Error analysis of an analog correlator for polarimetry

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Abstract: In this letter, we consider a performance evaluation of an analog correlator, and extract Stokes parameters from cross correlation between horizontal and vertical channels of electric fields for a fully polarimetric radiometer. To this end, we develop an analog correlator using 90° and 180° hybrid couplers and square-law detectors. In order for its performance analysis, we derive a closed form of a mathematical representation for Stokes parameters in terms of amplitude and phase responses of both channels, characteristics of hybrid couplers. As a result, the cross correlation of U and V of Stokes parameters is 0.92 and 0.96, respectively. And the maximum return loss of the proposed analog correlator is less than −23 dB for all directions. In addition, the peak-to-peak amplitude response of the proposed analog correlator is 1.9 dB, and for the phase response as 9°. We compare numerical and measured results of I and U of Stokes parameters, and obtain their average errors as 0.4% and 14.9%.

Keywords: analog correlator, polarimetric radiometer, stokes parameters

Classification: Microwave and millimeter-wave devices, circuits, and modules

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1 Introduction

In the remote sensing, many works using polarimetric radiometers have obtained Stokes parameters which measures horizontal and vertical polarizations of electric fields [1, 2]. The applicability of correlators is due to the linearly and circularly polarized thermal emission of objects, e.g., sea surfaces [3], cosmic background radiation [4]. For extracting Stokes parameters, both analog and digital approaches have been exploited to develop cross correlators [5]. While digital correlators have the benefits of controllable bandwidth and stability without cross coupling, analog correlators take the advantage of large bandwidth, high sensitivity, low cost, low power consumption, and tractable implementation [6, 7, 8].

Fully polarimetric radiometers have a great effect on obtaining the Stokes parameters depending on the amplitude and phase errors between the vertical and horizontal electric fields [2]. In fact, for the measurement of the Stokes parameters in the ocean surface, the peak-to-peak of Q and U of the Stokes parameters is about 4–6 Kelvin, while for the V parameter it is less [3]. In order to extract more accurate and sophisticated Stokes parameters, most of the studies of the polarimetric radiometers have been attempted to minimize their errors in the amplitude and phase responses. To this end, we propose a numerical approach for a performance analysis of a correlator which is one of the core components for the development of the fully polarimetric radiometer. The proposed approach can numerically evaluate the amplitude and phase responses of the correlator using input and output relations of components.

In this work, we develop an analog correlator with 2 GHz of bandwidth using 90° and 180° hybrid couplers and square-law detectors for the extraction of the Stokes parameters in the fully polarimetric radiometer. As a result, the cross correlation of U and V of Stokes parameters is 0.92 and 0.96, respectively. Also the maximum return loss of the proposed correlator is less than –23 dB for all ports of directions. The peak-to-peak amplitude and phase responses of the proposed correlator is 1.9 dB and 9°. In order for performance analysis, we propose a closed form of a mathematical representation for the Stokes parameters with characteristics

of the hybrid couplers. We evaluate the performance of the analog correlator with amplitude and phase differences between in-phase and out-phase directions for the hybrid couplers. And we compare performance results of the Stokes parameters between the numerical and measured outputs.

2 Stokes parameters

In this section, we define Stokes parameters in units of brightness ($\text{WHz}^{-1}\text{m}^{-2}\text{sr}^{-1}$). The radiation from an object is partly polarized, where the brightness temperatures for both horizontal and vertical polarizations are different with each other. In order to represent the polarization of these electric fields, a vector $\mathbf{S}$ of parameters called the Stokes parameters is defined as follows [9]:

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \frac{1}{z} \begin{bmatrix} \langle V_0^2 \rangle + \langle H_0^2 \rangle \\ \langle V_0^2 \rangle - \langle H_0^2 \rangle \\ 2V_0H_0 \cos \theta \\ 2V_0H_0 \sin \theta \end{bmatrix}, \quad (1)$$

where $\langle \cdot \rangle$ denotes the time average, H_0 and V_0 are the horizontal and vertical signals with incident angle θ , z is the impedance of the medium propagating the electric field, I describes the total intensity of the electric field, Q represents the difference of the vertical and horizontal electric intensities, and U and V describe the real and imaginary parts of the cross correlation of the electric field. Note that $I^2 = Q^2 + U^2 + V^2$ where I , Q , U , and V are the Stokes parameters in units of antenna temperature (K).

Taking Rayleigh-Jeans approximation, the Stokes vector $\mathbf{S}$ for the field in Eq. 1 can be rewritten as an equivalent brightness temperature T_B (K) [9]:

$$T_B = \frac{\lambda^2}{k} \mathbf{S}, \quad (2)$$

where λ is the wavelength and k is Boltzmann's constant. In this work, we will analyze a newly developed polarimetric radiometer: 94 GHz Fully Polarimetric Radiometer, see [10] for details. This instrument can measure all Stokes parameters by a cross correlation of horizontal and vertical polarization.

3 Design analog correlator

3.1 Description of correlator

Fig. 1 shows the structure of the analog correlator which consists of one phase discriminator, two hybrid couplers, and six square-law detectors. The first two hybrid couplers provide as power dividers for separating both intensities of the orthogonally polarized fields. The two inputs of the analog correlator are the vertically and horizontally polarized fields, i.e., V_0 and H_0 , respectively. The phase discriminator has three 90° and one 180° hybrid couplers which play a role of cross correlation for the extraction of U and V Stokes parameters. The outputs of six hybrid couplers are directly fed into the inputs of six square-law detectors which convert powers of electric fields to voltages with low-pass filters. From subtraction and addition of the outputs of the detectors, we can acquire all Stokes parameters as the equivalent brightness temperature.

Fig. 1. Structure of the analog correlator.

Now we present the mathematical representation for the outputs of six square-law detectors, i.e., from U_1 to U_6 as shown in Fig. 1. For simplicity, we assume that the 90° and 180° hybrid couplers have the following characteristics. The five 90° hybrid couplers have the same identical characteristics as coupling loss and phase shift between in-phase and out-phase directions. For the 90° hybrid couplers, let A_1 and A_2 be the coupling loss, and α and $\alpha + \frac{\pi}{2}$ be the phase shift for the in-phase and out-phase directions, respectively. For the 180° hybrid coupler, let B_1 and B_2 be the coupling loss, and β and $\beta + \pi$ be the phase shift for the in-phase and out-phase directions. Suppose that two input signals of the correlator are V_0 and $H_0 e^{-i\theta}$ for the vertical and horizontal channels. For this mathematical representation, we assume that effects of isolation and return loss for six hybrid couplers are ignored. Under these assumptions, we derive mathematical representation for the outputs of all the components along with the combination of the vertical and horizontal channels. For details, we describe as follows.

For instance, two outputs of the first two 90° hybrid couplers can be written by

$$V_1 = V_0 A_1 \exp^{-i\alpha}, \quad H_1 = V_0 A_1 \exp^{-i(\alpha+\theta)}. \quad (3)$$

Using the same manner, other outputs of the hybrid couplers are given by

$$\begin{cases} e_1 = H_0 A_2 \exp^{-i(\alpha+\theta+\frac{\pi}{2})}, \\ e_2 = V_0 A_1^2 A_2 \exp^{-i(3\alpha+\frac{\pi}{2})} + H_0 A_1^2 B_1 \exp^{-i(2\alpha+\beta+\theta)}, \\ e_3 = V_0 A_1^3 \exp^{-i3\alpha} + H_0 A_1 A_2 B_1 \exp^{-i(2\alpha+\beta+\theta+\frac{\pi}{2})}, \\ e_4 = V_0 A_1 A_2^2 \exp^{-i(3\alpha+\pi)} + H_0 A_1^2 B_2 \exp^{-i(2\alpha+\beta+\theta+\pi)}, \\ e_5 = V_0 A_1^2 A_2 \exp^{-i(3\alpha+\frac{\pi}{2})} + H_0 A_1 A_2 B_2 \exp^{-i(2\alpha+\beta+\theta+\frac{3\pi}{2})}, \\ e_6 = V_0 A_2 \exp^{-i(\alpha+\frac{\pi}{2})}. \end{cases} \quad (4)$$

By taking the average of all the outputs of the hybrid couplers for the detection, we can obtain U_1 to U_6 , and then represent a form of the following matrix as

Fig. 2. (a) Implementation of the proposed analog correlator with 90° and 180° hybrid couplers. (b) Output voltages of I , Q , U , and V of the Stokes parameters at 3 GHz with measured (dots lines) and ideal (solid lines) results.

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{bmatrix} = \begin{bmatrix} 0 & A_2^2 & 0 & 0 \\ A_1^4 A_2^2 & A_1^4 B_1^2 & 2A_1^4 A_2 B_1 \sin(\beta - \alpha) & 2A_1^4 A_2 B_1 \cos(\beta - \alpha) \\ A_1^6 & A_1^2 A_2^2 B_1^2 & -2A_1^4 A_2 B_1 \sin(\beta - \alpha) & -2A_1^4 A_2 B_1 \cos(\beta - \alpha) \\ A_1^2 A_2^4 & A_1^4 B_2^2 & 2A_1^3 A_2^2 B_2 \cos(\beta - \alpha) & 2A_1^3 A_2^2 B_2 \sin(\beta - \alpha) \\ A_1^4 A_2^2 & A_1^2 A_2^2 B_2^2 & -2A_1^3 A_2^2 B_2 \cos(\beta - \alpha) & -2A_1^3 A_2^2 B_2 \sin(\beta - \alpha) \\ A_2^2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_0^2 \\ H_0^2 \\ V_0 H_0 \cos \theta \\ V_0 H_0 \sin \theta \end{bmatrix}. \quad (5)$$

For the ideal correlator, the coefficients of the hybrid couplers are $A_1 = A_2 = B_1 = B_2 = 1/\sqrt{2}$ and $\alpha = \beta = 0$. And Stokes parameters are obtained from $I = U_6 + U_1$, $Q = U_6 - U_1$, $U = U_4 - U_5$ and $V = U_2 - U_3$.

3.2 Performance analysis of analog correlator

In this section, we analyze and compare the performance of the analog correlator. As shown in Fig. 2(a), we develop the wideband analog correlator with approximate 2 GHz of the effective bandwidth, i.e., 2 to 4 GHz. We used commercial 90° and 180° hybrid couplers provided by PULSAR microwave co. Ltd. For the phase balance, we compensate phase difference of both 90° and 180° hybrid couplers by using coaxial cables. All the square-law detectors having under 10 dB of return loss over 2 to 4 GHz are designed by Schottky diodes (HSCH9161) and precise amplifiers (AD620). Identically characteristic six square-law detectors are mounted on the analog correlator.

Fig. 2(b) shows that the output voltages for all the Stokes parameters with the phase sweep from 0 to 360° are measured. Comparison with the ideal Stokes parameters, this observation shows that the correlation coefficients of the U and V Stokes parameters are 0.92 and 0.96, respectively. This means that the cross correlation between the vertical and horizontal channels properly occurs.

Table I shows the results of other state of the arts including our work in order to compare the performance of the analog correlator. The analog correlators are mainly used to obtain vertical and horizontal polarizations of electromagnetic

Fig. 3. (a) Return loss of the vertical (solid line) and horizontal (dotted line) channels, and (b) Amplitude response of the output e_1 (solid line) and e_6 (dotted line) in the analog correlator.

waves in astronomy and radiometer. The maximum return loss of the analog correlator is less than -23 dB for the vertical and horizontal channels as shown in Fig. 3(a). And as shown in Fig. 3(b) and 4, the maximum peak-to-peak amplitude and phase responses of the proposed analog correlator obtained from this work are 1.9 dB and 9° , respectively. Recently, the studies of development of radiometer systems have been efforts to improve sensitivity by increasing bandwidth due to high frequencies, i.e., terahertz. Compared to other studies [5, 6, 7, 8], we achieve highly sensitive performance of the proposed analog correlator as shown in Table I.

Next we compare the performance of the analog correlator with respect to the amplitude and phase error responses using Eq. 5. Let ϵ and δ be the amplitude and phase differences between the in-phase and out-phase directions for the 90° and 180° hybrid couplers as $\epsilon = A_1 - A_2 = B_1 - B_2$, $\delta = \beta - \alpha$. We define the following coefficients of the errors for Stokes parameters as $\Delta I = |\frac{I-I_0}{I}|$, $\Delta Q = |\frac{Q-Q_0}{Q}|$,

Fig. 4. (a) Amplitude response of the output e_2 (black), e_3 (red), e_4 (blue), and e_5 (green) for the vertical and horizontal channels, (b) Phase response of the output e_2 (black), e_3 (red), e_4 (blue), and e_5 (green) for the phase difference between the vertical and horizontal channels.

Table I. Summary of the state of the arts for analog correlators.

Reference	Architecture	Bandwidth (GHz)	Correlation	Max. return loss (dB)	Peak-peak amplitude (dB)	Peak-peak phase (degree)	Application
Our work	Power divider	2–4	>0.92	<−23	1.9	9	Radiometer
[5]	Power divider	1.25/channel	-	<−6	2.0	-	Astronomy
[6]	Multiplier	0.09–0.52	-	<−17	3.9	4	Radiometer
[6]	Mixer	0.1–1.0	-	<−19	<1	17	Astronomy
[7]	Delay line	5.08–14.78	>0.95	-	-	-	Radiometer
[8]	Power divider	1.5–2.5	>0.96	<−8	<1.5	<11	Radiometer

$\Delta U = |\frac{U-U_0}{U}|$ and $\Delta V = |\frac{V-V_0}{V}|$ where I_0 , Q_0 , U_0 and V_0 denote the values of Stokes parameters in the ideal case. In Fig. 5, we show how varying the amplitude and phase differences affect the performance of Stokes parameters. While the amplitude difference ϵ of the hybrid coupler affects to I , U and V Stokes parameters, Q Stokes parameters does not change due to assumption that all 90° hybrid couplers are the same characteristics. Because I and Q Stokes parameters are not dependent of the phase difference, the errors of only U and V Stokes parameters in the phase difference occur as shown in Fig. 5(b) in which U and V are orthogonal with each other.

In order for comparison of the performance between the numerical and measured results for Stokes parameters, we evaluate the results of ΔI and ΔU as shown in Fig. 6. For ΔI , the numerical and measured results are well matched, then its average error is 0.4%. However, due to the non-ideal characteristics of all the 90° and 180° hybrid couplers, the measured cross correlation of the horizontal and vertical channels are different from the numerical approach in which the average error of ΔU is 14.9%.

Fig. 5. Errors of Stokes parameters with: (a) amplitude difference ϵ for I , Q , U , and V , (b) phase difference δ of the hybrid coupler for U .

Fig. 6. Comparison of the performance with the numerical (solid lines) and measured (marked) errors for: (a) ΔI , (b) ΔU .

4 Conclusion

We evaluated the performance of the analog correlator for the fully polarimetric radiometer, and extracted Stokes parameters from the cross correlation between the horizontal and vertical channels of electric fields. We designed the analog correlator using 90° and 180° hybrid couplers and square-law detectors. We derived the mathematical representation for Stokes parameters with respect to the characteristics of device components. We not only evaluated the effects on errors of Stokes parameters, but also compared the numerical and measured results in terms of amplitude and phase responses of both channels.

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