Analytical GMD formulas for mutual inductance calculation of multilevel interconnects

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Abstract: A set of new analytical formulas is proposed for determining the *geometric mean distance* (GMD) for use in the Greenhouse method to calculate the inductances of three-dimensional inductors composed of on-chip interconnects. The expressions are derived using a graphical method based on the integral form of the GMD equation. The mutual inductances of coupled interconnects are computed for given layout geometries using the proposed formulas, even when the conductors are partially overlapped in different metal layers. The inductances computed using the formulas agreed well with those generated by an accurate commercial field solver, differing by less than 1% across a variety of the conductor geometries.

Keywords: Greenhouse method, geometric mean distance, on-chip interconnect, three-dimensional inductor, mutual inductance, closed-form expression

Classification: Integrated circuits

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1 Introduction

Continued improvements in semiconductor process technology have enabled the design of advanced on-chip inductors [1]–[3] using multilevel interconnects. Greenhouse [4] reported a technique based on analytical expressions to calculate the inductance of planar rectangular spiral inductors. Accurate results can be obtained with this algorithm, but the expression for *geometric mean distance* (GMD) can only be applied to aligned conductors in the same layer that have the same width and negligible thickness. The pre-computed tables reported in Refs. [5] and [6] can be employed to calculate the GMD of two rectangular cross sections with finite thickness. The interconnection layout, however, is restricted to aligned cases. Expressions for aligned conductors of given widths and thicknesses were reported in Refs. [7] and [8], but expressions for interconnects placed in different metal layers were not addressed. Although three-dimensional (3D) field solvers [9]–[11] can accurately calculate inductances of multilevel conductors, they are time-consuming compared to the analytical technique due to the mesh generation algorithm [7, 12].

This paper presents a set of simple and accurate formulas for calculating the GMD of multilevel on-chip interconnects to be used in the Greenhouse method. The technique to derive the proposed expressions is based on the integral form of the GMD equation. The formulas include the geometric parameters of the metal lines, allowing for the independent scaling of conductor widths and thicknesses. The practical application of the formulas in the Greenhouse method is demonstrated with the results from commercial field solver Ansoft Q3D Extractor [9].

2 Geometric Mean Distance Expressions

The proposed set of GMD formulas for interconnects in different metal layers (see Figs. 1 (a)–(e)) was derived as a function of the GMD formula for aligned

Fig. 1. Cross sectional views of multilevel conductors used in VLSI circuits. The shaded rectangles depict the cross sections of the interconnects while the others denote the segments used to derive the GMD of the coupled interconnects. The geometric parameters of the conductors, the width w , the vertical and horizontal spacings s_v and s_h , and the thickness t , are shown in the figures. The subscripts on the widths and thicknesses indicate the corresponding rectangles.

conductors [7, 8]. The integral form of the GMD equation used to derive the formulas is given by

$$\log R_{ij} = \frac{1}{S_i S_j} \iint \log r \, dS_i dS_j \quad (i \neq j) \quad (1)$$

where R_{ij} is the GMD of rectangular cross sections i and j , r is the distance between every pair of points in the sections i and j , and S_i and S_j are the areas of the respective sections.

The concept for the derivation is explained using the example in Fig. 1 (a). Aligned stacked rectangles α and β are decomposed into aligned stacked segments including the two conductors α_1 and β_2 of rectangular cross section. It should be noted that the GMD for case (a) is derived from Eq. (1) when the unknown term, the double integral of $\log r$ over the pair of rectangular cross sections (i, j) , is obtained for the pair of rectangles $(i, j) = (\alpha_1, \beta_2)$. This decomposition yields the relationship between the double integral of $\log r$ over the pair of rectangles $(i, j) = (\alpha_1, \beta_2)$ and those of aligned rect-

Fig. 2. Graphical expressions for deriving proposed GMD formulas in cases (a)–(e) in Fig. 1. The terms in the expressions represent the double integrals in Eq. (1) over the pairs of segments denoted by the solid lines connecting the segments between aligned rectangles α and β . The details are described in the text.

angles $(i, j) = (\alpha, \beta)$, (α_1, β_1) and (α_2, β_2) as the *graphical expression* shown in Fig. 2(a). The terms in the *graphical expression* represent double integrals over the pairs of rectangles indicated by solid lines connecting the segments. For instance, the term in the left hand side of the *graphical expression* for case (a) denotes twice the double integral of $\log r$ over the segments $(i, j) = (\alpha_1, \beta_2)$. It should be noted that the integral value over the pair of cross coupled segments $(i, j) = (\alpha_2, \beta_1)$ is equal to that of the cross coupled segments $(i, j) = (\alpha_1, \beta_2)$ even if the segments have different thicknesses, as in Fig. 1 (a). The integration regions indicated by the each term in the right hand side of the expression are composed of coupled aligned rectangles to obtain the GMDs of aligned rectangles. This graphical description facilitates the derivation of the GMD formula because the relationship between double integrals over cross coupled and aligned rectangles can be found easily by performing arithmetic operations directly on the corresponding solid lines between the terms shown in the figure. The *graphical expression* for case (a) can be reduced to the following mathematical scheme:

$$2F_{\alpha_1\beta_2} = F_{\alpha\beta} - F_{\alpha_1\beta_1} - F_{\alpha_2\beta_2} \quad (2)$$

where

$$F_{ij} \equiv \iint \log r \, dS_i dS_j \quad (i \neq j) \quad (3)$$

where i and j denote the regions of the rectangles to be integrated. Using Eqs. (1), (2) and (3), the resulting formula for case (a) is given by

$$\log R_{\alpha_1\beta_2}^a = \frac{1}{2S_{\alpha_1}S_{\beta_2}} (S_{\alpha}S_{\beta} \log R_{\alpha\beta} - S_{\alpha_1}S_{\beta_1} \log R_{\alpha_1\beta_1} - S_{\alpha_2}S_{\beta_2} \log R_{\alpha_2\beta_2}) \quad (4)$$

where $R_{\alpha_1\beta_2}^a$ is the GMD for case (a). $R_{\alpha\beta}$, $R_{\alpha_1\beta_1}$ and $R_{\alpha_2\beta_2}$ are the GMDs for aligned stacked rectangles $(i, j) = (\alpha, \beta)$, (α_1, β_1) and (α_2, β_2) , respectively.

The GMD formulas for cases (b)–(e) are derived using the graphical method as described in case (a). Figures 2(b)–(e) show the graphical expressions derived by performing arithmetic operations on the solid lines. The resulting formulas are given by

$$\log R_{\alpha\beta_2}^b = \frac{1}{2S_{\alpha}S_{\beta_2}} (S_{\alpha}S_{\beta} \log R_{\alpha\beta} - S_{\alpha_1}S_{\beta_1} \log R_{\alpha_1\beta_1} + S_{\alpha_2}S_{\beta_2} \log R_{\alpha_2\beta_2}) \quad (5)$$

$$\log R_{\alpha\beta_2}^c = \frac{1}{2S_{\alpha}S_{\beta_2}} (S_{\alpha_{12}}S_{\beta_{12}} \log R_{\alpha_{12}\beta_{12}} - S_{\alpha_1}S_{\beta_1} \log R_{\alpha_1\beta_1} + S_{\alpha_{23}}S_{\beta_{23}} \log R_{\alpha_{23}\beta_{23}} - S_{\alpha_3}S_{\beta_3} \log R_{\alpha_3\beta_3}) \quad (6)$$

$$\log R_{\alpha_{12}\beta_{23}}^d = \frac{1}{2S_{\alpha_{12}}S_{\beta_{23}}} (S_{\alpha}S_{\beta} \log R_{\alpha\beta} - S_{\alpha_1}S_{\beta_1} \log R_{\alpha_1\beta_1} + S_{\alpha_2}S_{\beta_2} \log R_{\alpha_2\beta_2} - S_{\alpha_3}S_{\beta_3} \log R_{\alpha_3\beta_3}) \quad (7)$$

$$\log R_{\alpha_1\beta_3}^e = \frac{1}{S_{\alpha_1}S_{\beta_3}} (S_{\alpha}S_{\beta} \log R_{\alpha\beta} - S_{\alpha_{12}}S_{\beta_{12}} \log R_{\alpha_{12}\beta_{12}} - S_{\alpha_{23}}S_{\beta_{23}} \log R_{\alpha_{23}\beta_{23}} + S_{\alpha_2}S_{\beta_2} \log R_{\alpha_2\beta_2}) \quad (8)$$

where $R_{\alpha\beta_2}^b$, $R_{\alpha\beta_2}^c$, $R_{\alpha_{12}\beta_{23}}^d$ and $R_{\alpha_1\beta_3}^e$ are the GMDs for the respective cases (b)–(e). α_{12} , α_{23} , β_{12} and β_{23} represent rectangles $\alpha_1 \cup \alpha_2$, $\alpha_2 \cup \alpha_3$, $\beta_1 \cup \beta_2$, and $\beta_2 \cup \beta_3$, respectively. Note that Eqs. (4)–(8) are determined from the layout geometries of the coupled conductors in Fig. 1 because the GMD for aligned conductors can be computed from a known analytical expressions that include the geometric parameters [7, 8].

3 Verification

Figure 3 shows the mutual inductances for cases (a)–(e) calculated using the analytical expression in the Greenhouse method [4] in conjunction with the proposed formulas. The mutual inductances were simulated using Ansoft Q3D Extractor [9]. Note that the GMD for aligned conductors used in the formulas were determined from the expression in Ref. [7] that uses the *GMD of a rectangle from itself* with geometric parameters [5, 6]. The vertical spacing s_v on the x -axis and the widths w_{β_2} , $w_{\beta_{23}}$ and w_{β_3} of each configuration (see Fig. 1) are varied. The results from the proposed expressions agree well with those of the field solver, differing by less than 1% even for the

Fig. 3. Mutual inductances calculated using the proposed formulas and a field solver for cases (a)–(e). A length of $100\mu\text{m}$ is used in all cases, together with the following in the individual cases: (a) $w_{\alpha_1} = 10\mu\text{m}$, $w_{\beta_2} = w$, $t_\alpha = 1\mu\text{m}$ and $t_\beta = 2\mu\text{m}$ (b) $w_\alpha = 20\mu\text{m}$, $w_{\beta_2} = w$, $t_\alpha = 1\mu\text{m}$ and $t_\beta = 2\mu\text{m}$ (c) $w_\alpha = 20\mu\text{m}$, $w_{\beta_2} = w$, $s_h = 2\mu\text{m}$, $t_\alpha = 1\mu\text{m}$ and $t_\beta = 2\mu\text{m}$ (d) $w_{\alpha_{12}} = 10\mu\text{m}$, $w_{\beta_{23}} = w$, $s_h = 6\mu\text{m}$, $t_\alpha = 1\mu\text{m}$ and $t_\beta = 2\mu\text{m}$ (e) $w_{\alpha_1} = 10\mu\text{m}$, $w_{\beta_3} = w$, $s_h = 5\mu\text{m}$, $t_\alpha = 1\mu\text{m}$ and $t_\beta = 2\mu\text{m}$.

narrow spacing case. The inductance decreases with increasing conductor spacing because the magnetic coupling between the conductors is inversely proportional to the spacing of the metal lines. These plots indicate that the Greenhouse method can be extended to multilevel interconnects to a high degree of accuracy by applying the proposed formulas.

4 Conclusion

Accurate analytical formulas for obtaining the GMD of multilevel conductors for the Greenhouse method have been presented. The integral form of the GMD equation was used to derive the GMD expressions for multilevel metal lines. The mutual inductances of multilevel conductors calculated using the proposed formulas matched those derived with a commercial field

solver within 1%. The proposed expressions are fully scalable and computationally efficient for calculating mutual inductances of interconnects for 3D inductors in VLSI circuits.