Equivalent circuits and transmission zeros of the coupled square-loop resonator

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Abstract: The purpose of this paper is to present various equivalent circuits of the coupled square-loop resonator which can be used for realization of narrow bandstop filters like notch filters. Furthermore, it is made clear that those circuits vary by the relation of characteristic impedances of component lines in the resonator.

Keywords: microwave notch filters, network synthesis, square-loop resonator, equivalent circuits, transmission zeros

Classification: Microwave and millimeter wave devices, circuits, and systems

References


1 Introduction

As is well known, the use of microwave bandstop filters can be seen for many applications such as communications transceivers and radar systems in order to suppress undesired RF signals. Recently, in order to realize narrow bandstop filters like notch filters the coupled square-loop resonator was proposed in [1]. Thereafter, the design approach of such a resonator was discussed in [2]. Its approach is based on the assumption that the resonator is a lossy
resonator circuit consisting of a series RLC circuit. This, of course, means its stopband attenuation is finite. However, even if the component lines of resonator are lossless, there is a case in which the stopband attenuation is finite. Hence, elucidating characteristics of such a lossless case is very important for making use of the square-loop resonator and is an issue arousing interest. Furthermore, for the filter design using such a resonator it is wished to find its equivalent circuits because it is extremely convenient to use its equivalent circuits.

In this paper, we present equivalent circuits that the coupled square-loop resonator has, and also make clear that those circuits vary variously according to characteristic impedances of component lines in this resonator. Those equivalent circuits are obtained by using synthesis techniques in circuit theory.

2 Transmission Zeros

Figure 1 shows the configuration of the coupled square-loop resonator that is considered in this paper. This consists of a square-loop coupled to a transmission-line having input and output ports. It is assumed that for the considered resonator all of its component lines have commensurate lengths and are lossless TEM lines. Furthermore, the lengths are one quarter wavelength. In the figure, \( z_1 \), \( z_2 \) and \( z_3 \) are characteristic impedances of u.e. (unit element)’s, and \( z_e \) and \( z_o \), respectively, are characteristic impedances with even and odd excitations of the parallel coupled-line.

In the following, we investigate transmission zeros that the considered resonator has. As a result, we see that those zeros depend on the relations between a group of characteristic impedances of the non-coupled part and that of the parallel coupled part.

2.1 Network Analysis

Consider obtaining an open-circuit impedance matrix \( Z = [z_{ij}](i, j = 1, 2) \) of a 2-port network \( N \) in Fig. 2 (a) in order to obtain an open-circuit impedance matrix \( Z(p) \) of the resonator of Fig. 1. The term “open-circuit” will hereinafter be omitted in order to simplify description. This \( N \) is assembled with a 4-port network \( N_A \) and a 2-port network \( N_B \). Two ports of \( N_A \), port 1 and port 4, become the input and output ports of \( N \), respectively.

![Fig. 1. Coupled square-loop resonator.](image)
Let $Z_A = [z_{Aij}] (i,j = 1,\ldots, 4)$ be an impedance matrix of $N_A$ and similarly, let $Z_B = [z_{Bij}] (i,j = 1, 2)$ be that of $N_B$. Then $Z$ can be written as

$$Z = Z_{A11} - Z_{A12} (Z_{A22} + Z_B)^{-1} Z_{A21}$$

where

$$Z_{A11} = \begin{bmatrix} z_{A11} & z_{A14} \\ z_{A41} & z_{A44} \end{bmatrix}, \quad Z_{A12} = \begin{bmatrix} z_{A13} & z_{A12} \\ z_{A43} & z_{A42} \end{bmatrix}, \quad Z_{A21} = \begin{bmatrix} z_{A31} & z_{A34} \\ z_{A21} & z_{A24} \end{bmatrix},$$

$$Z_{A22} = \begin{bmatrix} z_{A33} & z_{A32} \\ z_{A23} & z_{A22} \end{bmatrix}.$$ 

Let $p$ be $p = j \tan (\pi w/2)$ ($j = \sqrt{-1}$), where $w$ is the real variable. When $N_A$ and $N_B$, respectively, are the parallel coupled-line and a cascade of three u.e.'s, $Z_A$ and $Z_B$ are given as follows:

$$Z_A = \frac{1}{p} \begin{bmatrix} a & b & tb & ta \\ b & a & ta & tb \\ tb & a & b & ta \\ ta & b & a & tb \end{bmatrix}, \quad Z_B = \frac{1}{z_2^2p^3+k_1p} \begin{bmatrix} k_2z_1p^2+k_3 & k_3t^3 \\ k_3t^3 & k_4z_3p^2+k_3 \end{bmatrix}.$$ 

Here,

$$a = (z_e + z_o)/2, \quad b = (z_e - z_o)/2, \quad t = \sqrt{1-p^2}, \quad k_1 = z_1z_2 + z_2z_3 + z_3z_4,$$

$$k_2 = z_1z_2 + \frac{z_2}{z_4} + z_1z_3, \quad k_3 = z_1z_2z_3, \quad k_4 = z_1z_3 + z_2^2 + z_2z_3,$$ 

$k_5 = z_1 + z_2 + z_3$.

Thus, from (1) and (2), $Z(p)$ can be written as

$$Z(p) = \frac{1}{m(p)} \begin{bmatrix} n_{11}(p) & n_{12}(p) \\ n_{12}(p) & n_{22}(p) \end{bmatrix}$$

where

$$m(p) = (az_2 - z_1z_3)^2 p^3 + (a + k_5) (k_3 + ak_1) p,$$

$$n_{11}(p) = \left\{ \left( b^2 z_3 k_4 - 2k_3z_3z_5 + a \left( \frac{z_2}{z_4} \frac{z_2}{z_4} + \frac{z_2}{z_4} \right) \right) p^2 + (a + k_5) (ak_3 + k_1z_eez_o) \right\},$$

$$n_{12}(p) = t(az_2 - z_1z_3) \left( a\bar{z}_1z_3 - z_2z_eez_o \right) p^2 + (a + k_5) (ak_3 + k_1z_eez_o),$$

$$n_{22}(p) = \left\{ b^2 z_1k_2 - 2k_3z_eez_o + a \left( \frac{z_2}{z_4} \frac{z_2}{z_4} + \frac{z_2}{z_4} \right) \right\} p^2 + (a + k_5) (ak_3 + k_1z_eez_o).$$

### 2.2 Transmission Zeros

Zeros of the (1,2)-element of $Z(p)$ are transmission zeros of the resonator under consideration. Hence, those are given as

$$p = \pm 1$$ \text{ and } $$p = \pm j \sqrt{\frac{(a + k_5) (ak_3 + k_1z_eez_o)}{(az_2 - z_1z_3) (a\bar{z}_1z_3 - z_2z_eez_o)}}.$$ 

Especially, we note that the latter in (4) is

- **Case 1**: $\frac{2}{y_o + y_e} < \frac{z_1z_3}{z_2} < \frac{z_e + z_o}{2}$,
- **Case 2**: $\frac{z_1z_3}{z_2} < \frac{2}{y_o + y_e}$ or $\frac{z_e + z_o}{2} < \frac{z_1z_3}{z_2}$,
- **Case 3**: $\frac{z_1z_3}{z_2} = \frac{y_o + y_e}{2}$ or $\frac{z_e + z_o}{2} = \frac{z_1z_3}{z_2}$,

where $y_o = z_2^{-1}$, $y_e = z_e^{-1}$. In a word, the transmission zeros vary by the magnitude correlation of two impedance ratios, $z_1z_3/z_2$ and $(z_e + z_o)/2$ (or
2/(y_o + y_e) ). With respect to the component lines of resonator, the former depends on the term of the non-coupled part and the latter depends on the term of the parallel coupled part.

3 Equivalent Circuits

To obtain one possible equivalent circuit of Fig. 1, consider an impedance matrix $Z^{(12)} = [z_{ij}^{(12)}] (i, j = 1, 2)$ of a 2-port network $N^{(12)}$ (Fig. 2 (b)) in which two 2-port networks, $N^{(1)}$ and $N^{(2)}$, are connected in cascade. For this purpose, let an impedance matrix of $N^{(k)} (k = 1, 2)$ be defined as $Z^{(k)} = [z_{ij}^{(k)}] (i, j = 1, 2)$. Then the elements of $Z^{(2)}$ can be written as follows:

$$
z_{11}^{(2)} = \frac{z_{12}^{(12)} z_{11}^{(1)}}{z_{11}^{(12)} - z_{11}^{(1)}}, \quad z_{12}^{(2)} = \frac{z_{12}^{(12)} z_{12}^{(1)}}{z_{11}^{(12)} - z_{11}^{(1)}}, \quad z_{22}^{(2)} = z_{22}^{(12)} - \frac{z_{12}^{(12)} z_{11}^{(1)}}{z_{11}^{(12)} - z_{11}^{(1)}}. \quad (5)$$

The above can be used to obtain the unknown $Z^{(2)}$ from already-known $Z^{(12)}$ and $Z^{(1)}$. For the resonator under consideration, we now are at a position of realizing $Z(p)$. In (3), put $p = 1$, which is one of the already mentioned transmission zeros. The result is

$$
Z^{(1)} = \begin{bmatrix}
\frac{az_3 + z_e z_o}{a + z_3} & 0 \\
0 & \frac{az_1 + z_e z_o}{a + z_1}
\end{bmatrix}.
$$

From the above, we define the following matrix $Z^{(1)}(p)$.

$$
Z^{(1)}(p) = \frac{Z_0}{p} \begin{bmatrix} 1 & t \\ t & 1 \end{bmatrix}, \quad \text{where} \quad Z_0 = \frac{az_3 + z_e z_o}{a + z_3}. \quad (6)
$$

$Z^{(1)}(p)$ is the impedance matrix of the u.e. having the characteristic impedance $Z_0$. Substituting (6) and (3) into (5) yields

$$
Z^{(2)}(p) = p \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} + \frac{1}{p C} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}
$$

where

$$
L_1 = \frac{(a z_2 - z_1 z_3)^2 (a z_3 + z_e z_o)^2}{b^2 z_3^2 (a + k_5) (a + z_3) (z_1 + z_2)}, \quad L_2 = \frac{(a + z_3) (a z_1 z_3 - z_2 z_e z_o)^2}{b^2 z_3^2 (a + k_5) (z_1 + z_2)}
$$

$$
M = \pm \sqrt{L_1 L_2}, \quad C = \frac{(a z_3 + z_e z_o) (a k_3 + k_1 z_e z_o)}{b^2 z_3^2 (z_1 + z_2)}.
$$

(7)
$L_1, L_2$ and $C$ are always positive, whereas $M$ can become positive, zero or negative. Since $az_3 + ze_0$ is positive, the sign of $M$ depends on the relation between two impedance ratios already described as seen from (7). For the considered resonator, this means there exist distinct equivalent circuits corresponding to positive, negative or zero’s $M$. We will show those circuits in the following.

**Case 1:** \[ \frac{2}{y_0 + ye} < \frac{z_1 z_3}{z_2} < \frac{z_e + ze}{2} \]

$M$ when the impedance ratios are this case becomes negative and, as seen from (4), the corresponding transmission zeros lie on the real axis of the $p$-complex plane. Figure 3 (a) indicates an equivalent circuit of this case. This circuit consists of a cascade of a u.e. and a microwave type C section in the $p$-variable. As was mentioned before, this case yields a finite stopband attenuation at the transmission zeros.

**Case 2:** \[ \frac{z_1 z_3}{z_2} < \frac{2}{y_0 + ye} \quad \text{or} \quad \frac{z_e + ze}{2} < \frac{z_1 z_3}{z_2} \]

$M$ when the impedance ratios are this case is positive and, as seen from (4), the corresponding transmission zeros lie on the imaginary axis of the $p$-complex plane. The circuit shown in Fig. 3 (b), generally, is an equivalent circuit of this case and consists of a cascade of a u.e. and a microwave Brune section in the $p$-variable. However, when values of $z_1 z_3/z_2$ are zero or infinity, as stated below, such values yield special patterns of the resonator of Fig. 1 and also yield degenerate-equivalent circuits of Fig. 3 (b). Those patterns are

![Equivalent circuits and special patterns of the considered resonator.](image-url)
listed in Fig. 3 (g).

(2A) \( z_1 z_3 / z_2 = 0 \)

For this case there are five possibilities: (A1) \( z_1 = 0 \), (A2) \( z_1 = 0 \), \( z_2 \to \infty \), (A3) \( z_2 \to \infty \), (A4) \( z_3 = 0 \), (A5) \( z_3 = 0 \), \( z_2 \to \infty \). On these notations, for example, (A1) implies both impedances \( z_2 \) and \( z_3 \) are \( 0 < z_2, z_3 < \infty \). As was mentioned already, Fig. 3 (g)(2A) lists each pattern in (A1), \( \sim \), (A5). The corresponding equivalent circuits can be divided into two groups: one is a group of (A1), (A2) and (A3), the remainder is a group of (A4) and (A5). The former is the same as the equivalent circuit shown in Fig. 3 (b), whereas the latter becomes that shown in Fig. 3 (c). Element values of the former are obtained by applying the corresponding conditions into (6) and (7). On the other hand, those of the latter can be obtained by applying a already mentioned similar method to the admittance matrix \( Z^{-1}(p) \) instead of \( Z(p) \).

As a result, element values on (A4) become as follows:

\[
Z_0 = \frac{2 y_o z_o}{a}, \quad L = \frac{b^2 (z_1 + z_2)}{a (a + z_1 + z_2)}, \quad C = \frac{a^2 z_2}{b^2 z_1 (z_1 + z_2)}.
\]

Those on (A5) are obtained from the above equations when \( z_2 \to \infty \).

We, of course, must note there exist other combinations with respect to \( z_1 \), \( z_2 \) and \( z_3 \). However, those combinations will be omitted here because those are a trivial case in which each of the corresponding equivalent circuits consists of only a u.e..

(2B) \( z_1 z_3 / z_2 \to \infty \)

Except for trivial cases in which equivalent circuits become only a u.e., there are the following five possibilities as well as (2A): (B1) \( z_1 \to \infty \), (B2) \( z_1 \to \infty \), \( z_2 \to 0 \), (B3) \( z_3 \to \infty \), (B4) \( z_2 \to 0 \), (B5) \( z_2 \to 0 \), \( z_3 \to \infty \). Figure 3 (g)(2B) lists configurations corresponding to (B1), \( \sim \), (B5). The corresponding equivalent circuits can be divided into two groups. (B1), (B2) and (B4) are one of the groups, and (B3) and (B5) are the remainder. Equivalent circuits of the former are the same as that of Fig.3 (b), however as shown in Fig. 3 (d) those of the latter are distinct. We can get both the element values of the former and the latter by applying the corresponding conditions into (6) and (7).

Case 3: \( \frac{z_1 z_3}{z_2} = \frac{2}{y_o + y_e} \) or \( \frac{z_e + z_o}{2} = \frac{z_1 z_3}{z_2} \)

\( M \) when the impedance ratios are this case is zero, and transmission zeros exist at the infinite point of the \( p \)-complex plane.

(3A) \( z_1 z_3 / z_2 = 2/(y_o + y_e) \)

Equation (7) for this case yields \( L_2 = M = 0 \). Figure 3 (e) shows an equivalent circuit of this case, and can be obtained by doing both \( L_2 \) and \( M \) in Fig. 3 (b) to zeros. Other element values can be obtained from (6) and (7).

(3B) \( z_1 z_3 / z_2 = (z_e + z_o)/2 \)

In similar fashion, this case yields an equivalent circuit being \( L_1 = M = 0 \), as shown in Fig. 3 (f).

In Fig. 4 is shown the relation of two impedance ratios, and the range of noteworthy realizable equivalent circuits.
Fig. 4. The range of realizable equivalent circuits.

4 Conclusions

Equivalent circuits and transmission zeros of the coupled square-loop resonator have been presented. It has been made clear those circuits and zeros vary variously by the relation between a group of characteristic impedances of the non-coupled part and that of the parallel coupled part.