Optimization of synthesis filters for hybrid filter bank DACs

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Abstract Hybrid filter bank (HFB) is a promising technology that both improve the resolution and speed of digital-to-analog conversion. In this letter, we present a wideband waveform generation method using HFB architecture. Mixers are adopted to break through the limitation caused by zero-order-hold (ZOH) of DACs. However, the imperfect match of analysis filters and synthesis filters significantly degrade the system performance. To solve this problem, we analyze the influence of filter mismatch. An analysis filter design based on weighted least squares (WLS) and an optimization design for synthesis filter is presented. The optimization method invokes the WLS method at each iteration. Performance evaluation shows that these two methods work together and perform a better reconstruction system than the WLS method which just focus on analysis filter bank.

Keywords: hybrid filter bank, digital-to-analog converter, analysis filter, synthesis filter, optimization design, perfect reconstruction

Classification: Circuits and modules for electronic instrumentation

1. Introduction

Modern test and measurement system exhibit a desire for wideband signal and hence resulting the demand for high speed digital-to-analog converters (DACs) [1, 2]. However, it is difficult to achieve high resolution and high speed in an individual DAC chip with existing fabrication techniques [3]. Hybrid filter bank (HFB) is recognized as one of the solutions to overcome this problem.

HFB is a promising technology that both improves the resolution and speed of analog-to-digital or digital-to-analog conversion [4]. It is an unconventional type of filter bank which employs both digital and analog filters. Several studies on HFB ADC have been carried out [5, 6, 7, 8, 9, 10], while HFB used in DAC array is relatively less studied. Conventional HFB ADC architecture [3] is not feasible for HFB DAC. This is mainly because of the different sample-and-hold circuits between ADC and DAC. Generally speaking, the most commonly used ADC architecture is the zero-order-hold (ZOH) circuit which has the same sample-and-hold bandwidth with Nyquist bandwidth [11, 12, 13, 14], which is much narrow as compared with ADC. To solve this problem, in this work, mixers are adopted in conjunction with hybrid-filter-bank architecture. Based on this method, a wideband signal could be generated with an DAC array of slower speed and would not be restricted by the sinc roll-off caused by ZOH.

The HFB system suffer from the aliasing error if the analysis filters and synthesis filters are not well matched [17, 18, 19, 20, 21]. Several filter match design methods such as linear programming [22], second-order cone programming [23] have been proposed to eliminate the aliasing error and minimize the distortion. Most works [24, 25, 26, 27, 28, 29] focus on designing the digital filter to match a pre-set analog filter bank while the parameter of these analog filters are not discussed. Once an analog filter bank is chosen, the system performance would be determined using a specific design method. However, the system performance would change if a different analog filter is adopted. Hence it gives the possibility to further improve the performance if an appropriate analog filter bank is found. In this work, an optimization design to find suitable parameters for analog synthesis filter is proposed. The optimization method invokes the weighted least squares (WLS) method at each iteration to calculate the analysis filter coefficients. Simulations in section 5 shows that these two methods work together and lead to a better performance than the WLS method which just focus on the analysis filters.

2. Hybrid-filter-bank DACs

\[ X_{\text{in}}(e^{j\omega}) = \frac{1}{M} \sum_{m=0}^{M-1} X_{\text{in}}(e^{j\omega/m})F_{m}(e^{j\omega/m}) \] (1)

Fig. 1 shows the block diagram of the proposed hybrid-filter-bank DACs. For an M-channel system, the input signal \( x[n] \) is first segmented by the digital analysis filter bank \( F_m(z) \) and then down-sampled to get the subband signal \( x_m[n] \). The output spectrum is given by

**Fig. 1.** Architecture of the hybrid-filter-bank DACs
\( p \) indicates the frequency-shifted replicas due to decimation, \( F_m(e^{j\omega}) \) is the digital analysis filter of the \( m \) th subchannel, \( \omega = M\Omega/f_s \). The whole system sampling rate is \( f_s \), while the DAC sampling period is \( MT \), \( T = 1/f_s \). The spectrum of DAC output could be represented as

\[
S_m(j\Omega) = X_m(e^{j2\pi jMT})H_{st}(j\Omega)H_{f}(j\Omega)
\]

(2)

where \( H_{st}(j\Omega) \) stands for the DAC sample-and-hold character and \( H_{f}(j\Omega) \) is the anti-image lowpass filter followed by DAC. Before sending to the synthesis filter, the output is up-converted in order to shift for the first Nyquist zone to a specific frequency region. The mixer output is given by

\[
Y_m(j\Omega) = \frac{1}{2} \left( S_m(j\Omega - 2\pi f_{lo,m}) + S_m(j\Omega + 2\pi f_{lo,m}) \right)
\]

(3)

\( f_{lo,m} \) is the frequency of mixed cosine signal, \( f_{lo,m} = \lfloor m/2 \rfloor f_s/M, \lfloor \cdot \rfloor \) means taking the integer upwardly. Substituting Eq. (2) into Eq. (3) and it gives:

\[
Y_m(j\Omega) = \frac{1}{2} \left( X_m(e^{j2\pi jMT} - 2\pi f_{lo,m}MT) \right.

\times H_{st}(j\Omega - 2\pi f_{lo,m})H_{f}(j\Omega - 2\pi f_{lo,m})

+ X_m(e^{j2\pi jMT} + 2\pi f_{lo,m}MT) \times H_{st}(j\Omega + 2\pi f_{lo,m})H_{f}(j\Omega + 2\pi f_{lo,m})

(4)

Note that \( f_{lo,m}MT \) is an integer, hence Eq. (4) could be written as

\[
Y_m(j\Omega) = \frac{1}{2} X_m(e^{j\Omega})

\times \left( H_{st}(j\Omega - 2\pi f_{lo,m})H_{f}(j\Omega - 2\pi f_{lo,m})

+ H_{st}(j\Omega + 2\pi f_{lo,m})H_{f}(j\Omega + 2\pi f_{lo,m}) \right)

(5)

These signals are summed together after synthesis filter bank. Using Eq. (1) and Eq. (5), the spectrum of the reconstructed signal is given by

\[
Y(j\Omega) = \sum_{m=0}^{M-1} Y_m(j\Omega)G_m(j\Omega)

= \sum_{m=0}^{M-1} X(e^{j\Omega - j2\pi})T_p(j\Omega)

(6)

where

\[
T_p(j\Omega) = \frac{1}{2M} \sum_{m=0}^{M-1} F_m(e^{j\Omega - j\Omega})G_m(j\Omega)

\times \left( H_{st}(j\Omega - 2\pi f_{lo,m})H_{f}(j\Omega - 2\pi f_{lo,m})

+ H_{st}(j\Omega + 2\pi f_{lo,m})H_{f}(j\Omega + 2\pi f_{lo,m}) \right)

G_m(j\Omega) \text{ is the analog synthesis filter bank. In this design, easily feasible analog filters such as second-order Butterworth filters are chosen as the synthesis filter bank. This will be discussed in Section 4.}

Eq. (6) shows that the output spectrum contains both the desired component \( X(e^{j\Omega - j\Omega}) \) and unwanted images \( X(e^{j\Omega - j\Omega + j2\pi}) \), \( p = 1, \ldots, M - 1 \). \( T_p(j\Omega) \) is usually referred to as the distortion function which characterizes the gain and phase response of desired signal. \( T_p(j\Omega), p = 1, \ldots, M - 1 \) are the transfer function of images which correspond to the aliasing errors. If \( T_p(j\Omega) \) satisfies Eq. (7)

\[
T_p(j\Omega) = D_p(j\Omega) = \begin{cases}
    a e^{-jM\Omega i}, & p = 0 \\
    0, & p = 1, \ldots, M - 1
\end{cases}

(7)

where \( a \) and \( d \) is the desired system gain and delay respectively, the HFB DAC could be regarded as a perfect-reconstruction system.

![Illustration of spectrum for four channel hybrid filter bank system](image)

If the analysis filters do not match the synthesis filters well, aliasing components would significantly degrade the system performance. Taking four-channel \( (M = 4) \) as an example, Fig. 2 illustrates the spectrum composition of the output when input is a real-valued sinusoid signal. It can be seen that the output at frequency \( \Omega = \omega_0 f_s \) is the desired signal. Components at frequency \( \pi f_s/2 + \omega_0, \pi f_s + \omega_0 + 3\pi f_s/2 + \omega_0 \) are the aliasing images.

3. Analysis filter bank design

To satisfy the Eq. (7), considering a N-tap finite impulse response (FIR) realization for the analysis filter. The frequency response of \( m \) th analysis filter is given by

\[
F_m(e^{j\omega}) = \sum_{n=0}^{N-1} f_m(n)e^{-jn\omega}

(8)

where \( f_m(n) \) is the coefficient of \( m \) th analysis filter, \( \omega = \Omega T \). Substituting Eq. (8) into the transfer function \( T_p(j\Omega) \) and it gives

\[
T_p(j\Omega) = \frac{1}{2M} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} G_m(j\Omega)f_m(n)e^{-jn\Omega T - j2\pi/M}

\times \left( H_{st}(j\Omega - 2\pi f_{lo,m})H_{f}(j\Omega - 2\pi f_{lo,m})

+ H_{st}(j\Omega + 2\pi f_{lo,m})H_{f}(j\Omega + 2\pi f_{lo,m}) \right)

(9)

Discretizing the frequency into \( K \) frequencies, \( i = 0, 1, \ldots, K - 1 \), Eq. (9) can be rewritten into a matrix form

\[
T_{MK\times 1} = G_{MK\times MN} f_{MN\times 1}

(10)

where
where

\[ G_{MK\times MN} = \begin{bmatrix} G_{0,0} & G_{0,1} & \cdots & G_{0,M-1} \\ G_{1,0} & G_{1,1} & \cdots & G_{1,M-1} \\ \vdots & \vdots & \ddots & \vdots \\ G_{M-1,0} & G_{M-1,1} & \cdots & G_{M-1,M-1} \end{bmatrix} \]

\[ f_{MN,1} = [f_0^T, f_1^T, \ldots, f_{M-1}^T]^T \]

\[ f_m = [f_m(0), f_m(1), \ldots, f_m(N-1)]^T \]

\[ [G_{pm}]_{mn} = \frac{1}{2M} G_m(j\Omega)e^{-jn(\Omega_{c,1}T-2\pi p/M)} \]

\[ \times \{ H_{Sf,1}(j\Omega_i - j2\pi f_{10,m})H_{g,f}(j\Omega_i - j2\pi f_{10,m}) + H_{Sf,1}(j\Omega_i + j2\pi f_{10,m})H_{g,f}(j\Omega_i + j2\pi f_{10,m}) \} \]

The analysis filter coefficients could be obtained by solving the following equation:

\[ \sqrt{W}G_{MK\times MN}f_{MN,1} = \sqrt{W}D_{MN,1} \]  

(11)

\[ D = [D_0^T, \ldots, D_{M-1}^T]^T, \quad D_0 = [D_p(\Omega_0), \ldots, D_p(\Omega_{M-1})]^T \]

\( W \) is a weight matrix specifying the “don’t care” band and relative weight between \( T_0(\Omega) \) and \( T_p(\Omega) \), \( p = 1, 2, \ldots, M-1 \). A solution in weighted least squares sense [8] is given by

\[ \hat{f}_{MN,1} = (G^H W G)^{-1} G^H W D_{MN,1} \]

(12)

where \( G^H \) means complex-conjugate transpose. When calculating the filter coefficients, cares must be taken to ensure that the filter coefficients are real numbers. If the discrete frequency \( \Omega_i \) and desired frequency response \( D_0 \) are chosen such that the conjugate symmetry is satisfied, the \( f_{MN,1} \) obtained by Eq. (11) will be real valued.

4. Synthesis filter design

It has been shown that analog filters with passband ripple and nonconstant delay (e.g., Chebyshev and elliptic filter) complicate the demands of digital filter part and result in a high order FIR filter [30]. Hence, the Butterworth filters with no passband ripple and nearly constant group delay are chosen as the synthesis filter. The frequency response of the synthesis filter is then given by

\[ G_0(j\Omega) = H_0(\Omega/\Omega_c) \]

\[ G_m(j\Omega) = H_0 \left( \frac{(\Omega)^2 + \Omega_{m1,2}\Omega_{m2}}{\Omega \times (\Omega_{m2} - \Omega_{m1})} \right), \quad m = 1, \ldots, M-1 \]

(13)

where \( \Omega_c \) is the cut-off frequency (approximately the -3 dB frequency) of the first channel lowpass filter, \( \Omega_{m1} \) and \( \Omega_{m2} \) are the lower and upper cut-off frequency of \( m \) th channel bandpass filter respectively. \( H_0(\Omega) \) is the n-order Butterworth prototype filter. In this letter, the synthesis parameters are specified in a vector

\[ g = [\Omega_c, \Omega_{1,1}, \Omega_{1,2}, \ldots, \Omega_{M-1,1}, \Omega_{M-1,2}]^H \]

(14)

Given the synthesis filter bank \( g \), the digital analysis filter coefficients \( \tilde{f} \) could be calculated using the method in section 3, and hence the system transfer function \( \tilde{T}_p(g, \Omega_i) \) determined accordingly. The approximation error is given by \( e_p(j\Omega) = |T_p(g, \Omega_i) - D_p(\Omega_i)| \). To further prove the system performance with the variable \( g \), we solve the following optimization problem:

\[ \min_{\arg\min g} \epsilon \quad s.t. \quad \epsilon \geq W(\Omega)|\tilde{T}_p(g, \Omega_i) - D_p(\Omega_i)| \]

(15)

where

\[ \tilde{T}_p(g, \Omega_i) = \frac{1}{2M} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} G_m(g, \Omega_i) \tilde{f}_m(n)e^{-jn(\Omega_{c,1}T-2\pi p/M)} \]

\[ \times \{ H_{Sf,1}(j\Omega_i - j2\pi f_{10,m})H_{g,f}(j\Omega_i - j2\pi f_{10,m}) + H_{Sf,1}(j\Omega_i + j2\pi f_{10,m})H_{g,f}(j\Omega_i + j2\pi f_{10,m}) \} \]

(16)

\[ G_m(g, \Omega_i) = \left\{ \begin{array}{l} H_0(j\Omega_i/g(1)), \\ H_0 \left( \frac{(\Omega_i)^2 + (g(2m+1))}{\Omega \times (g(2m+1) - g(2m))} \right), \quad m = 1, \ldots, M-1 \end{array} \right. \]

(17)

This is a nonlinear optimization problem and could be solved by the sequential quadratic programming [31] which is implemented using the MATLAB Optimization Toolbox.

5. Simulation result

A four-channel hybrid filter bank DAC with an overall sampling rate \( f_s = 2 \) GS/s is demonstrated. DACs with ZOH circuit are adopted and each works at the sampling rate of 500 MS/s.

\[ H_{Sf,1}(\Omega) = MT \cdot \sin^2(\Omega MT/2) \cdot e^{-j(\Omega MT/2)} \]

(18)

Second-order Butterworth filter is chosen as the analog synthesis filter and hence, \( H_0(\Omega) = 1/(\Omega^2 + \sqrt{2} \Omega + 1) \). The initial configuration of the parameter vector is \( g_0 = 2\pi \times [250, 250, 500, 750, 750, 1000]^H \) (Mrad/s). After the optimization, the filter parameter vector is \( g_{opt} = 2\pi \times [373.2, 212.1, 625.3, 485.4, 739.1, 641.5, 962.9]^H \) (Mrad/s). Fig. 3 shows the magnitude frequency response of the synthesis filter, the dashed lines represent the initial configuration of synthesis filter bank, the solid lines denote the optimized filters.

![Fig. 3. Magnitude frequency response of the synthesis filters \( G_m(\Omega) \)](image-url)

The length of digital analysis filter, the system gain \( a \) and delay \( d \) is set to 50, 1 and 25 respectively. The “don’t care” band is \([-\pi f_s, -0.9\pi f_s] \) and \([0.9\pi f_s, \pi f_s] \) which means the system output spectrum is artificially restricted within \( 0.9 f_s \). Fig. 4 and Fig. 5 shows the simulation results.
It could be seen in Fig. 4 that the WLS method lead to an almost unit magnitude response of distortion function, the maximum ripple is about 0.06 dB (dashed line). The aliasing components \( T_p(j\Omega) \) are suppressed under \(-59.4\) dB shown in Fig. 5 (dashed line). When the synthesis filter bank optimization is adopted together with WLS method. A better performance in distortion is achieved, the maximum ripple drops to 0.03 dB. Meanwhile the aliasing components are further suppressed under \(-76.5\) dB as shown in Fig. 5 (solid line).

![Fig. 4. Frequency response of the distortion \( T_0(j\Omega) \)](image)

![Fig. 5. Frequency response of the aliasing \( T_p(j\Omega) \)](image)

![Fig. 6. ENOB versus the analysis filter length \( N \)](image)

Table 1. Performances of WLS method and WLS with synthesis filter optimization

<table>
<thead>
<tr>
<th>Method</th>
<th>Filter length</th>
<th>Max magnitude distortion (dB)</th>
<th>Max phase distortion (rad)</th>
<th>Mean magnitude aliasing (dB)</th>
<th>Max magnitude aliasing (dB)</th>
<th>ENOB (bit)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>60</td>
<td>0.0293</td>
<td>0.0032</td>
<td>-69.00</td>
<td>-61.99</td>
<td>11.6</td>
</tr>
<tr>
<td></td>
<td>70</td>
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<td>0.0009</td>
<td>-78.45</td>
<td>-73.52</td>
<td>13.3</td>
</tr>
<tr>
<td></td>
<td>80</td>
<td>0.0032</td>
<td>0.0009</td>
<td>-87.35</td>
<td>-81.27</td>
<td>14.7</td>
</tr>
<tr>
<td>WLS with Optimization</td>
<td>60</td>
<td>0.0044</td>
<td>0.0006</td>
<td>-92.05</td>
<td>-86.33</td>
<td>15.5</td>
</tr>
<tr>
<td></td>
<td>70</td>
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<td>0.0003</td>
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<td>16.5</td>
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<tr>
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<td>0.0001</td>
<td>-99.65</td>
<td>-96.07</td>
<td>16.8</td>
</tr>
</tbody>
</table>

method bring an 1-2 bit improvement of ENOB compared with the WLS method. Table 1 presents the detail performance evaluation.

6. Conclusion

In this work, a waveform generation method based on HFB DAC is presented. Mixers are adopted and hence the system would not be restricted by the ZOH characteristic of DAC. An analysis filter design method based on WLS and a synthesis filter optimization method is proposed to further improve the performance. Simulations shows that these two methods work together and achieve a better performance.

References


