Abstract: A new method for formulating the equivalent distributed capacitance of NFC coil antenna is proposed. This method is based on an analytical approach by conformal mapping for quantifying the capacitance per unit length and then applied the capacitance network method for calculating the equivalent distributed capacitance. The results of this method are verified with simulated and measured results of two different types of antenna and the agreement is >91%. It is also tested that the capacitance between non-adjacent turns cannot be ignored in the equivalent distributed capacitance computation. Furthermore, the effects of the coil width and gap between the two adjacent coils are predicted in calculating the equivalent distributed capacitance for a specific coil antenna. The proposed method will be useful for designing NFC coil antennas with a better accuracy of capacitance.

Keywords: capacitance network, conformal mapping, distributed capacitance, NFC

Classification: Circuits and modules for electronic instrumentation

References


1 Introduction

Near field communication (NFC) has received much attention in recent years due to its applications in online banking, ticketing, and access control and peer-to-peer data transfer [1, 2, 3]. It is a bidirectional proximity communication technology that operates within an interaction distance of approximately 5–10 centimeters.

NFC works via inductive coupling between two coil antennas by tuning the parameters of network that holds the antenna circuits close to its resonant frequency. In the resonant circuit, it follows the basic rule, \(\omega_0 = 1/\sqrt{LC}\) [2]. Research endeavors concerning NFC appear to concentrate mainly on inductance of NFC enabling to get more magnetic flux for purpose of increasing the power efficiency [4, 5, 6, 7]. It is clear that the coil antenna will produce distributed capacitance, which depends on structure, nature and thickness of the coil material. Tunable performance of antenna mainly depends upon quality factor \(Q(w) = w \times (\text{Maximum Energy Stored/Power Loss})\) which is very important and generally approximated by neglecting the capacitance (storage component) of coil. At higher operating frequencies, distributed capacitance among the coils will affect antennas more significantly. So calculation of distributed capacitance associated with different dimensions of NFC antenna should also be well elaborated.

There are specifically 3 methods for obtaining the distributed capacitance of inductor winding and similar situations, finite element method [8], derivations from
the results of impedance test [9, 10] and analytical method [11, 12, 13]. Each of these approaches has advantages and shortcomings that must be balanced according to the application. FEM is more accurate and can get the frequency-dependent capacitance, but it is computationally intensive, time consuming and do not have insight to optimize the inductor design. The second method depends on model complexity and test precision. Analytical method can be used to design and optimize the inductors if certain physical and geometrical parameters are provided. Its due to the obvious relation between physical field variable, geometrical parameters and the ease of computation.

In this paper, we simply consider the capacitance arising from the adjacent and non-adjacent coils of NFC antenna without considering the effect from neighboring capacitances of surrounding conductors. We presented a new method for calculating the equivalent distributed capacitance of a coil antenna. Formulating capacitance per unit length between two adjacent and non-adjacent turns, using the lumped node to node capacitance network method [14], the equivalent distributed capacitance is obtained. Then compared results are verified with simulated and experimental results. The correlations among them are practicable. Thus dependency of distributed capacitance on the increasing width of coil and gap between adjacent coils are predicted by the proposed method.

2 Methodology

2.1 Modeling of distributed capacitance

A lumped capacitance connected between the terminals of the coil can represent the equivalent distributed capacitance of a coil antenna [15]. Coil antenna can be simply modeled as shown in Fig. 1. Where R is the equivalent series resistance, L is series inductance, and C is parallel capacitance.

![Fig. 1. Equivalent lumped circuit of NFC antenna.](image)

According to the service conditions of this coil antenna, it has no grounded conductors nearby hence no coil-to-ground parasitic capacitance. As a result, the effects of the grounding on the antenna’s capacitance can be ignored. The equivalent distributed capacitance of N-turn planar rectangular loop coil consists of the following two parts:

1) The turn-to-turn capacitance between two adjacent coil turns
2) The turn-to-turn capacitance between non-adjacent coil turns

Firstly, we have to find out all capacitances between two coil turns of N-turn planar rectangular coil antenna, adjacent or non-adjacent, and formulate the distributed capacitance $C$, which is given by (1). Fig. 2(a) shows the geometrical parameters of a NFC coil antenna. Fig. 2(b) shows the capacitance between two
coil turns. Here $C_f$, the resulting fringing field has a strong effect on the value of capacitance and should, therefore, be taken into consideration. While $C_o$, is the distributed capacitance denoting the 3D existence between the parallel coil side-walls which is a negligible small amount. Therefore, the value of $C_o$ is too small as compared to $C_f$ then its effect can be ignored.

$$C = 2C_f + C_o \quad \ldots(1)$$

Now consider the equivalent distributed capacitances among the N-turn planar coil antenna. Fig. 3 shows the model of capacitance network in coil antenna based on the network of lumped node-to-node capacitance elements, where each node represents one turn. Here, $C_{ij}$ represents the capacitance between the turns $i$ and $j$.

2.2 Calculation of distributed capacitance
To quantify capacitance $C_f$ due to fringing field between two coils, we map the coplane surfaces of coil to an equivalent parallel-plate system using conformal mapping technique [16]. It can prevent the variation in angles, a well-known transformation that maps the problem with specific boundaries along with a complicated geometry into another equivalent simpler and known geometry.

To perform this conformal mapping, the boundaries of concentric circles represent the electric field lines and the confocal hyperbolas represent the electric potential, as shown in Fig. 4. Therefore, the original model can be mapped to an equivalent parallel-plate model by transforming the $x$-$y$ coordinates to $u$-$v$ coordinates, as shown in Fig. 5. There is a suitable function to map circle geometry into linear geometry [17], which provides
\[ u + jv = \ln(x + jy). \]  
\[ x + jy = re^{j\theta}. \]  
Solving (2 & 3), we get
\[ u = \frac{1}{2} \ln(x^2 + y^2). \]  
\[ v = \tan^{-1} \frac{y}{x}. \]  
The fringing electric field in the \( z \) plane is mapped into a linear electric field in the \( \xi \) plane (Fig. 5) by using the above transformation. In the \( \xi \) plane, we get the capacitance per unit length as below
\[ C = \varepsilon \frac{d(C, D)}{d(C, B)}. \]  
where \( d(C, D) \) is the distance between \( C \) and \( D \), \( d(C, B) \) is the distance between \( C \) and \( B \), and \( \varepsilon \) is the permittivity of air or the substrate which could be ceramic, polyimide, or FR4.

Substituting the coordinates of all those points in (6), we get
\[ C = \frac{\varepsilon}{\pi} \ln \left(1 + \frac{2W}{S}\right). \]  
The total capacitance \( (C_{\text{total}}) \) of antenna can be calculated by integrating the entire length according to equation (8).
\[ C_{\text{total}} = \int \frac{\varepsilon}{\pi} \ln \left(1 + \frac{2W}{S}\right) \cdot dl \]  
where \( l \), \( W \) and \( S \) is the length, width of coil, gap between two adjacent coils.
The capacitance network in Fig. 3 can be represented by a $N \times N$ capacitance
matrix as in (9), and represented by a node current equation as in (10). Here $I_i$ is a
current, $V_j$ is voltage and $Y_{ij}$ is admittance, where $i, j = 1 \ldots N$. In general, (10)
gives an expression about the capacitive couplings among those coils.

\[
C = \begin{bmatrix}
\sum_{K=1}^{K=N} C_{1K} & -C_{12} & \cdots & -C_{1N} \\
-C_{21} & \sum_{K=1}^{K=N} C_{2K} & \cdots & -C_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
-C_{N1} & -C_{N2} & \cdots & \sum_{K=1}^{K=N} C_{NK}
\end{bmatrix}
\]  

(9)

\[
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix}
= \begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1N} \\
Y_{21} & Y_{22} & \cdots & Y_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{N1} & Y_{N2} & \cdots & Y_{NN}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix}
\]  

(10)

The equivalent distributed capacitance between the terminals of the coil can be
obtained, provided that all the intermediate nodes are eliminated. Reduction of
nodes can be accomplished by the specific matrix transformation. Here, the matrix
in (10) can be rewritten as below by using the matrix operation, where corresponding
changes are made in the current and voltage matrices.

\[
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix}
= \begin{bmatrix}
Y_{11} & Y_{12} & \cdots & Y_{1N} \\
Y_{21} & Y_{22} & \cdots & Y_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
Y_{N1} & Y_{N2} & \cdots & Y_{NN}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix}
\]  

(11)

The admittance matrix $Y$ in (11) consists of the following four matrices:
1) Upper left corner is replaced with $Y_{xx}$,
2) Upper right corner is replaced with $Y_{xy}$,
3) Lower left corner is replaced with $Y_{yx}$ and
4) Lower right corner is replaced with $Y_{yy}$.

Using the conversion and derivation process of Qin. Y et al. [8], we obtained the
matrix $Y_x$ as shown in (12),

\[
Y_x = Y_{xx} - Y_{xy} Y_{yy}^{-1} Y_{yx}.
\]  

(12)

The admittance at non-diagonal member of matrix $Y_x$ is equal to $-j\omega C_{eq}$. As a result,
the equivalent distributed capacitance $C_{eq}$ of a NFC coil antenna can be calculated
precisely.
3 Calculations and measurements

To evaluate the accuracy of the proposed method, we simulated and measured the equivalent distributed capacitance of two different appropriate sized antenna, as shown in Fig. 6. The model parameters of each antenna and results are tabulated in Table I. Here, W, S and N is width of coil, gap between two adjacent coils and the number of coils respectively. L is the length and G is the width of outermost loop of each antenna. We use HFSS to simulate those two antennas. Fig. 7 shows one of impedance profiles. Equivalent inductance of the antenna was obtained at the point of frequency $m_1$ (13.56 MHz), then the equivalent distributed capacitance was obtained from $C = 1/(2\pi f)^2L$ at the point of self-resonant frequency $m_2$. Agilent 4396B impedance analyzer measured equivalent distributed capacitance using the above same method. By comparing the measured, simulated and predicted results given in Table I, agreement between simulated and predicted results is obtained within 94%, and agreement between measured and predicted results is obtained within 91%. It means that the proposed method for calculating the distributed capacitance of NFC antenna is accurate for the design purposes.

Fig. 6. Two different sizes of antenna.

Table I. NFC coil antenna parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Antenna #1</th>
<th>Antenna #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{eq}$ (predicted)</td>
<td>1.7976 pF</td>
<td>1.4624 pF</td>
</tr>
<tr>
<td>$C_{eq}$ (simulated)</td>
<td>1.7360 pF</td>
<td>1.3795 pF</td>
</tr>
<tr>
<td>$C_{eq}$ (measured)</td>
<td>1.9140 pF</td>
<td>1.5730 pF</td>
</tr>
<tr>
<td>$N$</td>
<td>6 mm</td>
<td>7 mm</td>
</tr>
<tr>
<td>$S$</td>
<td>0.5 mm</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>$W$</td>
<td>1 mm</td>
<td>0.8 mm</td>
</tr>
<tr>
<td>$L$</td>
<td>76 mm</td>
<td>40 mm</td>
</tr>
<tr>
<td>$G$</td>
<td>45 mm</td>
<td>40 mm</td>
</tr>
</tbody>
</table>

As mentioned above, all non-adjacent capacitances are considered, while calculating the simulated and predicted results given in Table I. Fig. 8 shows distributed capacitance versus the change of adjacent and non-adjacent coil, where a means the capacitance $C_{ij}^1$ between two adjacent coils, b means the capacitance of $C_{ij}^1$ and also non-adjacent $C_{ij}^2$. c means the adjacent capacitance of $C_{ij}^1$, and non-adjacent capacitance $C_{ij}^2$ and $C_{ij}^3$ (d, e and f also be defined in the same
The number of non-adjacent coil nodes have great influence on distributed capacitance. As a consequence, capacitances between non-adjacent turns cannot be ignored in the equivalent distributed capacitance computation.

The equivalent distributed capacitance of #2 type antenna calculated by the proposed method presented in Table I is 1.4716 pF. We can apply the proposed model on varying parameters and the dependence of the distributed capacitance on the coil width W and the gap S between two adjacent coils.

Fig. 9(a) shows equivalent distributed capacitance versus the change of width of coil, whereas the size of innermost coil and gap values are invariant. Fig. 9(b) shows equivalent distributed capacitance versus the change in gap between adjacent coils, while the size of innermost coil and the width of coil are invariant. Increasing the value of W, value of S reduces correspondingly, the distributed capacitance obtained is shown in Fig. 10, while the equivalent area and size of antenna is invariant with a fixed value of width and gap size. It means that distributed capacitance increases with increase of the width of coil, and reduces with increasing of the gap between adjacent coils.
4 Conclusion

In this paper, we presented a new method for calculating the equivalent distributed capacitance. Formulating distributed capacitance per unit length between two adjacent, and two non-adjacent turns, using the lumped node to node capacitance network method, the equivalent distributed capacitance is obtained. To evaluate the accuracy of the proposed method, we simulate and measure the actual distributed capacitance of two different types of antenna, then compared with predicted results. The correlations among them are practical. The distributed capacitance between non-adjacent turns cannot be ignored in the equivalent distributed capacitance computation. The dependency of distributed capacitance on the increasing width of coil and gap between adjacent coils are predicted, indicating that the proposed method will be useful for designing antennas with a better accuracy of capacitance.

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Fig. 9. Dependency of equivalent distributed capacitance on (a) the change of width of coil W and (b) gap S between two adjacent coils.

Fig. 10. Dependency of equivalent distributed capacitance on the change in the values of Width (W) of antenna #1.