Efficient generation of holographic movies with frame interpolation using a coherent neural network

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Abstract: Computer-generated hologram (CGH) is recently expanding its application fields. However, the calculation cost is very high, in particular, in the generation of CGH streams for three-dimensional movies. This paper proposes a small-calculation-cost method to generate CGH streams based on a coherent neural network (CNN) that deals with complex-amplitude information with generalization ability in the carrier-frequency domain. After carrier-frequency-dependent learning, we can generate a CGH stream, by sweeping a virtual carrier frequency in the CNN, with neural interpolation thanks to the frequency-domain generalization. Experiments demonstrate a successful stream generation with 1/6 the conventional calculation time.

Keywords: coherent neural network (CNN), computer generated hologram (CGH), optical tweezers, generalization, spatial light modulator (SLM)

Classification: Photonics devices, circuits, and systems

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1 Introduction

Hologram records optical interference fringe, and reconstructs the optical wavefront. Computer-generated hologram (CGH) obtains the fringe by numerical calculation in a computer without any physical object. CGH is widening the application fields rapidly in particular in recent three-dimensional movies, including optical tweezers to manipulate small particles and biological cells in liquid [1, 2]. Phase CGH is most suitable for this purpose because of the extremely low optical-power loss [3].

The generation of even a single CGH requires a large calculation cost. To generate a movie, which is a stream of still CGH images, spends huge amount of time. However, in most cases of realistically meaningful movies, such as the optical-tweezers case, the CGH stream should generate continuously deforming lightbeams. Therefore, if we can interpolate a set of CGH images generated sparsely in time, the calculation cost will greatly be reduced.

In this paper, we propose a small-calculation-cost method to generate a CGH stream using a coherent neural network (CNN) that deals with phase and amplitude information with neural generalization ability in the carrier-frequency domain. A few related works have been reported so far, such as CGH optimization using a neural network where a real-valued neural network deals with phase values adaptively to reduce CGH noise [4]. Here, in this paper, we construct a CNN in which the delay and transparency of neural connections are changed in the phase-value learning to construct a desired CGH. Thereby, the learning dynamics correspond directly to the interference-recording process in a physical hologram. This fact brings advantages such as straightforward learning characteristics and automatic fading of unnecessary or harmful neural connections, if any.

2 CGH interpolation using CNN

Neural networks possess generalization ability. CNNs, having a carrier frequency f , can change the learning and processing behavior depending on f . Then the frequency-domain generalization will bring smoothly-changing phase signals [5] required for CGH stream. Inspired by this frequency-domain generalization ability, we propose a method to interpolate a set of sparsely generated holograms to generate a smooth three-dimensional movie at a low

Fig. 1. (a) Schematic diagram of CGH and (b) coherent neuron with multiple input connections.

calculation cost.

Figure 1 (a) is the schematic illustration of CGH wavefront forming. We use a two-dimensional phase modulator, i.e., a parallel-aligned spatial light modulator (PAL-SLM), as a dynamic hologram that is capable of generating interference fringes time-sequentially. Every pixel at (j, k) on the SLM surface is fed with an output signal of a coherent neuron described below.

Figure 1 (b) shows an amplitude-phase-type complex-valued neuron that forms a simple CNN. To enhance the flexibility in frequency-dependent behavior, it has multiple connections w_h with different initial time delays to a common input terminal. The relationship between the input $x(j, k)$ and output $y(j, k)$ is given as [6]

$$y(j, k) = \tanh(s(j, k)) \exp(i\beta(j, k)) \quad (1)$$

where $i \equiv \sqrt{-1}$, and $s(j, k)$ and $\beta(j, k)$ are amplitude and phase of the summation of weighted inputs, i.e.,

$$s(j, k) \exp(j\beta(j, k)) \equiv \sum_h w_h(j, k) x(j, k) \quad (2)$$

The connection weight w_h of the CNN is expressed by transparency (amplitude) $|w_h(j, k)|$, delay time $\tau_h(j, k)$, and (virtual) carrier frequency f as [5, 6]

$$w_h(j, k, f) = |w_h(j, k)| \exp(i2\pi f \tau_h(j, k)) \quad (3)$$

The CGH-stream generation is conducted as follows.

Procedure to generate a CGH stream:

1. Plan a stream of images to be obtained at position (m, n, z) in Fig. 1 (a) at sequential time t .
2. Choose a limited number of time points t_p in the stream. Calculate the desirable CGH phase images $\hat{\theta}(j, k, t_p)$ at time t_p , where (j, k) is the position on the SLM, with a conventional CGH generation method.
3. Unwrap the phase image $\hat{\theta}(j, k, t_p)$ [7]. That is, we develop (unwrap) the phase value, which is wrapped within $[-\pi, \pi)$ in radians, into values in $(-\infty, +\infty)$. Since the CGH phase image is generally simple, we can easily conduct the unwrapping by adding or subtracting 2π , when we find a “ 2π jump” in the phase image, to connect the phase pixel values smoothly in space. We denote the unwrapped phase image as $\hat{\phi}(j, k, t_p)$.
4. Let the CNN learn the respective teacher image $\hat{y}(j, k, t_p)$ determined by the unwrapped CGH image $\hat{\phi}(j, k, t_p)$ as

$$\hat{y}(j, k, t_p) = \exp(i\hat{\phi}(j, k, t_p)) \quad (4)$$

That is, we determine $|\hat{y}(j, k, t_p)| = 1$ for the present phase-hologram case. The neural input signals are chosen, only for simplicity, as

$$\hat{x}(j, k, t_p) = 1 \quad (\text{for all } (j, k)) \quad (5)$$

Discrete time t_p is chosen appropriately (e.g., evenly) in the stream-time domain. The learning is based on the carrier-frequency-dependent complex-valued Hebbian rule [8], which is expressed in terms of the virtual carrier frequency $f = f_p$ at the sampled time points t_p for the teacher CGHs, with a learning time constant T , as [9]

$$T \frac{d |w_h(j, k)|}{d t_{\text{learn}}} = -|w_h(j, k)| + |\hat{y}(j, k, t_p)| |\hat{x}(j, k, t_p)| \times \cos(\hat{\phi}(j, k, t_p) - 2\pi f_p \tau_h(j, k)) \quad (6)$$

$$T \frac{d \tau_h(j, k)}{d t_{\text{learn}}} = \frac{1}{2\pi f_p} \frac{|\hat{y}(j, k, t_p)| |\hat{x}(j, k, t_p)|}{|w_h(j, k)|} \times \sin(\hat{\phi}(j, k, t_p) - 2\pi f_p \tau_h(j, k)) \quad (7)$$

Note that, only in (6) and (7), time t_{learn} is not the time in the CGH stream but the time in the learning process. We present teacher signals $\hat{y}(j, k, t_p)$ one after another for sequential t_p repeatedly. The learning process changes the transparency (amplitude) $|w_h(j, k)|$ and the delay time $\tau_h(j, k)$ gradually. When the error has decreased to a sufficiently small value, we stop the learning process.

5. To generate a CGH stream, feed the phase values $\phi(j, k, t)$ of the neural output signals $y(j, k, t)$ to the SLM, i.e.,

$$\phi(j, k, t) = \arg(y(j, k, t)) \quad (8)$$

where $y(j, k, t)$ is calculated by (1)–(3). With the neural inputs being unity, (i.e., $x = 1$ constantly for all (j, k)), we sweep the frequency

$f(t)$ continuously to generate a phase-CGH stream. Then we obtain interpolated frames even at time points without learning, $t \neq t_p$, based on the frequency-domain generalization ability in the CNN.

Because of the neural interpolation, we can reduce the cost to calculate CGH frames in a conventional manner and, consequently, decrease the total calculation cost.

3 Experiments and results

First, we conducted simulation experiment assuming the realization of optical tweezers. We assume a so-called Fourier hologram using an additional convex lens. In this experiment, we restricted the movement of the image point (focal point) within a plane. However, the proposed method is effective also for three-dimensional movement without any modification. Because of the two-dimensionality, the conventional CGH calculation is simplified. Even in such a case, it is found below that our proposal reduces the calculation cost very effectively. Parameters in the experiments are shown in Table I. We determined the initial delays and other parameters referring to Ref. [9].

Figure 2 shows a simulation result when a single bright point is intended to move on an arc. The upper half of each image shows the CGH phase values on the SLM in gray scale, while the lower half shows the reconstructed image on the screen (image plane) at some time points. The numbers below the images show the time evolution of the CGH stream ($t = 0, 1, 2, \dots$). Underlines indicate the learning time points t_p . It is found that the bright point moves on the arc, as we expected. The moving focal points trace the learning points very smoothly. The CGHs are also naturally changing. The moving points at interpolation are occasionally ill-focused in comparison with those at the learning points, which presents room for improvement in learning.

Next, Fig. 3 shows movies obtained in optical experiments. Figure 3(a) shows the stream only of conventionally calculated CGH, while (b) shows the interpolated stream when f is swept ($t = 0, 1, 2, \dots$). In each images, the point moving in lower-right part is the target bright point. Another one existing at almost point-symmetric position is a negative first-order diffraction, which is often observed in holography. We find that a smoothly varying movie has been obtained with the neural interpolation successfully.

The calculation time to synthesize an interpolated CGH image is about

Table I. Parameters in experiments.

Number of pixels on SLM	128 × 128	
Number of pixels on screen	128 × 128	
Virtual optical carrier frequency	400.000~400.012	THz
Virtual teacher frequency interval	2	GHz
Number of parallel input connections (max h)	100	
Initial $\tau(j, k)$ distribution range	−0.25~0.25	ns
Learning time constant T	100

Fig. 2. Simulation result of interpolation when a single point is intended to move on an arc. In each image: (Upper) CGH phase images in gray scale, and (Lower) brightness on screen. Underlined t : learning points t_p .

Fig. 3. (a) Non-interpolated and (b) interpolated movies.

1.3 s in the present experiment. On the other hand, if we use conventional hologram-generation method, it takes about 7.6 s. We find that the calculation time has been reduced to $1/6$. This reduction implies that the proposal, together with extrapolation based on identical idea, is helpful to realize an al-

most real-time and interactive generation of CGH streams, which is desirable for optical tweezers and other various applications.

4 Conclusion

We have proposed a small-calculation-cost method to generate CGH movies based on the CNN. Experiments demonstrated that a smoothly varying image stream is obtained with the neural interpolation successfully.

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