Design of a new DFT filter bank with modified sampling kernels and its application to arbitrary image scaling

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Abstract: A new M-channel DFT filter bank with its analysis and synthesis parts being modified is proposed for arbitrary image scaling, whereby down/up sampling kernels are newly introduced along with a compactly supported sampling function of degree 2. In particular, while only an even number of channels is allowed in the conventional MDFT FB, arbitrary (even/odd) number of channels can be applied in the proposed approach. Furthermore, it is shown that images of arbitrary size can be obtained from an original image without changing the number of channels. Finally, simulation results demonstrate that high-quality images of arbitrary size can be achieved by the proposed approach.

Keywords: arbitrary scalability, modified DFT (MDFT) filter bank (FB), closed-form sampling kernel, compactly supported sampling function (CSSF)

Classification: Science and engineering for electronics

References


1 Introduction

Digital filter banks have been widely utilized in practical applications including a subband coding for image/audio processing and a TDM-FDM converter in digital communications (e.g., M-band filter banks). With recent increase of their multimedia applications, a wide range of scalabilities in image/video coders have become an important issue [1]. For that purpose, several filter bank (FB) approaches to image scaling by an integer/fractional factor have been studied during the last decade (e.g., a subband coding with dyadic scalability) [1, 2]. However, when a different fractional scaling factor is given, the number of channels in the analysis part of the FB structure should be changed in the conventional methods. Thus, both analysis and synthesis filters, satisfying perfect reconstruction conditions, should be redesigned. To solve those problems, an arbitrary image scaling method, utilizing a DFT FB with closed-form sampling kernels, was recently reported [3]. While the DFT FB has some advantages such as high-speed implementation, it is not suitable for subband coding, since it is not easy to eliminate aliasing effects due to subsampling [4]. Also, another DFT filter bank designed by employing an optimized adaptive interpolation (OAI) [3, 5] was proposed to compensate for such quality degradation in the image reconstruction. However, the OAI technique requires high computational burden [5, 6].

In this paper, a new modified DFT (MDFT) FB is proposed for arbitrary image scaling, whereby analysis and synthesis parts of the conventional MDFT [4] are revised in an efficient manner. More specifically, down/up sampling kernels are newly introduced along with a compactly supported sampling function (CSSF) of degree 2. In particular, while only an even number of channels is allowed in the conventional MDFT FB, the proposed approach allows us to design an MDFT FB with arbitrary (even/odd) number of channels. Finally, it is demonstrated that a high-quality image of arbitrary size can be obtained by the proposed MDFT FB.
2 Closed-form sampling kernels using a CSSF of degree 2

The down/up sampling formulae for a single-step conversion of a sampling rate by an arbitrary factor \( a \) \((a > 1)\) were derived in [7]:

\[
x_{1a}[n] = \frac{1}{2a\pi} \sum_{k=-\infty}^{\infty} x[k] \int_{-\pi}^{\pi} e^{j(n-k/\alpha)w} dw = \frac{1}{a} \sum_{k=-\infty}^{\infty} x[k] \cdot \text{sinc} \left( \frac{n-k}{\alpha} \right) \quad (1)
\]

\[
x_{1a}[n] = \frac{a}{2\pi} \sum_{k=-\infty}^{\infty} x[k] \int_{-\pi/a}^{\pi/a} e^{j(n/\alpha-k)w} dw = \sum_{k=-\infty}^{\infty} x[k] \cdot \text{sinc} \left( \frac{n}{\alpha} - k \right) \quad (2)
\]

In (1)-(2), the scaling factor \( a \) \((a > 1)\) can be any real number, and the upper/lower limits of the integral in (1) are chosen as \([-\pi \sim \pi]\) to avoid aliasing arising in the down-sampling process. Also, the upper/lower limits of the integral in (2) are set to \([-\pi/a \sim \pi/a]\) to eliminate unwanted images due to up-sampling. By utilizing (1)-(2) further for the M-band low-pass prototype pre/post filters as in [4], the following down/up samplers with a factor \( L \) \((L < M; \text{i.e., non-critically sampled case})\) can be newly derived as

\[
x^{(M)}_{1L}[n] = \frac{1}{2L\pi} \sum_{k=-\infty}^{\infty} x[k] \int_{-L\pi/M}^{L\pi/M} e^{j(n-k/\alpha)Lw} dw = \frac{1}{M} \sum_{k=-\infty}^{\infty} x[k] \cdot \text{sinc} \left( \frac{L}{M} \left( n - \frac{k}{L} \right) \right) \quad (3)
\]

\[
x^{(M)}_{1L}[n] = \frac{L}{2\pi} \sum_{k=-\infty}^{\infty} x[k] \int_{-\pi/M}^{\pi/M} e^{j(n-L-k)w} dw = \frac{L}{M} \sum_{k=-\infty}^{\infty} x[k] \cdot \text{sinc} \left( \frac{1}{M} (n-L \cdot k) \right) \quad (4)
\]

For the down/up sampling by \( L \) \((L > 1)\) in the M-band FB, the scaling factor \( a \) in (1)-(2) can be replaced by \( L \). Also, the upper/lower limits in the integral of (1) and (2) are changed to \([-L\pi/M \sim L\pi/M]\) and \([-\pi/M \sim \pi/M]\) respectively as in (3) and (4). Furthermore, a CSSF of degree 2, \( \psi^3_{[s],0}(t) \), with a finite support of \([-2 \sim 2]\) [5], is employed, instead of the \text{sinc} function [7], as a sampling kernel, since the \text{sinc} function has some problems in practical implementation (e.g., truncation effects) [3, 8]. Accordingly, for the design of M-band low-pass prototype pre/post filters, the following down/up samplers with a factor \( L \) \((L < M)\) can be utilized:

\[
x^{(M)}_{1L,\text{CSSF}}[n] = \frac{1}{M} \sum_{k} x[k] \cdot \psi^3_{[s],0} \left( \frac{L}{M} \left( n - \frac{k}{L} \right) \right), \quad L \cdot n - 2 \cdot M < k < L \cdot n + 2 \cdot M, \ k \neq L \left( n \pm \frac{M}{L} \right) \quad (5)
\]

\[
x^{(M)}_{1L,\text{CSSF}}[n] = \frac{L}{M} \sum_{k} x[k] \cdot \psi^3_{[s],0} \left( \frac{1}{M} (n - L \cdot k) \right), \quad \frac{1}{L} (n - 2 \cdot M) < k < \frac{1}{L} (n + 2 \cdot M), \ k \neq \frac{1}{L} (n \pm M) \quad (6)
\]
The proposed M-channel modified DFT Filter bank

The conventional MDFT FB [4] is designed by modifying the DFT FB structure to cancel all odd-alias components causing quality degradation in the image reconstruction by the DFT FB. Also, it is known that the MDFT FB requires lower computational burden than the DFT FB [9]. In this paper, we propose a new MDFT FB for arbitrary image scaling by modifying further the conventional MDFT FB in a more efficient manner. In particular, the proposed MDFT FB does not employ the OAI technique which was employed to compensate for the image quality degradation [3] for further reduction of computational complexity. More specifically, the proposed MDFT FB is designed by modifying the analysis and synthesis parts of a conventional MDFT FB [4] (i.e., see (5)-(6)) as presented in Fig. 1. In the analysis part of the proposed FB, (i) each input signal is modulated by $W^{-k_n}_M$ , (ii) the modulated signals are directly down-sampled by a scale factor $M/2$ (i.e., $2/M$ of the original size), (iii) the down-sampled signals are further down-sampled by a factor 2 without and with a delay of one sampling period, and, finally, (iv) real and imaginary parts of the subband signals are separated respectively. In the modified synthesis part, the closed-form down-sampling formula as in Table I (i.e., circular block) is applied to subband signals, followed by a modulation with $W^{k_n}_{2/\beta}$ when $0 < \beta < 1$. On the other hand, when $1 \leq \beta$, the closed-form up-sampling formula as in Table I is applied to subband signals, followed by a modulation with $W^{k_n}_{2/\beta}$. Note that the closed-form down/up sampling formulae for the input-output relationship of each triangular (and circular) block are presented in Table I, whereby each triangular block in the analysis part of Fig. 1 corresponds to a direct down-sampling by a factor $M/2$ (or equivalent to (5) when $L = M/2$), and each circular block in the synthesis part of Fig. 1 corresponds to a direct down or up sampling by a factor $\beta$ (i.e., equivalent to (5) when $0 < \beta < 1$, $L = 1/\beta$ and $M = 2/\beta$, or equivalent to (6) when $1 \leq \beta$, $L = \beta$ and $M = 2\beta$). Moreover, when a desired scaling factor $\alpha$ and the number of channel (i.e., $M$ $(M \geq 2)$) are given, the

![Fig. 1. The proposed MDFT FB with its modified analysis and synthesis parts](image-url)
down/up sampling factor $\beta$ of the circular block can be determined by

$$\beta = \frac{M}{2} \times \alpha \quad (7)$$

When $\alpha = 1$, the proposed MDFT filter bank approach yields a near perfect reconstruction (also, see Fig. 2(b)). That is, it enables us to obtain images of arbitrary size directly from the original image, without post-resampling process and without changing the number of channels in the analysis part. Note that the number of channels (i.e., $M$ ($M \geq 2$)) can be even or odd in the proposed MDFT filter bank, while only an even number of channels is allowed in the conventional MDFT FB.

Table I. Closed-form down/up sampling formulae for the input-output relationship of each block

<table>
<thead>
<tr>
<th>Parts</th>
<th>Blocks</th>
<th>Closed-form formulae for the input-output relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis part</td>
<td>$\frac{M}{2}$</td>
<td>$y(n) = \frac{1}{M} \sum_{k} x(k) \cdot \psi_{\frac{M}{2},0} \left( \frac{1}{2} (n - 2k) \right)$</td>
</tr>
<tr>
<td>Synthesis part</td>
<td>$\beta$</td>
<td>$0 &lt; \beta &lt; 1$ \quad $y(n) = \frac{\beta}{2} \sum_{k} x(k) \cdot \psi_{\frac{M}{2},0} \left( \frac{1}{2} (n - \beta \cdot k) \right)$ \quad $1 \leq \beta$ \quad $y(n) = \frac{1}{2} \sum_{k} x(k) \cdot \psi_{\frac{M}{2},0} \left( \frac{1}{2\beta} (n - \beta \cdot k) \right)$</td>
</tr>
</tbody>
</table>

Fig. 2. (a) The original ‘Barbara’ image, the images reconstructed (b) by the 3-ch proposed MDFT FB when $\alpha = 1$ and (c) by the 3-ch DFT FB with optimized adaptive interpolation kernels [3] when $\alpha = 1$, and by the proposed 3-ch MDFT FB when (d) $\alpha = 3/8$, (e) $\alpha = 7/9$, and (f) $\alpha = 9/8$
4 Simulation results

To validate the performance of the proposed MDFT FB approach, 8-bit gray scale ‘Lena’ and ‘Barbara’ images of 256 × 256 size are utilized as test images, and the proposed 3-ch MDFT filter bank is applied in horizontal and vertical directions, respectively.

More specifically, Fig. 2 (a) is the original ‘Barbara’ image, and Fig. 2 (b) and Fig. 2 (c) correspond to ‘Barbara’ images, reconstructed from the original image by the proposed 3-ch MDFT FB and by the conventional 3-ch DFT FB with optimized adaptive interpolation (OAI) kernels [3] (here, $\alpha = 1$). We can see from Fig. 2 (b) that the proposed approach yields a near perfect reconstruction when $\alpha = 1$.

Also, quality comparisons (i.e., PSNR values) are presented in Table II, for the images reconstructed by the conventional 2-ch/3-ch DFT FBs without or with OAI kernels [3] and by the proposed 2-ch/3-ch MDFT FBs. Note that the proposed approach yields better image reconstruction performance (i.e., higher PSNR in both 2-ch/3-ch cases) than the conventional DFT FB [3], even though the optimization (i.e., OAI) technique [3] is not utilized in the proposed approach. This is due to the fact that the MDFT filter bank has the “structure-inherent” alias cancellation property. Furthermore, ‘Barbara’ images, scaled by various scaling factors (i.e., 3/8, 7/9, and 9/8) and obtained by applying the proposed 3-ch MDFT FB, are shown in Fig. 2 (d)-(f), respectively. The simulation results verify that high-quality images of arbitrary size can be obtained directly from an original image by the proposed M-channel MDFT FB approach.

Table II. PSNR values of ‘Lena’ and ‘Barbara’ images reconstructed by the conventional 2-ch and 3-ch DFT FBs without/with optimized adaptive interpolation (OAI) kernels [3] and by the proposed 2-ch/3-ch MDFT FB when $\alpha = 1$

<table>
<thead>
<tr>
<th>Image</th>
<th>Lena (256 × 256)</th>
<th>Barbara (256 × 256)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DFT [3]</td>
<td>MDFT (proposed)</td>
</tr>
<tr>
<td></td>
<td>without OAI</td>
<td>with OAI</td>
</tr>
<tr>
<td>2-ch</td>
<td>31.5477</td>
<td>31.7703</td>
</tr>
<tr>
<td></td>
<td>30.8976</td>
<td>31.0086</td>
</tr>
<tr>
<td>3-ch</td>
<td>30.9989</td>
<td>31.0194</td>
</tr>
<tr>
<td></td>
<td>30.5697</td>
<td>30.6573</td>
</tr>
</tbody>
</table>

5 Conclusion

In this paper, an M-channel MDFT FB is proposed for arbitrary scaling of images, whereby new closed-form sampling kernels are introduced by employing a CSSSF of degree 2 and utilized in the analysis and synthesis parts of the proposed FB. In particular, arbitrary (even/odd) number of channels can be implemented in the proposed MDFT. Finally, simulation results demonstrate
that high-quality images of arbitrary size can be obtained from an original image, without a post-resampling process and without changing the number of channels in the analysis part.

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