Modeling and analysis of nonlinear behavior for synchronous switching Z-source topology

Yan Chen¹, Zhiyang Zhou¹, Yong Zheng²,a), and Lin Zhong¹

Abstract As a high-performance step-up/step-down converter, the synchronous switching Z-source overcomes the theoretical obstacles and limitations of traditional voltage source and current source converters. However, due to the existence of switching devices, the system has strong nonlinear characteristics. At the same time, it belongs to a high-order system, which is easily affected by changes in environment and structural parameters, causing local system instability, namely bifurcation and chaos. To solve the above problems, the discrete iterative model is established on the basis of the state equation of the system, the steady-state operating point of the system was analyzed by nonlinear theory, and the mechanism of the nonlinear behavior was revealed, so as to determine the parameter conditions of the system stability region. Compared with solving the differential equation directly, this paper maps the discrete iterative model to the complex frequency domain for solution, which can greatly reduce the amount of calculation while ensuring the accuracy of the model. The analysis provides the parameter basis for the stable operation of the system and provides theoretical support for further optimization of the system design.

Keywords: synchronous switching converter, bifurcation, nonlinear behavior, Z-source topology

1. Introduction

As the core component of power transformation, switching converter plays an important role in energy transfer, and is widely used in new energy power generation, rail transit equipment, new energy electric vehicles and other fields. With the deepening of power electronics technology research, some phenomena such as unknown electromagnetic noise, intermittent instability or sudden collapse of the system in the circuit system under certain conditions have gradually attracted the attention of researchers. With the gradual establishment of nonlinear dynamics theory [1, 2] and the continuous in-depth study of converters, it is gradually realized that these phenomena are the manifestations of bifurcation and chaos in nonlinear behavior in power electronic converters. The switching converter has a nonlinear variable structure control system. The on-off or off of the power switch tube makes the system switch periodically in different structures, which will lead to complex nonlinear behaviors of the switching converter, such as period doubling bifurcation [3, 4], Hopf bifurcation [5, 6], Flip bifurcation [7, 8], boundary collision bifurcation, and even chaos [9, 10], subharmonic oscillation [11]. As a new type of converter, Z-source converter was first proposed by Professor Peng Fangzheng in 2002 [12]. Z-source converter [13, 14] introduces a unique impedance source network to connect the input and output, so that any loop in the converter topology contains inductive elements, which can effectively suppress the spikes generated by the switching commutation. Therefore, many improved derivative topologies have emerged [15, 16, 17]. However, the introduction of Z-source network also increases the order of the circuit, which makes the nonlinear dynamic behavior of Z-source converter more complex [18]. Compared with the traditional boost, buck converter, the nonlinear behavior has been studied maturely [19, 20], the current Z source nonlinear behavior research is still in its infancy, only appeared in some topologies [21, 22, 23]. At present, the existing researches on Z-source DC conversion are based on the combination of diode and switch tube complementary conduction control [24, 25, 26]. Replacing diode with switch tube for synchronous control is more conducive to efficiency improvement, but it is easy to produce nonlinear behavior.

As a kind of Z-source group, synchronous Z-source converter is more likely to produce nonlinear behavior because of its two switches are controlled and the equivalent circuit is different from the traditional Z-source. In reference [27], a mapping model is established by solving the second-order differential equation for the nonlinear behavior analysis of synchronous switch Z source, the iterative equation [28] is too complex, and the research will be hindered by the increase of system complexity. In this paper, an accurate strobe iterative model was established on the basis of the system state equation, and the model was solved in the mapping to the complex frequency domain [29, 30]. While ensuring the accuracy of the model, the computational complexity was greatly reduced and the processor memory space was saved. Furthermore, the critical value trajectory of the system in the stability domain with the change of parameters was further analyzed. This provides the parameter basis for the stable operation of the system, and provides the theoretical support for the further optimization of the system design.

¹ School of Electrical and Electronic Engineering, Chongqing University of Technology, Chongqing, China
² Engineering Research Center of Mechanical Testing Technology and Equipment, Ministry of Education, Chongqing University of Technology, Chongqing, China
a) sdzzzy@cqut.edu.cn

DOI: 10.1587/elex.18.20210209
Received May 12, 2021
Accepted June 2, 2021
Publicized June 15, 2021
Copyedited July 10, 2021

Copyright © 2021 The Institute of Electronics, Information and Communication Engineers
2. Working principle and modeling of synchronous switching Z-source converter

2.1 Working principle of synchronous switching Z-source converter

The peak current controlled synchronous switching Z-source converter is shown in Fig. 1. It is composed of voltage source \( V_{in} \), switch tube \( Q_1, Q_2 \) and Z-source network and load. For the convenience of analysis, take \( L_1 = L_2, C_1 = C_2 \). In the inductor current continuous mode, the system has two operating modes, mode 1, two switches are switched on at the same time, the power \( V_{in} \) supply charges the inductors \( L_1, L_2 \) and capacitors \( C_1, C_2 \) in the Z-source network, the inductor current \( i_L \) rises, when the inductor current increases to the reference current \( I_{ref} \), it will reset the trigger through the comparator; in mode 2, when the switch tube \( Q_1 \) and \( Q_2 \) are turned off at the same time, the Z-source network supplies power to the load \( R \), and the inductance current \( i_L \) drops, after a operating cycle until the switch tube is turned on again.

\[
\begin{align*}
\dot{x} &= A_1 x + B_1 E \\
\dot{x} &= A_2 x + B_2 E
\end{align*}
\]

Where, \( A_2 = \begin{bmatrix}
-2(RL + r_c) & 1/L \\
-1/C & 0
\end{bmatrix}, \quad B_2 = \begin{bmatrix}
0 \\
0
\end{bmatrix}.
\]

3. The establishment of discrete iterative equation

Since the synchronous switching Z-source converter is a piecewise nonlinear system under the action of the periodic switching of the power switch, it is difficult to analyze the stability of its nonlinear dynamic behavior directly through the state equation, so it is necessary to establish its discrete mapping equation to analyze it. The stroboscopic sampling method can be used to solve the discrete iterative equation of the system. Let \( d_n \) be the duty cycle of the \( nth \) cycle, the state variable of the \( nth \) cycle is \( x_n = x(nT) \), and the definition of other discrete parameters can be deduced by analogy, then the discrete iterative equation can be written as

\[
x_{n+1} = f(x_n, d_n)
\]

The two state equations of the switching converter can be expressed as

\[
\begin{align*}
\dot{x} &= A_1 x + B_1 E \\
\dot{x} &= A_2 x + B_2 E
\end{align*}
\]

Where \( E \) represents the input voltage, namely \( E = V_{in} \). For the system described by the state equation \( \dot{x} = Ax + BE \), the complex frequency domain solution is

\[
X(s) = \phi(s)[x(0_-) + BE(s)]
\]

Where \( \phi(s) = (sI-A)^{-1} \). Then the inverse Laplace transform is carried out to obtain the solution of the state equation

\[
X(t) = \phi(t)X(0_-) + \int_0^t \phi(t - \tau)BE(\tau)d\tau
\]

Where \( \phi(t) = e^{At} \). The discrete model under stroboscopic map is

\[
x(n + 1) = e^{AT}x(n) + \int_0^T e^{A(T-t)}BE(\tau)d\tau
\]

From the state equations (1) and (2) of the system, the discrete model of the system in the \( nth \) period can be obtained as

\[
x(t_n + d_nT) = \phi_1(d_nT)x(t_n) + \int_{t_n}^{t_n + d_nT} \phi_1(t_n + d_nT - \tau)B_1 E d\tau
\]

\[
= \phi_1(d_nT)x(t_n) + \int_0^{d_nT} \phi_1(t_n + d_nT - \tau)B_1 E d\tau \quad (nT < t < nT + d_nT)
\]

\[
x(t_{n+1}) = \phi_2(\bar{d}_nT)x(t_n + d_nT) + \int_{t_n + d_nT}^{t_{n+1} + d_nT} \phi_2(t_n + d_nT - \tau)B_2 E d\tau
\]

\[
= \phi_2(\bar{d}_nT)x(t_n + d_nT) + \int_0^{d_nT} \phi_2(t_n + d_nT - \tau)B_2 E d\tau \quad (nT + d_nT < t < (n + 1)T)
\]
\( \phi_1(\bar{d}_n T) = e^{A_1 \bar{d}_n T}, \quad \phi_2(\bar{d}_n T) = e^{A_2 \bar{d}_n T}, \quad \bar{d}_n = 1 - d_n \). Note that \( A_1 \) is irreversible and \( A_2 \) is reversible, so the integral terms in the above two formulas are simplified

\[
x(t_n + d_n T) = \phi_1(\bar{d}_n T) x(t_n) + B_1 E d_n T
\]

(11)

\[
x(t_{n+1}) = \phi_2(\bar{d}_n T)x(t_n + d_n T) + (\phi_2(\bar{d}_n T) - 1) A_2^{-1} B_2 E
\]

(12)

The discrete mapping equation is obtained by substituting (11) into (12)

\[
x_{n+1} = f(x_n, d_n)
\]

\[
= \varphi_2(\bar{d}_n T) \varphi_1(d_n T) x_n + \varphi_2(\bar{d}_n T) B_1 E d_n T
\]

\[
+ (\varphi_2(\bar{d}_n T) - 1) A_2^{-1} B_2 E
\]

(13)

Define switch function

\[
s(x_n, d_n) = i_{r, e} - i_L
\]

\[
= i_{r, e} + k_1 x_n = i_{r, e} + k_1 x(t_n + d_n T)
\]

\[
= i_{r, e} + k_1 \phi_1(d_n T) x_n + k_1 B_1 E d_n T
\]

(14)

Where \( k_1 = [-1, 0] \). Then when \( s(x_n, d_n) = 0 \), the state of switch \( S \) changes from \( Q = 1 \) to \( Q = 0 \). The discrete mapping equation of synchronous switching Z-source converter is constituted by (13) and (14), the discrete iterative model of synchronous switching Z-source converter is constituted. In Reference [27], the second-order differential equation is derived from the state equation, and the analytical formula parameters obtained by solving the ordinary differential equation theory are the complex function formula about time \( t \). Then, the iterative equation with complex and difficult calculation is derived by fitting all the results. Compared with the iterative equation deduced by solving in the complex frequency domain, the complexity of the formula is greatly reduced, the calculation amount of a single iteration is greatly reduced, and the overall iteration time of the computer is almost reduced by half. The figure 2 shows the time required to draw the bifurcation diagram, take the reference current change interval of 0.1A and iteration step length of 1mA, then the number of per iteration step length is 1000.

4. Stability analysis

The Jacobian matrix is an effective tool for analyzing stability. It is used here to analyze the stability of the static operating point of the converter. First, the Jacobian matrix is established.

Assuming that the controlled system is stable in a single-period state, and that the input voltage and reference voltage always remain unchanged, we have

\[
x_n = \delta_n + X_Q, d_n = \hat{d}_n + D
\]

(15)

\( X_Q \) and \( D \) as steady-state single-periodic solutions. The small signal linearization perturbation for Eq. (3) is

\[
\dot{\delta}_n = \frac{\partial f}{\partial x_n} \delta_n + \frac{\partial f}{\partial d_n} \hat{d}_n = 0
\]

(17)

According to Eq.(16), we can get

\[
\dot{\delta}_n = \frac{\partial f}{\partial x_n} \delta_n + \frac{\partial f}{\partial d_n} (-\frac{\partial s}{\partial d_n})^{-1} \frac{\partial s}{\partial x_n} \delta_n
\]

(18)

Then the corresponding Jacobian matrix is

\[
J(X_Q) = \left[ \frac{\partial f}{\partial x_n} - \frac{\partial f}{\partial d_n} (-\frac{\partial s}{\partial d_n})^{-1} \frac{\partial s}{\partial x_n} \right]_{(X_Q, D)}
\]

(19)

\[
\frac{\partial s}{\partial x_n} = k_1 e^{A_1 d_n T}, \quad \frac{\partial s}{\partial d_n} = k_1 T e^{A_1 d_n T} A_1 x_n + k_1 TBE
\]

\[
\frac{\partial f}{\partial d_n} = e^{A_1(1-d_n) T} e^{A_1 d_n T} \frac{\partial f}{\partial d_n} = T e^{A_1(1-d_n) T} e^{A_1 d_n T} A_1 x_n
\]

Let \( \lambda \) be the characteristic value of \( J \). From the system stability, when all values of \( \lambda \) fall within the unit circle, the system is stable. The reason is that the Jacobian matrix here is the ratio of small perturbations of two adjacent iterations, and is the ratio of the subsequent term to the preceding term. If the ratio is greater than 1, it means that with the increase of iteration times, the disturbance quantity gradually increases, and the system is in an unstable state; if the ratio is less than 1, the system is stable.

Before calculating the eigenvalues of the Jacobian matrix of the system, it is firstly necessary to solve the static operating point of the controlled synchronous switching Z-source converter system, that is, the steady-state periodic solutions \( X_Q \) and \( D \) of the system’s discrete iterative equations. The process is as follows.

\[
\text{set } x_{n+1} = x_n, \quad x(nT + d_n T) = X_D, \quad d_n = D
\]

According to equations (18) and (19), there is

\[
X_D = \phi_1 X_Q + BE DT
\]

(20)

\[
X_Q = \phi_2 X_D + (\phi_2 - I) A_2^{-1} BE
\]

(21)

According to the above two formulas, we can get

\[
X_Q = (I - \phi_2 \phi_1)^{-1} \left[ \phi_2 BE DT + (\phi_2 - I) A_2^{-1} BE \right]
\]

(22)

At the same time, the switching function satisfies

\[
s(X_D, D) = i_{r, e} + k_1 X_D = 0
\]

(24)

Combining (20) to (24), the steady-state periodic solutions \( X_Q \) and \( D \) can be obtained by numerical operations, and
on this basis, the eigenvalues of the Jacobian matrix can be obtained to judge the stability of the system.

According to the schematic diagram of the peak current mode controlled synchronous switching Z-source converter shown in Fig. 1, the eigenvalues of Jacobian matrix are calculated according to the following parameters.

\[ L = 2mH, R = 20\Omega, C = 2000\mu F, I_{ref} = 0.1 \sim 2.1A \]

For example, when \[ |\lambda_{max}| = \max\{|\lambda_1|,|\lambda_2|\} \]. As can be seen from Fig. 3, when the eigenvalue reaches the point \((-1,0)\) on unit circle, there is a boundary where the system enters into instability. That is, the system enters into an unstable periodic bifurcation state when \(I_{ref}\) increases to a certain value.

Fig. 3 Refers to the motion trajectory of system eigenvalues when the current changes.

In order to intuitively obtain the position coordinates of the bifurcation point, the maximum module characteristic value is plotted as a change trajectory map, as shown in Fig. 4. When the maximum eigenvalue changes to a certain point, \[ |\lambda_{max}| \] will exceed 1 when \(I_{ref}\) taking a value between 0.3A and 0.4A, thus causing the system to enter an unstable state.

Fig. 4 Trajectories of the maximum module characteristic values of the Jacobian matrix.

According to the characteristics of the synchronous switching Z converter system, the reference current is used as the bifurcation parameter for simulation, and the other parameters of the circuit remain unchanged. When the reference current \(I_{ref}\) changes in interval 0.1A to 2.1A, the bifurcation of inductor current \(I_L\) is shown in Fig. 5. When \(I_{ref} = 0.37A\), the system bifurcates from period 1 to period 2. When \(I_{ref} = 0.67A\), period-doubling bifurcation occurs again, and when \(I_{ref} = 1.2A\), the system enters into a chaotic state. The figure 4 shows the whole process of the system entering chaos through period-doubling bifurcation with the change of the reference current.

Fig. 5 Bifurcation diagram of inductance current with change of reference current \(I_{ref}\)

When \(I_{ref} = 2.1\), the system has entered a chaotic state. At this time, the change of state is observed through the phase diagram of inductance current and capacitor voltage of the Z-source network. Fig. 6 shows the operating state diagram of the phase trajectory.

Fig. 6 Inductance current and capacitance voltage of phase trajectory.

5. Parameter analysis of system stable operation region

The position of bifurcation point can be accurately judged by the change of eigenvalue of Jacobian matrix, and the system parameters can be set within a period to ensure that the system works in a stable region. As shown in Fig. 7,
the peak reference current $I_{ref}$, inductance current $L$ of Z-source and load $R$ were selected as the spatial distribution diagram of the stability region of variation.

6. Numerical simulation and experimental analysis

6.1 Dynamic response analysis
The simulation parameters are the same as the theoretical analysis. Fig. 8 shows the process of the system inductance current changing with the load. The initial load is 10Ω, and the load changes are 20Ω, 30Ω, and 40Ω at 0.03s, 0.06s, and 0.09s respectively. It can be seen that the process of inductive current in time domain from one period to period-doubling bifurcation and then to chaos.

![Fig. 8 Dynamic parameter simulation diagram.](image)

6.2 Semi-physical RT-LAB simulation
The RT-LAB was used to simulate the circuit, as shown in Fig. 9. And the inductance current waveform under three conditions as shown in Fig. 9 (a) (single periodic state), Fig. 9 (b) (period 2 state) and Fig. 9 (c) (chaotic state) were obtained respectively.

![Fig. 9 Experimental waveform of inductor current $I_L$.](image)

7. Conclusion
In this paper, the nonlinear behavior of synchronous switching Z-source converter under peak current mode control and the parameter domain of stable operation are studied. An accurate stroboscopic iterative model is established based on the state equation of the system, and the model is mapped to its complex frequency domain to solve the steady-state operating point of the system. Different from solving the differential equation directly, the analysis process greatly simplifies the equation and reduces the amount of calculation while ensuring the accuracy of the model remains unchanged. The bifurcation diagram and Jacobian matrix eigenvalues are used to analyze the condition of the bifurcation point under the fixed parameters of the converter, which is further extended to the determination of the system stability region when the converter parameters change. Finally, the nonlinear dynamic phenomenon and evolution process of the synchronous switching Z-source converter are observed through simulation and experiment. The results show that the conditions for the occurrence of bifurcation points in the synchronous switching Z-source converter are consistent with the theoretical analysis results. The study not only provides a reference for the stable operation and system parameter optimization of the synchronous switching Z-source converter, but also broadens the application of nonlinear dynamics theory in the field of power conversion.

Acknowledgments

This work is supported by the Science and Technology Research Program of Chongqing Municipal Education Commission under Grant KJQN202001150 and Science & Technology Special Funds of Banan District of Chongqing under Grant 2020TJZ023.

References


