Short envelope pulse propagation in composite right- and left-handed transmission lines with regularly spaced Schottky varactors

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Abstract: Composite right- and left-handed (CRLH) transmission lines with regularly spaced Schottky varactors are characterized for propagation of short envelope pulses. Due to strong dispersion, a short pulse cannot travel on a conventional CRLH line without distortion. This distortion is compensated for by the effects of Schottky varactors such that the line supports solitonic pulses, whose pulse width depends on the carrier frequency. It then becomes possible to find carrier frequencies at which the pulse width becomes minimal. This article describes the design criteria for propagating short envelope pulses in the lines.

Keywords: nonlinear Schrödinger solitons, CRLH lines, left-handed waves

Classification: Microwave and millimeter wave devices, circuits, and systems

References

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1 Introduction

It is well-known that a composite right- and left-handed (CRLH) transmission line defined as a lumped transmission line containing a series inductor, series capacitor, shunt inductor, and shunt capacitor in each section, exhibits left-handed properties [1]. Because of dispersion, a broadband pulse cannot travel on the line without distortion. Here, we consider a modified CRLH line, whose shunt capacitor is replaced by a Schottky varactor, hereafter called a Schottky CRLH line. The nonlinearity introduced by the varactors can compensate for the dispersion of CRLH lines, resulting in an envelope soliton governed by the nonlinear Schrödinger equation. Lately, there have been discussions on the solitonic pulses in Schottky CRLH lines using second-order long-wavelength approximation [2]. However, the discrepancy between the approximated and exact dispersions is so large at short wavelengths that the existing formulation cannot describe the properties of short solitonic pulses. This article discusses the method of short-pulse propagation on Schottky CRLH lines, by improving the approximation accuracy together with the design criteria obtained analytically. Several numerical evaluation results validate this method.

2 Short pulse in Schottky CRLH lines

Figure 1 (a) shows a representation of a unit cell of a Schottky CRLH line. L_R , C_L , and L_L represent the series inductor, series capacitor, and shunt inductor, respectively. C_R represents the shunt Schottky varactor. Ψ_n and I_n indicate the line voltage and current at the n th cell. V_0 shows the terminal voltage of the Schottky varactor. The capacitance-voltage relationship of a Schottky varactor is generally given by

$$C(V) = \frac{C_0}{\left(1 - \frac{V}{V_J}\right)^m}, \quad (1)$$

where C_0 , V_J , and m are the optimizing parameters. Note that $V < 0$ for reverse bias. Using these variables, the transmission equations of a Schottky CRLH line are given by

$$\left(L_R \frac{d^2}{dt^2} + \frac{1}{C_L}\right) I_n = \frac{d}{dt} (\Psi_{n-1} - \Psi_n), \quad (2)$$

$$\begin{aligned} \left(C_R(\Psi_n - V_0) \frac{d^2}{dt^2} + \frac{1}{L_L}\right) V_n &= \frac{d}{dt} (I_n - I_{n+1}) \\ &- \frac{dC_R(V)}{dV} \Big|_{V=\Psi_n-V_0} \left(\frac{d\Psi_n}{dt}\right)^2. \end{aligned} \quad (3)$$

When the pulse spreads over many cells, the discrete spatial coordinate n can be replaced by a continuous one x . For a broadband pulse, we series-expand $\Psi_{n\pm 1}$ around Ψ_n up to the sixth order of the cell length. Applying this long-wavelength approximation to eqs. (2) and (3), we obtain the evolution equation of the line voltage:

$$c_{RC} l_R l_L \frac{\partial^4 \Psi}{\partial t^4} + 4c_L l_R l_L \frac{dc_R}{dV} \frac{\partial \Psi}{\partial t} \frac{\partial^3 \Psi}{\partial t^3} + l_L \frac{dc_R}{dV} \left(\frac{\partial \Psi}{\partial t}\right)^2$$

Fig. 1. Line properties: (a) equivalent representation of Schottky CRLH lines, and (b) dispersion of a CRLH line.

$$\begin{aligned}
 & + 3c_L l_R l_L \frac{dc_R}{dV} \left(\frac{\partial^2 \Psi}{\partial t^2} \right)^2 + 6c_L l_R l_L \frac{d^2 c_R}{dV^2} \left(\frac{\partial \Psi}{\partial t} \right)^2 \frac{\partial^2 \Psi}{\partial t^2} \\
 & + c_L l_R l_L \frac{d^3 c_R}{dV^3} \left(\frac{\partial \Psi}{\partial t} \right)^4 + (c_L l_R + c_R l_L) \frac{\partial^2 \Psi}{\partial t^2} + \Psi \\
 & - c_L l_L \frac{\partial^4}{\partial t^2 \partial x^2} \left(\Psi + \frac{1}{12} \frac{\partial^2 \Psi}{\partial x^2} + \frac{1}{360} \frac{\partial^4 \Psi}{\partial x^4} \right) = 0, \quad (4)
 \end{aligned}$$

where $l_{L,R}$ and $c_{L,R}$ are the line inductance and capacitance per unit length, respectively. The function $\Psi = \Psi(x, t)$ is the continuous counterpart of Ψ_n . The shunt capacitance at $V = 0$, denote as $c_R^{(0)}$, determines the linear dispersion of the line. For brevity, we define ω_{se} and ω_{sh} as $1/\sqrt{C_L L_R}$ and $1/\sqrt{C_R^{(0)} L_L}$, respectively. Then, the line exhibits a left-handed (LH) property at frequencies lower than $\omega_l \equiv \min(\omega_{se}, \omega_{sh})$ and an ordinary right-handed (RH) property at frequencies higher than $\omega_u \equiv \max(\omega_{se}, \omega_{sh})$. The LH branch is continuously connected with the RH one at $\omega = \omega_u = \omega_l$ in the balanced line. In contrast, a stop band, where all supporting modes become evanescent, appears between ω_l and ω_u in the unbalanced line. Therefore, we call the Schottky CRLH lines *balanced* when $C_R^{(0)} L_L = C_L L_R$ is satisfied, otherwise we call them *unbalanced*. The dispersion relationship $\omega = \omega_{R,L}(k)$ of the modes propagating on a linear CRLH line at the present approximation

(ω_R is for the RH and ω_L for LH) is expressed as

$$\omega_R(k) = \sqrt{\omega_x^2(k) + \sqrt{\omega_x^4(k) - \frac{1}{c_L c_R^{(0)} l_L l_R}}}, \quad (5)$$

$$\omega_L(k) = \sqrt{\omega_x^2(k) - \sqrt{\omega_x^4(k) - \frac{1}{c_L c_R^{(0)} l_L l_R}}}, \quad (6)$$

where $\omega_x(k)$ is defined as

$$\omega_x(k) = \sqrt{\frac{k^6}{720 c_R^{(0)} l_R} - \frac{k^4}{24 c_R^{(0)} l_R} + \frac{k^2}{2 c_R^{(0)} l_R} + \frac{1}{2 c_R l_L} + \frac{1}{2 c_L l_R}}. \quad (7)$$

Figure 1 (b) shows the dispersion of a CRLH line. To obtain this, we set L_L , L_R , $C_R^{(0)}$ and C_L to $100 \mu\text{H}$, $100 \mu\text{H}$, 47 pF and 47 pF , respectively. Thus, the line is balanced. The black curve shows the exact dispersion, while the red curve shows the dispersion obtained by the present approximation. For reference, we indicate the dispersions obtained by the second- and fourth-order accuracy with green and blue curves, respectively. The latter two dispersions do not exactly match the one for short wavelengths, so they are inadequate for describing short pulses traveling on the line. Thus, we have to employ the sixth-order approximation. To quantify how the nonlinearity compensates for the dispersion, we extract the soliton equation from eq. (4) by reductive perturbation [3]. First, we prepare the spatial and temporal coordinates for the envelope and carrier waves. We use x and t as the spatial and temporal coordinates, respectively, to describe the carrier wave. For the envelope wave, $\xi \equiv \epsilon(x - V_g(k)t)$ and $\tau \equiv \epsilon^2 t$ are the spatial and temporal coordinates, respectively, where $V_g(k)$ represents the group velocity of the line explicitly given by

$$V_g(k) = \frac{k(k^4 - 20k^2 + 120)}{240 C_R L_R} \frac{\omega(k)}{\omega_x^2(k) - \omega^2(k)}, \quad (8)$$

where $\omega(k) = \omega_L(k)$ for the LH branch and $\omega(k) = \omega_R(k)$ for the RH branch. Next, we expand the voltage variable as

$$\Psi(x, t) = \sum_{m=1}^{\infty} \sum_{l=-\infty}^{\infty} \epsilon^m \psi_l^{(m)}(\xi, \tau) e^{i(\omega(k)t - kx)}, \quad (9)$$

for $\epsilon \ll 1$. Substituting this into eq. (4), the ϵ^m terms are collected to make them identically zero. By solving the resulting identities, we obtain the explicit solutions of $\psi_l^{(m)}$. Till the terms up to $O(\epsilon^3)$, it is found that $\psi_{-1}^{(1)}$ satisfies the following nonlinear Schrödinger (NS) equation,

$$i \frac{\partial \psi_{-1}^{(1)}}{\partial \tau} + P(k) \frac{\partial^2 \psi_{-1}^{(1)}}{\partial \xi^2} + Q(k) |\psi_{-1}^{(1)}|^2 \psi_{-1}^{(1)} = 0, \quad (10)$$

where the coefficients $P(k)$ and $Q(k)$ are given by

$$P(k) = \frac{(k^4 - 12k^2 + 24)\omega(k)}{96 C_R L_R (\omega_x^2(k) - \omega^2(k))} + \frac{V_g(k)^2 (5\omega_x^2(k) - 7\omega^2(k))}{2\omega(k) (\omega_x^2(k) - \omega^2(k))}, \quad (11)$$

Fig. 2. Temporal widths of the solitonic pulses in Schottky CRLH lines.

$$Q(k) = \frac{\omega(k)(-1 + C_L L_R \omega^2(k))}{8C_L L_R (\omega_x^2(k) - \omega^2(k))} \quad (12)$$

$$\times \frac{M}{(V_0 + V_J)^2} \left(1 + \frac{1}{1 - \frac{1}{\frac{1}{4C_R L_L \omega^2(k)} + \frac{4C_L k^2 (45 - 15k^2 + 2k^4)}{45C_R (-1 + 4C_L L_R \omega^2(k))}}} M \right). \quad (13)$$

The light one-soliton solution of eq. (10) is given by

$$\Psi(x, t) = A \operatorname{sech} \left(A \sqrt{\frac{Q(k)}{8P(k)}} (x - V_g(k)t) \right) e^{i(\omega(k)t - kx)}. \quad (14)$$

Note that $Q(k)/P(k) > 0$ for the development of light solitons. The estimated pulse width is shown in Fig. 2. As a model of Schottky varactors, we set $C_0 = 64.77$ pF, $V_J = 3.561$ V and $m = 1.259$. C_L is set to 47 pF. L_L and L_R are both set to 100 μ H. Moreover, V_0 is set to +1 V, and the pulse amplitude A is set to 1 V. The frequencies f_0 , f_1 , f_2 , and f_3 are 2.32, 1.82, 2.04, and 3.55 MHz, respectively. The line exhibits the RH and LH properties at frequencies above and below f_0 , respectively. The hatched regions in Fig. 2 bounded by $f_{1,2}$ or f_3 , represent the frequencies at which the light solitons cannot be allowed to develop. We can see that the pulse width becomes very short at frequencies either around f_1 or around f_3 . These observations are useful in designing the carrier frequency and for signal application.

3 Numerical evaluation

We numerically solve eqs. (2) and (3) using the standard finite-difference time-domain method [4] for a Schottky CRLH line. The total cell size is 2000. The parameters are the same as those used to obtain Fig. 2. The input pulse has the form of eq. (14) with an amplitude A of 0.3. The carrier frequency is 1.7 MHz. Note that the line exhibits the LH property at this frequency. We compared the pulse propagation in a nonlinear Schottky CRLH line with

Fig. 3. Numerical evaluations. The waveforms on (a) the linear and (b) nonlinear CRLH lines.

that in a linear CRLH line. For the linear line, we employ a 47 pF capacitor in place of the Schottky varactor.

Figure 3(a) shows the numerically obtained waveforms monitored for the linear line at cells $n = 1, 500, 1000$, and 1500. Due to the dispersion, the waveform gradually becomes distorted during propagation. Figure 3(b) shows the waveforms for the Schottky CRLH line. In contrast to the linear line, the pulse travels without distortions in every cell. By the calculations using the carrier frequency of 3.4 MHz, it was found that the line supported an RH solitonic pulse. Moreover, the calculations at 2.0 MHz led to the observation that both the dispersion and nonlinearity stretched out the input pulse, resulting in a greatly distorted waveform.

In conclusion, we described the propagation of short envelope pulses in Schottky CRLH lines. The Schottky varactors employed at C_R make the CRLH line a good platform for developing short envelope pulses and we expect to see them in a variety of broadband applications.