuits [4, 5]. The proposed circuit requires neither a diode, as it was required in [4], nor a two-opamp approach, as it was required in [5]. In addition, the proposed circuit offers a simpler current-reversible chaotic jerk circuit with only 6 devices and a single opamp compared to the current-reversible chaotic jerk circuit in [5] which requires 6 devices and two opamps.

Fig. 2. Trajectories using $I_0 = (5.20, 5.40, 5.70)$ mA, respectively.
a–c Numerical results on an $[X + Y, -Z]$ plane
d–f Experimental results on a $[v_{C1} + v_{C2}, i_{LR_L}]$ plane

Fig. 3. Two numerical trajectories of chaotic attractors with anti-symmetric images on a $[Y, Z]$ plane, using $I_0 = \pm 5.6$ mA.

Fig. 4. Bifurcation diagrams of $X_{\max}$ and LLEs of Fig. 1
a against I_0 from 5.4 mA to 5.9 mA
b against I_0 from -5.9 mA to -5.4 mA.

Fig. 5. Homoclinic orbit on $[X, Y, Z]$ plane in example 2.

4 Conclusions

A simple current-reversible chaotic jerk oscillator has been proposed based on a single opamp. In contrast to other existing opamp-based chaotic oscillators where a diode or two opamps are exploited, this paper has suggested use of inherently hyperbolic tangent nonlinearity of a single opamp for a chaotic jerk circuit. No other nonlinearity is required. Numerical and experimental results of chaotic attractors have been illustrated and compared. The chaotic behaviour is electronically tunable through a current source in both positive and negative directions resulting in mirror images of the LLEs and anti-symmetric images of chaotic attractors. The homoclinic connection exists through an illustration of a homoclinic orbit.

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