An accurate analytical memristor model for SPICE simulators

Sam Thomas^{1a)} and Savarimuthu Prakash²

¹ Research Scholar, Faculty of Information and Communication Engineering,
Anna University, Chennai-600073, Tamil Nadu, India

² Professor, CSE Department, Saveetha Institute of Medical and Technical Sciences,
Chennai-600077, Tamil Nadu, India

a) sam_thomas_st@yahoo.com

Abstract: Memristor holds great potential for memory technology, neuro-morphic systems and digital applications. For implementation in complex circuits, an accurate and predictive model is required to make significant progress. This paper introduces a model that is based on the physics of the device. The mathematical modeling helps in understanding the physical properties which determine the behavior of the memristor and also aid in the characterization of the device. Namely, i-v characterization and switching mechanism involved are examined. In our SPICE modeling, Simmons tunnel barrier is incorporated into the port and state equation. The SPICE equivalent circuit for the same is presented and discussed. The presented model satisfies the fingerprints of the memristor.

Keywords: memristor, modeling, SPICE, Simmons tunneling model

Classification: Electron devices, circuits and modules

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1 Introduction

For years, the field of electrical engineering has worked with three fundamental passive elements which are the resistor, the inductor and the capacitor. In 1976, Prof. Leon Chua mathematically [1] proved the existence of a fourth element which is passive and whose characteristics cannot be duplicated by the combination of any of the three elements. The generalized memristor system [2] is defined by the below equations.

$$v = R(w)i \quad (1)$$

$$\frac{dw}{dt} = F(w, i) \quad (2)$$

Equation (1) is the Ohm’s law which is dependent on the state w and the equation (2) states the evolution of w as function of current and the present state of w . The memristance R is a function of the state variable w and current i , thus memristor is a nonlinear element. The memory for the element is imparted by the state variable w . Memristors have application in the fields of non-volatile memory, digital, analog, neuromorphic, and chaotic systems.

2 Existing models

For memristor applications, it is important to properly model the device. In 2008, Strukov *et al.* proposed the Linear Ion Drift Model [3] where the memristance was controlled by the ratio of the state variable and the device thickness. Based on [3], other models were proposed by adding nonlinearity to the state equation [4, 5, 6, 7, 8]. Simmons tunneling barrier model [9] built on Simmons tunneling theory offers a memristor model which has a Metal–Insulator–Metal (MIM) junction in series with a resistor. The ThrEshold Adaptive Memristor model (TEAM) was proposed by Kvatinsky *et al.* [10] which is derived from Simmons tunnel barrier model.

3 Proposed physics based model

For developing an accurate and predictive memristor model, the equations must be based on physics of the device rather than an approximation. The analytical model must integrate different types of ionic transport and device dynamics.

For state-dependent Ohm' law (1), the drift of oxygen vacancies causes TiO_2 switching mechanism. On the application of a negative voltage at the terminal (top electrode) of $\text{Pt}/\text{TiO}_2/\text{TiO}_{2-x}/\text{Pt}$ (Fig. 1(a)) based memristor, positively charged vacancies are attracted toward the top electrode. This forms a high electrically conductive channel because of the vacancy dopant drift [11]. The device is switched ON when more channels penetrate the tunnel barrier. Conduction is controlled by the effective tunneling barrier width (state w) which varies with the applied input. The parameter under consideration is w which determines the state of the device. The i - v curve in the ON condition is an exponential function which is that of the electron tunneling characteristic and the OFF state is similar to that of a rectifier. Pulse state test was conducted in [12] to inspect the i - v characteristics. The measurements exhibited that these curves were defined by a resistor of $215\ \Omega$ in series (R_s) with Simmon's tunnel barrier circuit as shown in Fig. 1(a).

Fig. 1. (a) TiO_2 sandwiched between Pt electrodes. TiO_{2-x} formed as a result of electroforming process. (b) schematic used for circuit simulation, the resistance is used to account for the electrode resistance and the externally applied input is V_{ext} . ($k_1 = 0.01$, $\beta = 0.04$, $\alpha = 3.3$, $k_2 = 10^{-3}$, $\gamma = 5$)

The port equation (state-dependent Ohm's law) used in [9] is highly sensitive to changes in the input. Any changes to amplitude or frequency in the externally applied input often results in convergence errors in SPICE. This limits the usability of the model. To overcome the issue, we propose an improved port equation of the tunneling barrier, in the form

$$i(v_{mem}, w) = k_1^w \times [\beta \sinh(\alpha v_{mem}) + k_2 e^{(\gamma v_{mem} - 1)}] \quad (3)$$

The first term represents the ON state and the second term the OFF state. In the first term, the memristor state variable is w and $\beta \sinh(\alpha v_{mem})$ represents the electron tunneling for the memristor ON state (β and α are the fitting parameters for ON state). The second term is the i - v characterization of a rectifier (k_2 and γ are the fitting parameters for the OFF state). k_1 is used to adjust the characteristics of the ON and OFF switching. Equation (3) defines the voltage and current relationship,

which is similar to equation (1). The rate of change of state variable described by the OFF and ON switching equations is shown in (4) and (5). f_{OFF} , a_{OFF} , i_{OFF} , b and w_c are the fitting parameters for OFF switching and f_{ON} , a_{ON} , i_{ON} , b and w_c are the fitting parameters for ON switching.

$$\frac{dw}{dt} = f_{OFF} \sinh\left(\frac{i}{i_{OFF}}\right) \exp\left[-\exp\left(\frac{w - a_{OFF}}{w_c} - \frac{|i|}{b}\right) - \frac{w}{w_c}\right] \quad (4)$$

$$\frac{dw}{dt} = f_{ON} \sinh\left(\frac{i}{i_{ON}}\right) \exp\left[-\exp\left(\frac{w - a_{ON}}{w_c} - \frac{|i|}{b}\right) - \frac{w}{w_c}\right] \quad (5)$$

The change of state variable w is controlled by the applied voltage (or current). Equation (4) and (5) defines the state equation (evolution of state variable w as a function of current i and current state of w). This is similar to equation (2).

4 Modeling state and port equations

Fig. 2. (a) Circuit model shows the equivalent circuit of the proposed memristor model where the memristor is modeled as a voltage dependent current source. (b) i - v characteristics of the simulated memristor (red dots) and the experimental data (black dots) is shown, the inset shows the externally applied voltage.

Fig. 2(a) shows the equivalent circuit for the proposed voltage-dependent memristor. The memristor is modeled at the plus and minus terminals using the series connection of a current source G_{mem} and a 215Ω resistor (R_s). G_{mem} is modeled by the state-dependent Ohm's law equation (3). The change in the state variable w is modeled using G_{ON} (5) and G_{OFF} (4). The auxiliary resistor R is for the provision of a DC path from node w to the ground. The voltage across the capacitor C signifies the tunnel barrier width w (nm). The fitting parameters a_{OFF} , a_{ON} and w_c were scaled because w was in nm. The fitting parameter values used are $f_{OFF} = 3.5 \mu s$, $a_{OFF} = 1.2 \text{ nm}$, $i_{OFF} = 115 \mu A$, $f_{ON} = 40 \mu s$, $a_{ON} = 1.8 \text{ nm}$, $i_{ON} = 8.9 \mu A$, $b = 500 \mu A$ and $w_c = 107 \text{ pm}$.

To compare the proposed model with the experimental measurements in [13], the memristor is connected in series with a $2.4 \text{ k}\Omega$ and is excited by an external voltage source shown in Fig. 1(b). The electrode resistance in the experiment [13] is represented with the $2.4 \text{ k}\Omega$ series resistor. The simulation results for the SPICE model is shown in Fig. 2(b) and the excitation input is triangular signal shown in the inset. The simulated i - v curve is similar to the experimental data i - v characterization displayed in [13]. The root mean square error value is 0.3 mA between the simulated and the experimental data.

5 Simulation results and discussions

The i - v characteristic in Fig. 3(b) is termed by Chua as “pinched-hysteresis loop”. The lobe of the hysteresis loop must appear in the first and third quadrant for a memristor. The model exhibits bipolar switching because the device is switched ON and OFF when input voltages of opposite polarity are applied.

Fig. 3. (a) The characteristics of the memristor for a sinusoidal input of 1 V and frequency 1 Hz. (b) the lobes of the i - v characteristics reduces as the frequency increases. (c) change of state variable w is shown. (d) memristor varies between OFF and ON switching as the input voltage changes.

When $v(t) = 0$, $i(t)$ also reaches zero (Fig. 3(a)). Thus the hysteresis of the i - v characteristic is pinched at the origin. The change in the state variable with respect to the input voltage is shown in Fig. 3(c). Fig. 3(d) represents the two conduction states of the device. Increasing the voltage, increases the memristance and sets the device state to OFF and decreasing the voltage, decreases the memristance and sets the device state to ON. The pinched hysteresis loop of the proposed model holds true for any periodic signal. Fig. 3(b) shows the i - v characteristics of the memristor for a sinusoidal input. The characteristic shows a double-valued lissajous figure for all values of $v(t)$ and $i(t)$ except at the origin. As the input frequency increases, the hysteresis loop is reduced to a single-valued function. After a particular frequency, the memristance property is lost and the device acts as a linear resistor without hysteresis in the i - v characteristic. Thus the proposed model exhibits all the three fingerprints required for a memristor device model [14].

Fig. 4(a) shows the change in state variable w during OFF switching. The rise time for external voltage was set to 100 ns. The positive voltage increases the state variable and also the resistance of the device increases exponentially, thus the device is switched to the OFF state. As the amplitude of external voltage increases the switching time reduces. Accordingly, the change in w is also affected. The tunneling width (w) is held between the limits 1.2 nm to 1.8 nm, which agrees with

Fig. 4. The response of the memristor model for (a) OFF switching and (b) ON switching for different amplitudes of voltage step input with a rise time of 100 ns is shown. The legend indicates the applied voltage. The solid lines represent the proposed model and the scatter plot represents the experimental data.

the experimental data. For lower amplitude input, w change is lesser compared to that of higher amplitude signals. The root mean square error in OFF switching for all applied voltages is less than 0.05 nm.

Fig. 4(b) shows the change in state variable w during ON switching. The negative voltage applied decreases the state variable from the previous value of the resistance, hence, the resistance decreases exponentially and the device is switched to the ON state. The root mean square error in ON switching for both the applied voltages is less than 0.2 nm. For ON switching, the switching speed is sensitive to the amplitude of the externally applied signal. The switching speed of ON switching was faster compared to that of OFF switching. The simulated characteristics are similar to the experimental data [13]. The parameters of the simulated model are intrinsic to the device and are not dependent on the externally applied signal and the model does not diverge or oscillate for set voltages and currents. Since both the i - v characteristic and the switching speed are exponential, a double exponential function is used to model the device.

6 Conclusion

The SPICE model presented is based on analytical modeling. The state-dependent Ohm's law and the change in state variable equations are based on physical characterization. The model was validated with experimental data of $\text{TiO}_2/\text{TiO}_{2-x}$ memristor. This helps with an improved simulation of the characteristics of memristor compared to the models which use approximations for modeling. This significantly helps because of the absence of an actual physical device. The model presented also satisfies all the three fingerprints for a memristor model.