Speed global integral sliding mode control with a load sliding mode observer for PMSM

Liu Yubo\(^a\) and Wang Xudong

Institute of Electrical and Electronic Engineering,
Harbin University of Science and Technology, Harbin, China
\(^a\) yubo\_ryan@yahoo.com

Abstract: In this paper, a permanent magnet synchronous motor controller that improves the robustness to parameter perturbations and load disturbances is proposed for electric drive applications. The speed loop uses global integral sliding mode control strategy, chooses an appropriate global integral sliding surface, maintains a continuous control law, realizes the sliding mode motion throughout the entire motion of the system through a dynamic nonlinear sliding surface, effectively weakens the chattering inherent in the sliding mode control system, and significantly improves the dynamic quality. The load torque sliding mode observer is used to realize the feed-forward compensation of the load torque, the influence on the performance of control system caused by load torque disturbance can be inhibited. The proposed controller is found to be more effective than the conventional controller, as demonstrated through simulations and experiments.

Keywords: permanent magnet synchronous motor (PMSM), global integral sliding mode (GI-SMC), load torque, sliding mode observer (SMO)

Classification: Power devices and circuits

References


1 Introduction

The permanent magnet synchronous motor (PMSM) has been widely used in speed control systems because of its high efficiency, high power density and high torque/inertia ratio [1, 2]. The PMSM is a multi-variable, nonlinear, strong coupling system, that is very sensitive to internal parameter perturbation and external disturbance. Therefore, the traditional linear control method cannot accurately describe the system's steady state and dynamic process adapt to changes in system parameters. Hence, it is difficult to ensure the quality of the motor operation over a wide range of speed regulation. The sliding mode control (SMC) is a variable structure control system that realizes high performance control by frequently switching the system structure. The most notable feature of SMC is that the sliding surface and the sliding mode control rate are planned in advance, and the excellent control effect can be achieved when the system parameters are perturbed or the load is altered [3, 4]. When the SMC is used in PMSM speed control, the high frequency switching control not only guarantees a fast response speed, but also enhances the anti-interference performance of the system. However, chattering of the system is unavoidable in SMC frequent switching of the system structure, which has become an important factor affecting the control. Therefore, improving the implementation of SMC by chatter reduction has become a key research direction. In [5], a method is suggested to reduce chattering and increase the approach speed using the absolute value of the speed error, which is the system state variable. In [6], a hybrid sliding mode approach law is designed and applied to a speed controller, which introduces the system state variables and the terminal attractor to accelerate the system state to reach the sliding surface speed. Meanwhile, the proposed approach gives a detailed plan for the speed near the sliding surface of the system and weakens the chattering phenomenon. Moreover, many scholars have combined intelligent control theory and conventional sliding mode to achieve a better control
effect of the PMSM. In [7], a neural network is introduced in the SMC of the PMSM. The switching component of the sliding mode is adjusted in real time by the recurrent wavelet-based elman neural network, to improve the position tracking performance, which also reducing the steady-state error and enhancing the robustness of the system.

In the SMC control system, only the minimum switching gain increases with the increase of the load disturbance range to satisfy the existence and accessibility conditions of the sliding mode, so as to effectively resist the load disturbance. However, increased switching gain exacerbates system chattering. If the load observer is used to compensate the load disturbance, the switching gain will be significantly reduced and the shake of system will be greatly reduced [8]. The conventional closed-loop load observer can guarantee the asymptotic stability of the observation, but it depends on the tedious online identification of uncertain or time-varying parameters [9, 10]. The load observer based on sliding mode method can effectively suppress the influence of parameter disturbance and load disturbance on the system characteristics [11].

Fig. 1 shows the PMSM control block diagram. The speed controller is designed as a global integral sliding mode controller (GI-SMC). The nonlinear dynamic sliding surface of the GI-SMC realizes sliding motion throughout the entire system movement, and the entire process of the system response is included in the control range. The GI-SMC improves the full range of static and dynamic characteristics of the system. The state variables are speed and load torque, errors between the real and the observed speed are selected as a sliding mode hyper plane. The load torque sliding mode observer is used to realize the feed-forward compensation of the load torque.

2 Mathematical model of the PMSM

The stator voltage equation for the PMSM in the d-q rotating coordinate system is given as

\[
\begin{align*}
    u_d &= R_s i_d + \frac{d\psi_d}{dt} - \omega_r \psi_q \\
    u_q &= R_s i_q + \frac{d\psi_q}{dt} + \omega_r \psi_d
\end{align*}
\]  

where \(u_d\) and \(u_q\) are the d and q axis voltages, respectively; \(i_d\) and \(i_q\) are the d and q axis currents, respectively; \(\psi_d\) and \(\psi_q\) are the d and q axis components of the rotor.
permanent magnet flux linkage, respectively; $L_d$ and $L_q$ are the $d$ and $q$ axis inductances, respectively; $R_s$ is the stator phase resistance; and $\omega_e$ is the angular velocity.

The electromagnetic torque equation is given as

$$T_e = \frac{3}{2} P_n \psi_f i_q + (L_d - L_q) i_d i_q$$

(2)

The equation of motion is given as

$$T_e - T_L = J \frac{d\omega_m}{dt} + B_m \omega_m$$

(3)

where $T_e$ is the electromagnetic torque, $\psi_f$ is the rotor permanent magnet flux linkage, $T_L$ is the load torque, $P$ is the number of pole-pairs, $J$ is the equivalent moment of inertia, $B_m$ is the coefficient of friction torque, and $\omega_m$ is the mechanical angular speed of rotor.

3 Design of the sliding mode speed controller

3.1 State space equation

For speed control of the PMSM, the error of the given angular speed and the actual angular speed are chosen as state variables, because the output of the speed loop will be given as the input of the current loop; thus, the input is defined as the electromagnetic torque $T_e$.

$$x_1 = \omega_{\text{ref}} - \omega_m$$

(4)

$$u = T_e$$

(5)

where $\omega_{\text{ref}}$ is the reference motor speed and $\omega_m$ is the actual motor speed.

The state space equation of the speed control system can be obtained by the motor motion given in Eq. (3), Eq. (4), and Eq. (5).

$$\dot{x}_1 = -\frac{B_m}{J} x_1 - \frac{1}{J} u + \frac{(T_L + B_m \omega_{\text{ref}})}{J}$$

(6)

For the PMSM speed controller, the state equation is rewritten as follows:

$$\dot{x}_1 = bx_1 + cu + e$$

(7)

where $b = -\frac{B_m}{J}$, $c = -\frac{1}{J}$, and $e = \frac{(T_L + B_m \omega_{\text{ref}})}{J}$.

3.2 Design of the sliding surface

The integral SMC can reduce the steady-state error of the system. The integral action can also compensate for the uncertainty of the model, thus enhancing the anti-interference ability of the system.

For the PMSM speed controller, the integral sliding mode surface is taken as

$$s = k_0 x + k_1 \int_0^t x_1 dt$$

(8)

where $k_0$, $k_1$ are variable positive integers.

The global sliding places the approaching stage of the sliding mode motion into the control range, thereby reducing the chattering phenomenon. Its formula can be designed as
\[ s = k_0x_1 + k_1 \int_0^t x_1 \, dt + f(t) \]  \hspace{1cm} (9)

Where \( f(t) \) is the control in order to achieve global sliding mode. In order to guarantee the sliding mode control to satisfy the reachability condition in the whole range, \( f(t) \) must satisfy three characteristics as following:

1. \( f(0) = -(k_0x_1(0) + k_1 \int_0^0 x_1(r) \, dr) \);
2. when \( t \to \infty \), \( f(t) \to 0 \);
3. the derivative \( f(t) \) exists.

According to the above three conditions, \( f(t) \) will be designed as

\[ f = \lambda e^{-at} \]  \hspace{1cm} (10)

where \( f = \lambda e^{-at} \) and \( a > 0 \).

It follows from Eq. (10) and Eq. (9) that the global integral sliding surface can be obtained as

\[ s = k_0x_1 + k_1 \int_0^t x_1 \, dt + \lambda e^{-at} \]  \hspace{1cm} (11)

According to condition 1, \( \lambda = -k_0x_1(0) \) can be solved.

### 3.3 Design of the reaching law

The variable speed approach law can be chosen as

\[ \dot{s} = \varepsilon|x_1| \text{sgn}(s) \]  \hspace{1cm} (12)

where \( \varepsilon \) is the switching gain. The excessive value of \( \varepsilon \) will cause the large chattering of the system; when value is too small will increase the dynamic response time.

### 3.4 Design of the control law

When the system is in the sliding mode area, \( \dot{s} = 0 \). When the load torque and friction coefficient are disturbed, then

\[ \dot{s} = cx_1 = c \left( -\frac{B_m}{J} x_1 - \frac{1}{J} u \right) = 0 \]  \hspace{1cm} (13)

It is follows from Eq. (12) and Eq. (13) that the function switching control law can be obtained as

\[ u = -B_m x_1 + \varepsilon|x_1| \text{sgn}(s) \]  \hspace{1cm} (14)

For the design of the GI-SMC control law, the equivalent control must also be solved for \( \dot{s} = 0 \). Therefore, the derivative of Eq. (13) and \( \dot{s} = 0 \) are substituted into Eq. (14), the adopted control law is shown as

\[ u = -(k_0b + k_1)x_1 + a\lambda e^{-at} \frac{1}{k_0c} + \varepsilon|x_1| \text{sgn}(s) \]  \hspace{1cm} (15)

### 3.5 Continuous switching function

To further reduce the chattering, a continuous function in the switching control \( \text{sat}(s) \) replaces the sign function \( \text{sgn}(s) \):
In Eq. (16), $\varphi$ is a very small positive number, which can weaken chattering by proper tuning. It can be seen that the switching process of the continuous function $\text{sat}(s)$ is smoother and more flexible than that of the symbolic function $\text{sgn}(s)$, which can reduce chattering.

After the improvement, the SMC law is obtained as

$$u = -(k_0 b + k_1)x_1 + a\lambda e^{-at} + c|x_1|\text{sat}(s)$$

### 3.6 Analysis of the stability

The stability conditions of the global integral SMC are obtained, with the Lyapunov function $V(x) = s^2/2$. The reachability condition shows that if the system is stable, then

$$\dot{V}(x) = ss < 0$$

It follows from Eq. (13), Eq. (15) and Eq. (18) that

$$ss = s(k_0\dot{x}_1 + k_1x_1 - a\lambda e^{-at})$$
$$= s[k_0(b\dot{x}_1 + cu + d) + k_1x_1 - a\lambda e^{-at}]$$
$$= s[c\dot{x}_1|\text{sgn}(s) + d]$$
$$\leq (D|s| + c|\dot{x}_1||s|)$$

where $c < 0$ and $\varepsilon > 0$. Therefore, only $\varepsilon \geq D/(-c|x_1|)$ need to be satisfied to force $\dot{V}(x) < 0$, which ensures that system is stable.

### 4 Design of the load sliding mode observer

The sliding mode observer (SMO) is used to observe the torque value, compensates the torque disturbance and improves the performance of the system.

#### 4.1 Extended state space equation

With electrical angular speed $\omega_c$ and load torque $T_L$ as the state variables, the extended state space equation can be described from Eq. (1) to Eq. (3) as

$$\dot{\omega}_c = \frac{1.5p_2^2}{J} [\psi_f i_q + (L_d - L_q)i_d i_q] - \frac{p_n}{J} T_L - \frac{B}{J}\omega_c$$

$$\dot{T}_L = 0$$

#### 4.2 Observer model

With electrical angular speed $\hat{\omega}_c$ and load torque $\hat{T}_L$ as the observed objects, the model of the load sliding mode observer is established from Eq. (20) as

$$\dot{\hat{\omega}}_c = \frac{1.5p_2^2}{J} [\psi_f i_q + (L_d - L_q)i_d i_q] - \frac{p_n}{J} \hat{T}_L - \frac{B}{J}\hat{\omega}_c + u_{smo}$$

$$\dot{\hat{T}}_L = lu_{smo}$$

where $l$ is the feedback gain of the observer, and $u_{smo}$ represents the sliding mode control function, $\hat{\omega}_c$ is estimated speed, $\hat{T}_L$ is estimated torque.
Subtracting Eq. (20) from Eq. (21) yields the error equation of the observer.

\[
\begin{align*}
\dot{\omega}_e &= \dot{\omega}_e - \dot{\omega}_e = -\frac{p_n}{J} \dot{T}_L - B \dot{\omega}_e + u_{ smo} \\
\dot{T}_L &= \dot{T}_L - T = lu_{ smo}
\end{align*}
\]  

(22)

where \(\dot{\omega}_e\) is speed error, \(\dot{T}_L\) is torque error.

### 4.3 Design of the sliding surface

It follows from \(\dot{\omega}_e = \dot{\omega}_e - \omega_e\) that the sliding surface can be obtained as

\[
s = \ddot{\omega}_e = \dot{\omega}_e - \omega_e
\]  

(23)

### 4.4 Design of the reaching law

The variable speed approach law can be chosen as

\[
\dot{s} = \ddot{\omega}_e = \varepsilon_{\omega_0} |\dot{\omega}_e| \text{sgn}(s)
\]  

(24)

where \(\varepsilon_{\omega_0}\) is the switching gain. The excessive value of \(\varepsilon_{\omega_0}\) will cause the large chattering of the system; when value is too small will increase the dynamic response time.

### 4.5 Design of the control law

When the system is in the sliding mode area, \(\dot{s} = 0\). When the load torque and friction coefficient are disturbed, then

\[
\dot{s} = \frac{B}{J} \dot{\omega}_e - \frac{p_n}{J} \dot{T}_L + u_{ smo} = 0
\]  

(25)

It is follows from Eq. (24) and Eq. (25) that the function switching control law can be obtained as

\[
u_{ smo} = \frac{B}{J} \dot{\omega}_e + \frac{p_n}{J} \dot{T}_L + \varepsilon_{\omega_0} |\dot{\omega}_e| \text{sgn}(s)
\]  

(26)

### 4.6 Analysis of the stability

According to Lyapunov stability theory in Eq. (18), the sliding mode existence and accessibility condition of the load observer can be derived from Eqs. (22) to (26) as

\[
\varepsilon_{\omega_0} < \frac{p_n}{J} \dot{T}_L
\]  

(27)

where load torque observation error \(\dot{T}_L\) is large, necessary switching gain \(\varepsilon_{\omega_0}\) and the resulting chattering of the observed objects are also large.

### 5 Simulation and experimental results

The proposed design method for a controller is tested both in simulations and a laboratory prototype. The PMSM parameters include the output power \(P_e = 32\, \text{kW}\), rated speed \(N_e = 1500\, \text{r/min}\), rated torque \(T_e = 200\, \text{N·m}\), stator phase resistance \(R_s = 2.875\, \Omega\), permanent magnet flux linkage \(\psi_f = 0.175\, \text{Wb}\), pole pairs \(p = 4\), and moment of inertia \(J = 0.06\, \text{kg·m}^2\). The experimental prototype was built with the same design parameters used in the above simulation. The results of the output performance of test bench are shown in Fig. 2. The sliding mode speed controller...
parameters are a constant reaching coefficient \( \epsilon = 1.0 \), integral coefficient of the sliding surface \( k_0 = 0.01 \), proportionality coefficient \( k_1 = 1.0 \), and continuous coefficient of the control law \( \delta = 0.5 \). The current controller parameters are the q axis proportionality coefficient of the current controller \( k_{pq} = 2.0 \), the q axis integral coefficient \( k_{iq} = 0.003 \), the d axis proportionality coefficient of the current controller \( k_{pd} = 0.8 \), and the d axis integral coefficient \( k_{id} = 0.003 \). Additionally, the DC voltage was \( U_{dc} = 320 \) V, the sampling period was \( T_s = 0.0001 \) s, and the settling time was 1 s.

Figs. 3 show curves of the sliding mode controller and the SMO for the speed response when the system speed command changes suddenly at a load torque of \( T_L = 200 \) N·m. The system suddenly increases to a speed of 1500 r/min when \( t = 0 \) s. The conventional integral SMC and SMO can make the system have larger chattering time and amplitude. The GI-SMC with SMO accelerates the speed of approaching the sliding mode surface and weakens the chattering existing in the sliding mode control.
Fig. 4 show that the speed response curve of the system suddenly decreases to a speed of 750 r/min when $t = 1.2$ s, after which the speed was stabilized. Under the control of the GI-SMC, the convergence time is shortened, the chattering is reduced, the overshoot of the system is decreased, and the system has good steady and dynamic characteristics.

Figs. 5 and Figs. 6 show curves of the sliding mode controller and SMO in response to a load disturbance, for a given speed of 1500 r/min and a load torque of $T_L = 200$ N·m. Figs. 5 show that the response curve of the system suddenly increases the load to $T_L = 200$ N·m when $t = 0.4$ s. Figs. 6 show that the response curve of the system suddenly reduces the load to 100 N·m when $t = 1.6$ s. The speed controller uses GI-SMC, the speed pulsation is decreased, the response time...
of the d and q axis currents are shortened, and the anti-interference of the load is enhanced.

Figs. 7 and Figs. 8 show that the curves of the two sliding mode controllers and the SMO, for a load torque of $T_L = 200 \text{ N\cdot m}$. The system suddenly increases to a speed of 1500 r/min when $t = 0.6 \text{ s}$. Although the motor accelerates rapidly at the start of the motor, the GI-SMC speed controller can still maintain a large acceleration when the actual speed is close to the target speed, so that the response speed is shortened and the speed fluctuation that reach the steady state will be much smooth, the d and q axis current overshoot is also shortened. So that the GI-SMC speed controller should enable fast motor start-up.

Fig. 9. Responses of speed tracking experiment (SMC+SMO)

Fig. 10. Responses of speed tracking experiment (GI-SMC+SMO)

Fig. 11. Responses of the loading experiment (SMC+SMO)

Figs. 9 and Figs. 10 show that the curves of the two sliding mode controllers and the SMO, for a given speed of 1500 r/min and a load torque of $T_L = 200 \text{ N\cdot m}$. The system suddenly decreases to a speed of 750 r/min when $t = 2.6 \text{ s}$. The response of GI-SMC to speed and overshoot is significantly better than the
conventional integral SMC. GI-SMC incorporates the approach phase of sliding mode motion into the range of sliding mode control, the convergence time is shortened and the shake is reduced, thus the system overshoot is reduced. The response time of d and q axis current is shortened, overshoot significantly is reduced, and the ripple current significantly is reduced.

Figs. 11 and Figs. 12 show that curves of the two sliding mode controllers and the SMO of the load torque suddenly increase, while Figs. 13 and Figs. 14 show that curves of the two sliding mode controllers and the SMO of the load torque suddenly decrease. When the load torque is \( T_L = 100 \text{ N·m} \), and the given speed is \( 1500 \text{ r/min} \), the system suddenly increases the load to \( T_L = 200 \text{ N·m} \) when \( t = 0.6 \text{ s} \), and the system suddenly reduces the load to \( T_L = 100 \text{ N·m} \) when \( t = 2.6 \text{ s} \). The response speed of the GI-SMC is significantly better than the conventional integral SMC. Since the GI-SMC moves the approach stage of the sliding mode into the SMC range, the convergence time is shortened, and the overshoot of the system is decreased. The GI-SMC the entire campaign with SMC effect and the robustness of the SMC improves the system’s anti-interference.
ability. The d and q axes current response time of GI-SMC is shortened, the overshoot is shortened, and the phase current response to torque disturbance is more rapid and accurate.

6 Conclusion

To solve the chattering problem that exists in the conventional integral SMC, a nonlinear dynamic sliding surface GI-SMC that realizes sliding motion over the entire movement of the system is proposed. The entire process of system responses is included in the control range. The GI-SMC improves the system over the full range of static and dynamic characteristics, effectively weakening the chattering phenomenon and improve the system speed tracking effect. The load torque SMO is used to realize the feed-forward compensation of the load torque, the influence on the performance of control system caused by load torque disturbance can be inhibited. To the best of the author’s knowledge, the design method is simple and practicable. The simulation and experimental results show that the controller have a strong robustness and good dynamic performance, remain in a steady state, and are suitable for motor drive applications.

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